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ON *Pgpr* α-CLOSED SETS, *Pgpr* α-OPEN SETS AND *Pgpr* α-CONTINUOUS MAP WITH TOPOLOGICAL SPACES

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A set A in a topological space (X, τ) is said to be a regular generalized α-closed if $\alpha cl\,(A) \subset U$. Whenever $A \subset U$ and U is a regular α-open in X. In this paper we introduce $pgpr\ \alpha$- closed sets, pgpr α-open sets from a topological spaces.

KEYWORDS : $pgpr\ \alpha$-closed sets, $pgpr\ \alpha$-open sets and $pgpr\ \alpha$-continuous functions.

Introduction

Regular closed set have been introduced and studied by Palaniappan [6]. On pre generalized pre regular closed sets have been introduced an studied by Anitha [1]. On *pgpr* Regular and *pgpr* normal spaces have been introduced and Studied by Gnanachandra [2]. Generalized closed maps were introduced and studied by Malghan [5]. We also obtain some properties of $pgpr\ \alpha$-closed mappings.

Let us recall the following which we shall require later.

Definition 1.1. A subset A of a space (X, τ) is called generalized closed set (briefly *g*-closed) [4] if Cl $(A) \subseteq U$. Whenever $A \subseteq U$ and U is open in X.

Definition 1.2. Regular generalized closed set (briefly *rg*-closed [6] if Cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition 1.3. Generalized pre regular closed set (briefly *gpr*-closed) [3] if *p*Cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Pgpr α-CLOSED SETS AND *pgpr* α-OPEN SETS

We introduce the following definitions

Definition 2.1. Let (X, τ)be a topological space and $A \subseteq U$. Then A is pre generalized pre regular α-closed (briefly *pgpr* closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $rg\ \alpha$-open.

Definition 2.2. A function $f: (X, \tau) \longrightarrow (Y, \sigma)$ is called $pgpr\ \alpha$-open if the image of each open set in X is a $pgpr\ \alpha$-open set in Y.

Definition 2.3. If $f:(X,\tau) \longrightarrow (Y,\sigma)$then f is $pgpr\ \alpha$-continuous if for each open subset *V* of *y* the set $f^{-1}(V)$ is an $pgpr\ \alpha$-open subset of *X*.

Example: Let *X* = *Y* = *Z* = $\{a,b,c\}, = P(X), \sigma = \{Y, \emptyset, \ \{e\}, \{a,b\}\}$,

$$\eta = \{Z, \phi, \{a\}, \{b\}, \{a,b\}\}.$$

Define $f:(X,\tau) \longrightarrow (Y,\sigma)$ by $f(a) = a, f(b) = b,\ f(c) = c$ and $g:(Y,\sigma) \longrightarrow (Z,\eta)$ be the identity map. Then f and g are $pgpr\ \alpha$-closed maps, but their composition $g \cdot f:(X,\tau) \longrightarrow (Z,\eta)$ is not $pgpr\ \alpha$-closed map because $F = \{a\}$ is closed in (X,τ) but $g \cdot f(F) = g.f(\{a\}) = g(f(\{a\})) \ = g(\{a\}) = \{a\}$ which is not $pgpr\ \alpha$-closed in (Z,η).

Theorem 1. If $f:(X,\tau) \longrightarrow (Y,\sigma)\ pgpr\ \alpha$-open iff for any subset *S* of (Y,σ) and any closed set *F* of (X,τ) containing $f^{-1}(S)$ there exist a $pgpr\ \alpha$-closed set *K* of (Y,σ) containing *S* such that $f^{-1}(K) \subset F$.

Proof: Suppose *f* is $pgpr\ \alpha$-open set. Let *S* be a subset of (Y,σ) and *F* be a closed set of (X,τ) such that $f^{-1}(S) \subset F$. Then *X-F* is open in (X,τ) since *f* is $pgpr\ \alpha$- open in (Y,σ). That implies $f[(X-f)]^C$ is $pgpr\ \alpha$- closed in (Y,σ).

Take $K = [f(X-f)]^C$. Then *K* in a $pgpr\ \alpha$-close set in (Y,σ). Now $f^{-1}(S) \subset F$

$$\Rightarrow X - F \subset [f^{-1}(S)]^C = f^{-1}(S^C). \Rightarrow f(X-F) \subset f(f^{-1}(S^C)) \subset S^C.$$

$$\Rightarrow S \subset [f(X-F)]^C = K.$$

Also$f^{-1}(K) = f^{-1}[f(X-F)]^C = f^{-1}[Y - f(X-F)] = X - f^{-1}[f(X-F)] \subset F$,

conversely let *U* be a open set of (X,τ). Then *V* is closed in (X,τ) and $f^{-1}[f(U)^C] \ \subset U^C$. By hypothesis there exist a $pgpr\ \alpha$-closed set *K* of (*Y*, *σ*). Such that $[f(U)]^C \subset K$ and $f^{-1}(K) \subset U^C$.

Now $\quad [f(U)^C] \subset K \Rightarrow K^C \subset \ f(U) \quad \ldots (1)$

Also $\quad f^{-1}(K) \subset U^C \Rightarrow U \subset [f^{-1}(K)]^C = f^{-1}(K^C)$

$\quad \Rightarrow f(U) \subset f[f^{-1}(K^C)] \subset K^C \quad \ldots (2)$

From (1) and (2) $f(U) = K^C$.

Hence $f(U)$ is $pgpr\ \alpha$-open in (Y,σ) and hence f is $pgpr\ \alpha$-open.

Theorem 2. Let *X* and *Y* be topological spaces If $f: x \to y$ then the following are

1. f is $pgpr\ \alpha$-continuous.
2. For every subset *A* of *X* $(\bar{A}) \subset \overline{f(A)}$.
3. For every closed set *B* of *Y* the set $f^{-1}(B)\ pgpr\ \alpha$-closed in *X*.
4. For Each $x \in X$ and each neighbourhood *V* of $f(x)$ there is a neighbourhood *U* of *x* such that $f(U) \subset V$. For the point *x* of *X* we say that f is $pgpr\ \alpha$-continuous at the point *x*.

Proof: (1) $\Rightarrow$ (2) Assume that f is $pgpr\ \alpha$-continuous. Let *A* be a subset of *X* we show that if $x \in \bar{A}$ then $(x) \in \overline{f(A)}$. Let *V* be neitghbourhood of *f* (*x*). The $f^{-1}(V)$ is an $pgpr\ \alpha$-open set of *X* containing x it must intersect *A* in some point *y*. Then *V* intersects $f(A)$ in the point $f(y)$ so that $f(x)) \in \overline{f(A)}$.

(2) $\Rightarrow$ (3) Let *B* be $pgpr\ \alpha$-closed in *Y* and Let $A = f^{-1}(B)$. We prove that *A* is $pgpr\ \alpha$-closed in *X*. We show that $\bar{A} = A$. We have $f(A) = f[f^{-1}(B)] \subset B$.

∴ $\quad$ if $x \in \bar{A}$. $f(x) \in f(\bar{A}) \subset \overline{f(A)} \subset \bar{B} = B$. So that $x \in f^{-1}(B) = A$. Thus $\bar{A} \subset A$.

So that $\bar{A} = A$

(3) $\Longrightarrow$ (1). Let V an open set of Y.

Set $B = Y - V$. Then $f^{-1}(B)=f^{-1}(Y) - f^{-1}(V) = X - f^{-1}(V)$

Now B is a $pgpr\ \alpha$-closed set of Y. Then $f^{-1}(B)$ is $pgpr\ \alpha$-closed in X. So that $f^{-1}(V)$ is $pgpr\ \alpha$-Open in X.

(1) $\Longrightarrow$ (4). Let $x \in X$ and let V be a neighbourhood of $f(x)$ then the set

$U = f^{-1}(V)$ is neighbourhood of x such that $f(U) \subset V$.

(4) $\Longrightarrow$ (1). Let V be a open set of Y. Let x be a point of $f^{-1}(V)$ then $f(x) \in V$

$U_x \subset f^{-1}(V)$ If follows that $f^{-1}(V)$ can be union of open set $U_x$is $pgpr\ \alpha$-open.

Theorem 3: If $f: (X, \tau) \longrightarrow (Y, \sigma)$ is closed map and $g: (Y, \sigma) \longrightarrow (Z, \eta)$ is $pgpr\ \alpha$-closed then the composition. If $g \cdot f: (X, \tau) \longrightarrow (Z, \eta)$ is $pgpr\ \alpha$-closed map.

Proof: Let F be any closed set in (X, τ). Since f is closed map $f(F)$ is closed set in (Y, σ). Since g is $pgpr\ \alpha$-closed map $g[f(F)]$ is $pgpr\ \alpha$-closed set in (Z, η).

i.e. $g.f(F) = g[f(F)]$ is $pgpr\ \alpha$-closed and $\boldsymbol{g}.\boldsymbol{f}$ is $pgpr\ \alpha$-closed map.

Theorem 4: Let $(X, \tau), (Z, \eta)$ be topological spaces and (Y, σ) be Topological Spaces where every $pgpr\ \alpha$-closed subset is closed. Then the composition $g \cdot f: (X, \tau) \longrightarrow (Z, \eta)$ of the $pgpr\ \alpha$-closed maps $f: (X, \tau) \longrightarrow (Y, \sigma)$and $g: (Y, \sigma) \longrightarrow (Z, \eta)$is $pgpr\ \alpha$-closed.

Proof: Let A be a closed set of (X, τ). Since f is $pgpr\ \alpha$-closed. $f(A)$ is $pgpr\ \alpha$-closed in (Y, σ). Then by hypothesis. $f(A)$ is closed. Since g is $pgpr\ \alpha$-closed. $g(f(A))$ is $pgpr\ \alpha$-closed in (Z, η) and $g\big(f(A)\big) = g \cdot f(A)$. $\therefore g \cdot f\ pgpr\ \alpha$-closed.

Conclusion

In this paper $pgpr\ \alpha$-closed sets, $pgpr\ \alpha$-open sets and the $pgpr\ \alpha$-continuous map. Can be further characterized by using concept of Topological Spaces.

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