VISCOUS INCOMPRESSIBLE FLUID THROUGH MAGNETIC FIELD

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In the present paper we investigated the problem of steady flow of viscous incompressible fluid in a porous medium under magnetic field.

The problem is for the steady laminar flow of viscous incompressible fluid between two infinite parallel porous plates separated by a distance. The fluid flow freely and continuously.

KEYWORDS: Porous medium, Magnetic field, Incompressible fluid, Navier Stockes Equation.

Nomenclature

$\rho \Rightarrow$ Density of the fluid

$r \Rightarrow$ Radial coordinate

$P \Rightarrow$ Pressure

$v \Rightarrow$ Cross flow velocity

$x \Rightarrow$ Velocity along x axis

$y \Rightarrow$ Velocity along y axis

$\mu \Rightarrow$ Coefficient of viscosity

$u \Rightarrow$ Axial velocity

$B_0 \Rightarrow$ Magnetic field

Introduction

In the present paper we investigated "Study of viscous incompressible fluid in a porous medium under magnetic field". Many researchers have been investigated the problem related to this field.

Green and Rivlive (1) investigated the theory of second order fluids. Darcy's law (2) is the basis of Instability of flows in a porous medium.

Brikman (3) studies the modification of Darcy' law and another researchers investigated the problem in this field as Kent [4], Peter Gilbon [5], Beek [6], Schedegger [7], Kumar Sudhir, Singh, M.P. and Kumar Rajendra [8].

Many researchers investigated some problem Srivastava, A.C.and Sharma, B.R. [9]. Hundal, B.S. and Kumar Rajneesh [10]. Pozrikidis [11]. Sanyal, D.C. and Maji, N.K. [12].

MATHEMATICAL FORMULATION

Let x be the direction of main flow, y be the direction perpendicular to the flow and the width of the plates parallel to the z-direction.

The two plates are situated at a distance 2h from each other.

Continuity equation

$$\frac{\partial v}{\partial y} = 0 \qquad \ldots (1)$$

The body force is not considered for the problem of the present steady flow in absence of body forces the Navier stocks equations for x and y directions are given by

$$V_0 \frac{du}{dy} = \frac{-1}{l}\frac{dP}{dx} + v\frac{d^2u}{dy^2} + \frac{\sigma B_0^2 \quad \mu}{\rho} \qquad \ldots (2)$$

From (1) to use (2)

$$0 = \frac{-1}{l}\frac{dP}{dx} + \frac{\sigma B_0^2 \mu}{\rho} \qquad \ldots (3)$$

From (2)

$$\frac{dP}{dx} = l\left[v\frac{d^2u}{dy^2} + \frac{\sigma B_0^2 \mu}{\rho} - Vo\frac{du}{dy}\right] \qquad \ldots (4)$$

Differenting w.r.t. x of equation (4)

$$\frac{d^2P}{dx^2} = 0 \qquad \ldots (5)$$

Integrate it

$$\frac{dP}{dx} = A$$

From (4)

$$\frac{A}{l} = v\frac{d^2u}{dy^2} + \frac{\sigma B_0^2 \mu}{\rho v} - Vi\frac{du}{dy}$$

$$\frac{d^2u}{dy^2} - \frac{\mathrm{Vo}}{v}\frac{du}{dy} + \frac{\sigma B_0^2 \mu}{\rho v} = \frac{A}{lv} \qquad \ldots (6)$$

Integrate it (6)

$$\frac{du}{dy} - \frac{V_0}{v}u + \frac{\sigma B_0^2 \mu}{\rho v}y = \frac{A}{lv}y + B \qquad \ldots (7)$$

The boundary condition

$$y = 0, \quad \frac{du}{dy} = 0 \qquad \dots (8)$$

$$y = 0, \quad u = 0$$

$$0 = B$$

So equation (7) reduces to

$$\frac{du}{dy} - \frac{V_0}{v}u + \frac{\sigma B_0^2 \mu}{\rho v} y = \frac{A}{lv} y$$

$$\frac{du}{dy} - \frac{V_0}{v}u = \frac{A}{lv} y - \frac{\sigma B_0^2 \mu}{\rho v} y \qquad \dots (9)$$

Equation (9) to solve it

$$I.F. = e^{\int -\frac{V_0}{v} dy}$$

$$= e^{-\frac{V_0 y}{v}} \qquad \dots (10)$$

Solution is

$$u.e^{-\frac{V_0 y}{v}} = \int \left(\frac{A}{lv} y - \frac{\sigma B_0^2 \mu}{\rho v} y \right) e^{-\frac{V_0 y}{v}} dy + C$$

$$u = \left(\frac{A}{lv} - \frac{\sigma B_0^2 \mu}{\rho v} \right) \int y e^{-\frac{V_0 y}{v}} dy + C$$

$$u = \left(\frac{A}{lv} - \frac{\sigma B o^2 \mu}{\rho v} \right) \left(-\frac{v}{V_0} \right) \left[y + \frac{v}{V_0} \right] + Ce^{\frac{V_0 y}{v}}$$

$$u = \frac{Dv}{V_0} y + \frac{Dv^2}{V_0^2} + Ce^{\frac{V_0 y}{v}}$$

where

$$D = -\left(\frac{A}{lv} - \frac{\sigma B_0^2 \mu}{\rho v} \right)$$

$$u = E + \frac{Dv}{V_0} y + Ce^{\frac{V_0 y}{v}} \qquad \dots (11)$$

where

$$E = \frac{Dv^2}{V_0^2}$$

The boundary condition

$$u = 0, \text{ at } y = -h$$

$$u = u, \text{ at } y = h$$

$$0 = E - \frac{Dv}{V_0} h + Ce^{\frac{-V_0 h}{v}} \qquad \dots (12)$$

$$u = E + \frac{Dv}{V_0} h + Ce^{\frac{V_0 h}{v}} \qquad \dots (13)$$

Solve (12) and (13) using

$$Re = \frac{V_0 h}{v}$$

$$u = \frac{2Dvh}{V_0} + C\left[e^{\frac{V_0 h}{v}} - e^{\frac{-V_0 h}{v}}\right]$$

$$u = \frac{2Dh^2}{Re} + 2C \sinh \frac{V_0 h}{v}$$

$$u = \frac{2Dh^2}{Re} + 2C \sinh Re \qquad \ldots (14)$$

$$u = 2\left(\frac{A}{lv} - \frac{\sigma B_0^2 \mu}{\rho v}\right)\frac{h^2}{Re} + 2C \sinh Re \qquad \ldots (15)$$

Case – I

If there are no magnetic field means that

$Bo = 0$, then

$$u = \frac{2Ah^2}{lvRe} + C \sinh Re \qquad \ldots (16)$$

Case – II

If we take the distance of two plate are 0 then $h = 0$

$$u = 2C \sinh Re$$

$$u = 2C \sinh u \frac{V_0\, h}{v}$$

$$u = 0 \qquad \ldots (17)$$

So there are no velocity means that there are no any motion.

Case – III

If both plates are taken at rest $v = 0$ and $Vo \to \infty$

$$u = 2\left(\frac{A}{l} - \frac{\sigma B_0^2 \mu}{\rho}\right)\frac{h}{\infty} + 2C \sinh Re$$

$$u = 0 + 2C \sinh Re$$

$$u = 2C \sinh Re \qquad \ldots (18)$$

Result and discussion

In the present paper we investigated viscous incompressible fluid in a porous medium under magnetic field. We find many condition for it by equations (15), (16), (17), (18).

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