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A PERSPECTIVE ON MINIMAL & MAXIMAL *b*-OPEN AND *b*-CLOSED SETS IN TOPOLOGICAL SPACES

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In this paper contains the discourse on a new type of open and closed sets namely minimal and maximal *b*-open sets as well as minimal and maximal *b*-closed sets. The basic properties of the concerned sets are here elaborated and studied which open the new horizon for obtaining results in regular & normal spaces in topology.

Keywords : Minimal *b*-open set, Maximal *b*-closed set, Minimal *b*-closed set and Maximal *b*-open set.

Introduction

In 1986, D. Andrijivic introduced and investigated semi-pre-open sets [1] and in 1996, he conceptulised *b*-open sets [2] which are some of the weak forms of open sets. In recent years a number of generalizations of open sets have been considered in the literature. Three of these notions were defined similarly using the closure operator (cl) and the interior operator (int) in the following manner and which are useful in the sequel :

Definition (1.1) : A subset A of space (X, T) is called

(a) a pre-open [4,7] set if $A \subset \text{int}(\text{cl}(A))$ and pre-closed set if $\text{cl}(\text{int}(A)) \subset A$.

(b) a semi-open [5] set if $A \subset \text{cl}(\text{int}(A))$ and semi-closed set if $\text{int}(\text{cl}(A)) \subset A$

(c) an α-open set [6] if $A \subset \text{int}(\text{cl}(\text{int}(A)))$ and α-open closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subset A$

(d) a β-open set [8] if $A \subset \text{cl}(\text{int}(\text{cl}(A)))$ and β-closed set [1] if $\text{int}(\text{cl}(\text{int}(A))) \subset A$.

The semi-pre-open set, called by D. Andrijeric [1], was introduced under the name *B*-open set by M.E. Abd. El-Monsef, S.N. El-Deeb and R.A. Mahmond [8] as :

Definition (1.2) : A subset A of a space (X, T) is called a semi-pre-open set [1] or β-open set [8] if $A \subset \text{cl}(\text{int}(\text{cl}(A)))$ and a semi-pre-closed set or β-closed set if $\text{in}(\text{cl}(\text{int}(A))) \subset A$.

Now, a new class of generalized open sets given by D. Andrijenic under the name *b*-open sets is as :

Definition (1.3) [2, 3, 12] : A subset A of a space (X, T) is called a *b*-open set [2] if $A \subset \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$ and a *b*-closed set [3] if $\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subset A$.

All the above given definitions are different and independent. The classes of pre-open, semi-open, α-open, semi-pre-open and b-open sets of a space (X, T) are usually denoted by $PO\ (X, T)$, $SO\ (X, T)$, T^{α} , $SPO\ (X, T)$ and $BO\ (X, T)$ respectively. All of them are larger than T and closed under forming arbitrary unions.

O. Njastad [6] showed that T^{α} is a topology on X. In general, anyone of the other classes need not be a topology on X. However the intersection of a semi-open set with an open set is semi-open. The same holds for $PO\ (X, T)$ $SPO\ (X, T)$ and $BO\ (X, T)$.

In 1996, D. Andrijevic made the fundamental observation :

Proposition (1.4) : [[12] (Prop. 1.1)]

For every space (X, T), $PO\ (X, T)\ U\ SO\ (X, T) \subseteq BO\ (X, T) \subseteq SPO\ (X, T)$ holds but none of these implications can be reversed.

In 2001 and 2003, F. Nakaoka and N. Oda [9-11] introduced and analysed minimal open/closed and maximal open/closed sets which are subclasses of open/closed sets. Naturally the compliments of minimal open sets and maximal open sets are called maximal closed and and minimal closed sets respectively. We, here, project minimal b-closed sets, maximal b-open sets, minimal b-open sets and maximal b-closed sets and also analyze their basic properties.

The following definitions are the tools & motivation for our purpose :

Definition (1.4) : A Proper non-empty open subset U of a space (X, T) in said to be :

(a) a minimal open sets [9] if any open set contained in U is either ϕ or U.

(b) a maximal open set [10] if any open set containing U is either X or U.

Definition (1.5) : A proper non-empty closed subset F of a space (X, T) is said to be

(a) a minimal closed set [11] if any closed set contained in F is either ϕ or F.

(b) a maximal closed set [11] if any closed set containing F is either X or F.

Definition (1.6) : A topological space (X, T) is said to be

(a) $T_{\min}$ Space if every non-empty proper open subset of X is a minimal open set.

(b) $T_{\max}$ space if every non-empty proper open subset of X is a maximal open set.

Minimal *b*-open sets and maximal *b*-open sets

Definition (2.1) : A proper non-empty b-open subset G of a space (X, T) is said to be

(a) a minimal b-open set if any b-open set contained in G is either ϕ or G.

(b) a maximal b-open set if any b-open set containing G is either X or G.

Remark (2.2) :

(a) Every minimal open set is a minimal b-open set but the converse is not true.

Let $X = \{a, b, c, d\}$, $T = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, X\}$, then $\{a\}$ is both minimal open and minimal b-open set but $\{b\}$ and $\{c\}$ are minimal b-open sets but not minimal open sets.

(b) Every maximal open set is a maximal b-open set but the converse is not true.

Let $X = \{a, b, c, d\}$, $T = \{\phi, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$, the $\{a, b, c\}$ is both maximal open and maximal b-open set but $\{a, b, d\}$ and $\{b, c, d\}$ are maximal b-open sets but not maximal open sets.

Theorem (2.3) : A proper non-empty subset G of X is a minimal b-open (resp. maximal b-open) set iff G^c is a maximal b-closed (resp. minimal b-closed set).

Proof : (i) Let G be a minimal b-open set in a space (X, T) and $\phi \neq G \subset X$. If possible, let G^c not be a maximal b-closed set. Then there exists a b-closed set $E \neq G^c$ such that $G^c \subset E \neq X$. This means that $\phi \neq E^c \subset (G^c)^c = G$ and E^c is b-open which contradicts G to be minimal b-open. Hence, G^c is a maximal b-closed set.

Conversely, let G^c be a maximal b-closed set in a space (X, T) and $\phi \neq G^c \subset X$. If possible, let G not be a minimal b-open set. Then, there exists a b-open set $U \neq G$ such that $\phi \neq U \subset G$. This means that $G^c \subset U^c \neq X$ and U^c is b-closed which contradicts G^c to be maximal b-closed. Hence G is a minimal b-open set.

(ii) Let G be a maximal b-open set in a space (X, T) and $\phi \neq G \subset X$. On the contrary, let G^c not be a minimal b-closed set, then there exists a b-closed set $F \neq G^c$ such that $\phi \neq F \subset G^c$. This means that $G \subset F^c \neq X$ and F^c is b-open which contradicts G to be maximal b-open. Hence G^c is a minimal b-closed set conversaly, let G^c be a minimal b-closed set.

Conversely, let G^c be a minimal b-closed set. Suppose that G is not a maximal b-open set then there exists a b-open set E such that $E \neq G$ and $G \subset E \neq X$. This means that $\phi \neq E^c \subset G^c$ and E^c is a b-closed set which contradicts G^c to be minimal b-closed. Hence, G is a maximal b-open set.

Hence, the theorem.

Theorem (2.4) (A) : (i) If G be a minimal b-open set and W, a b-open set, then $G \cap W = \phi$ or $G \subset W$.

(ii) If G and H are minimal b-open sets, then $G \cap H = \phi$ or $G = H$.

Proof : (i) Let if G be a minimal b-open set and W, a b-open set. If $G \cap W = \phi$, then there is nothing to prove. If $G \cap W \neq \phi$, then $G \cap W \subset G$. but G is a minimal b-open set so $G \cap W = G$. This means that $G \subset W$.

(ii) Let G and H be minimal b-open sets. If $G \cap H = \phi$, nothing is left to prove. If $G \cap H \neq \phi$, then using (i), we have $G \subset H$ and $H \subset G$. Hence, $G = H$.

Hence, the theorem.

Theorem (2.4) (B) : (i) If G be a maximal b-open set and W, a b-open set, then either $G \cup W = X$ or $W \subset G$.

(ii) If G and H are maximal b-open sets, then either $G \cup H = X$ or $G = H$.

Proof : (i) Let G be a maximal b-open set and W, a b-open set. If $G \cup W = X$, nothing is left to prove. If $G \cup W \neq X$, then $G \subset G \cup W$. But G is a maximal b-open set so $G = G \cup W$ *i.e.* $W \subset G$.

(ii) Let G and H be maximal b-open sets. If $G \cup H = X$, nothing is left to prove. If $G \cup H \neq X$, then we have $G \subset H$ and $H \subset G$ by (i). Hence, $G = H$.

Hence, the theorem.

Theorem (2.5) (A) : For every $x \in U$ and U is a minimal b-open set, there exists a b-open set V_x containing x such that $U \subset V_x$.

Proof : Let U be a minimal b-open set and $x \in U$. Let V_x be a b-open set for which $x \in V_x$ *i.e.* $\{x\} \subset U \cap V_x$.

On the contrary, let $U \not\subset V_x$. Then, using theorem (2.4) (A) (i), we have $U \cup V_x = \phi$. This contradicts the fact that $\{x\} \subset U \cap V_x$.

Hence, $U \subset V_x$.

Hence, the theorem.

Theorem (2.5) (B) : For every $x \in U^c$ and U is a maximal b-open set, there exists a b-open set V_x containing x such that $U^c \subset V_x$.

Proof : Let U be a maximal b-open set and $x \in U^c$. Let V_x be a b-open set for which $x \in V_x$.

Obviously, $V_x \not\subset U$ so that $U \cup V_x = X$, by theorem (2.4) (B) (i). This provides that $V_x{}^c = U$ and consequently, $U^c \subset V_x$.

Hence, the theorem

Theorem : (2.6) (A) : If U is a non-empty b-open set in a space (X, T), then the following statements are equivalent

(a) U is a minimal b-open set;

(b) $U \subset \text{bcl}\,(S)$ for any non-empty subset S of U,

(c) $\text{bcl}\,(U) = \text{bcl}\,(S)$ for any non-empty subset S of U.

Proof : Let U be a non-empty b-open set in a space (X, T).

(a) $\Rightarrow$ (b) :

Let U be a minimal b-open set and S, a non-empty subset of U.

Let $x \in U$, then by theorem (2.5) (A), for any b-open set W containing x, $U \subset W$.

Therefore, $S \subset U \subset W \Rightarrow S \subset W$

Now, $S = S \cap U$ & $S \cap U \subset S \cap W$. This means that $S \cap W \neq \phi$ as $S \neq \phi$.

Again, since, W is any b-open set containing x, hence $x \in \text{bcl}\,(S)$ *i.e.* $x \in U \Rightarrow x \in \text{bcl}\,(S)$. Hence, for any non-empty subset S of U, $U \subset \text{bcl}\,(S)$.

(b) $\Rightarrow$ (c)

Let $S \neq \phi$ and $S \subset U$ such that $U \subset \text{bcl}\,(S)$.

Now, $U \subset \text{bcl}\,(S) \Rightarrow \text{bcl}\,(U) \subset \text{bcl}\,\{\text{bcl}\,(S)\}$

$$\Rightarrow \text{bcl}\,(U) \subset \text{bcl}\,(S) \qquad \text{... (i)}$$

And $$S \subset U \Rightarrow \text{bcl}\,(S) \subset \text{bcl}\,(U) \qquad \text{... (ii)}$$

Hence, from (i) and (ii), we have $\text{bcl}\,(U) = \text{bcl}\,(S)$ where $S \subset U$ and $S \neq \phi$.

(c) $\Rightarrow$ (a)

Let $$\phi \neq S \subset U \Rightarrow \text{bcl}\,(U) = \text{bcl}\,(S) \qquad \text{... (iii)}$$

If possible, let U be not a minimal b-open set. Then there exists a non-empty b-open set V such that $V \subset U$ and $V \neq U$. This provides that there exists an element $a \in U$ but $a \notin V$. This means that $a \in V^c$.

Now, $\{a\} \subset V^c \Rightarrow \text{bcl}(\{a\}) \subset \text{bcl}(V^c) = V^c$, as V is a b-open set.

$$\Rightarrow \text{bcl}(\{a\}) \neq \text{bcl}(U)$$

That is, we have, $\{a\} \subset U \Rightarrow \text{bcl}(\{a\}) \neq \text{bcl}(U)$, which stands as a contradiction for the condition (c). Hence, U is a minimal b-open set.

Hence, the theorem.

Theorem (2.6) (B) : If U is a non-empty b-open set in a space (X, T) then the following statement are equivalent :

(a) U is a maximal b-open set;

(b) $b\ \text{int}(S) \subset U$ for any superset S of U and $S \neq X$;

(c) $b\ \text{int}(U) = b\ \text{int}(S)$ for any super set S of U and $S \neq X$.

Proof : Let U be a non-empty b-open set in a space (X, T).

(a) $\Rightarrow$ (b) :

Let $x \in U^c$ and U be a maximal b-open set and also $U \subset S \neq X$. By theorem (2.5) (B), for any b-open set V_x containing X, $U^c \subset V_x$. Thus $S^c \subset U^c \subset V_x$.

Now, $S^c = S^c \cap U^c \subset S^c \cap V_x$. Since, $S^c \neq \phi$ hence $S^c \cap V_x \neq \phi$. Again, as V_x is any b-oepn set containing x, $x \in \text{bcl}(S^c)$.

This provides that $x \in U^c \Rightarrow x \in b\ \text{cl}(S^c)$

i.e. $U^c \subset b\ \text{cl}(S^c)$

i.e. $U^c \subset (b\ \text{int}(S))^c$

i.e. $b\ \text{int}(S) \subset U$

(b) $\Rightarrow$ (c) :

Let $U \subset S$ and $S \neq X$ such that $b\ \text{int}(S) \subset U$.

This provides that $b\ \text{int}(b\ \text{int}(S)) \subset b\ \text{int}(U)$

i.e. $$b\ \text{int}(S) \subset b\ \text{int}(U) \quad ...(i)$$

Next, $$U \subset S \Rightarrow b\ \text{int}(U) \subset b\ \text{int}(S) \quad ...(ii)$$

Hence, from (i) and (ii), it concludes that $b\ \text{int}(U) = b\ \text{int}(S)$ for any superset S of U and $S \neq X$.

(c) $\Rightarrow$ (a) :

Given that $b\ \text{int}(U) = b\ \text{int}(S)$ for any superset S of U and $S \neq X$, where U is a b-open set.

Suppose that U is not a maximal b-open set. Then there exists a non-empty b-open set V such that $U \subset V$ and $V \neq X$.

Now, there exists an element $a \in U$ such that $a \notin V$. This means that $a \in V^c$ *i.e.* $b\ \text{int}(\{a\}) \subset b\ \text{int}(V^c)$. Since, $U \notin V^c$, hence, $b\ \text{int}(\{a\}) \neq b\ \text{int}(U)$ for any $\{a\} \subset U$. Consequently U is a maximal b-open set.

Hence, the theorem.

Theorem (2.7) (A) : In a space (X, T), if G_α, G_β, G_δ are minimal b-open sets, then

(a) $G_\alpha \neq G_\beta$ and $G_\delta \subset G_\alpha \cup G_\beta \Rightarrow$ Either $G_\alpha = G_\delta$ or $G_\beta = G_\delta$

(b) $G_\alpha \neq G_\beta \neq G_\delta \Rightarrow (G_\alpha \cup G_\delta) \not\subset (G_\alpha \text{ U } G_\beta)$

Proof : Let G_α, G_β and G_δ be minimal b-open sets in a space (X, T).

(a) Let $G_\alpha \neq G_\beta$ and $G_\delta \subset G_\alpha \cup G_\beta$

If $G_\alpha = G_\delta$, then there is nothing to prove.

If $G_\alpha \neq G_\delta$, then it is to prove that $G_\beta = G_\delta$

Now,

$G_\beta \cup G_\delta = G_\beta \cup (G_\delta \cup \phi)$

$= G_\beta \cup [G_\delta \cup (G_\alpha \cap G_\beta)]$ [By theorem (2.4) (A) (ii)]]

$= G_\beta \cup [(G_\delta \cup G_\alpha) \cap (G_\delta \cup G_\beta)]$

$= (G_\beta \cup G_\delta \cup G_\alpha) \cap (G_\beta \cup G_\delta \cup G_\beta)$

$= (G_\alpha \cup G_\beta) \cap (G_\beta \cup G_\delta)$ [$\because$ $G_\delta : G_\alpha \cup G_\beta$]

$= (G_\alpha \cap G_\delta) \cup G_\beta$

$= \phi \cup G_\beta$ [$G_\alpha \neq G_\delta$ & they are minimal b-open sets, so $G_\alpha \cap G_\delta = \phi$]

$= G_\beta$

$\Rightarrow G_\delta \subset G_\beta$

Since, G_δ and G_β are minimal b-open sets, hence, $G_\beta = G_\delta$.

(b) Let $G_\alpha \neq G_\beta \neq G_\delta$.

If possible, let $(G_\alpha \cup G_\delta) \subset (G_\alpha \cup G_\beta)$

$\Rightarrow (G_\alpha \cup G_\delta) \cap (G_\delta \cup G_\beta) \subset (G_\alpha \cup G_\beta) \cap (G_\delta \cup G_\beta)$

$\Rightarrow (G_\alpha \cap G_\beta) \cup G_\delta \subset (G_\alpha \cap G_\delta) \cup G_\beta$

$\Rightarrow \phi \cup G_\delta \subset \phi \cap G_\beta$ [By the theorem (2.4) (A) (ii)]

$\Rightarrow G_\delta \subset G_\beta$

Since, G_δ and G_β are minimal b-open sets, hence $G_\beta = G_\delta$. But it is a contradiction to the fact that $G_\beta \neq G_\delta$. Hence, $G_\alpha \cup G_\delta \not\subset G_\alpha \cup G_\beta$.

Hence, the theorem.

Theorem (2.7) (B) : In a space (X, T), if G_α, G_β & G_δ are maximal b-open sets, then

(a) $G_\alpha \neq G_\beta$ and $G_\alpha \cap G_\beta \subset G_\delta \Rightarrow$ Either $G_\alpha = G_\delta$ or $G_\beta = G_\delta$.

(b) $G_\alpha \neq G_\beta \neq G_\delta \Rightarrow (G_\alpha \cap G_\beta) \not\subset (G_\alpha \cap G_\delta)$.

Proof : Let G_α, G_β & G_δ be maximal *b*-open sets in a space (X, T).

(a) Let $G_\alpha \neq G_\beta$ and $G_\alpha \cap G_\beta \subset G_\delta$.

If $G_\alpha = G_\delta$, then there is nothing to prove.

If $G_\alpha \neq G_\delta$, then it is to prove that $G_\beta = G_\delta$.

Now,
$$G_\beta \cap G_\delta = G_\beta \cap (G_\delta \cap X)$$
$$= G_\beta \cap [G_\delta \cap (G_\alpha \cup G_\beta)] \qquad \text{[By theorem (2.4) (B) (ii)]}$$
$$= G_\beta \cap [(G_\delta \cap G_\alpha) \cup (G_\delta \cap G_\beta)]$$
$$= (G_\beta \cap G_\delta \cap G_\alpha) \cup (G_\beta \cap G_\delta \cap G_\beta)$$
$$= (G_\alpha \cap G_\beta) \cup (G_\beta \cap G_\delta) \qquad [G_\alpha \cap G_\beta \subset G_\delta]$$
$$= (G_\alpha \cup G_\beta) \cap G_\beta$$
$$= X \cap G_\beta$$
$$= G_\beta$$
$$\Rightarrow G_\beta \subset G_\delta$$

Since, G_β & G_δ are maximal *b*-open sets, hence, $G_\beta = G_\delta$.

(b) Let $G_\alpha \neq G_\beta \neq G_\delta$.

If possible, let $(G_\alpha \cap G_\beta) \subset (G_\alpha \cap G_\delta)$

$$\Rightarrow (G_\alpha \cap G_\beta) \cup (G_\delta \cap G_\beta) \subset (G_\alpha \cap G_\delta) \cup (G_\delta \cap G_\beta)$$
$$\Rightarrow (G_\alpha \cup G_\delta) \cap G_\beta \subset (G_\alpha \cup G_\beta) \cap G_\delta$$
$$\Rightarrow X \cap G_\beta \subset X \cap G_\delta \qquad \text{[By theorem (2.4) (B) (ii)]}$$
$$\Rightarrow G_\beta \subset G_\delta$$

Since, G_β and G_δ are maximal *b*-open sets, hence $G_\beta = G_\delta$.

But it is a contradiction to the fact that $G_\beta \neq G_\delta$.

Hence, $(G_\alpha \cap G_\beta) \not\subset (G_\alpha \cap G_\delta)$.

Hence, the theorem.

MINIMAL *b*-CLOSED SETS AND MAXIMAL *b*-CLOSED SETS

Definition (3.1) : A proper non-empty *b*-closed subset F of a space (X, T) is said to be.

(a) a minimal *b*-closed set if any *b*-closed set contained in F is either ϕ or F.

(b) a maximal *b*-closed set if any *b*-closed set containing F is either X or F.

Remark (3.2) :

(a) Every minimal closed set is a minimal *b*-closed but not conversely.

Let $X = \{a, b, c, d\}$, $T = \{\phi, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$, then $\{d\}$ is both minimal closed & minimal b-closed set but $\{a\}$ and $\{c\}$ are minimal b-closed but not minimal closed.

(b) Every maximal closed set is a maximal b-closed but not conversely.

Let $X = \{a, b, c, d\}$, $T = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, X\}$, then $\{b, c, d\}$ is both maximal closed and maximal b-closed set but $\{a, b, d\}$ & $\{a, c, d\}$ are maximal b-closed but not maximal closed sets.

Theorem (3.3) (A) : If V is a non-empty finite b-closed set then there exists at least one finite minimal b-closed set U such that $U \subset V$.

Proof : Suppose that V is a non-empty finite b-closed set. If V is a minimal b-closed set, it may be set $U = V$. If V is not a minimal b-closed set, then there exists a finite b-closed set V_1 such that $\phi \neq V_1 \subset V$. If V_1 is a minimal b-closed set, it may be set $U = V_1$. If V_1 is not a minimal b-closed set, then there exists a finite b-closed set V_2 such that $\phi \neq V_2 \subset V_1$.

Continuing the process, a sequences of b-closed sets $V \supset V_1 \supset V_2 \supset \ldots \supset V_k \supset \ldots.$ is obtained. Since, V is finite, this process is repeated finite number of times.

Hence, we get a minimal b-closed set $U = V_n$ for some positive integer n.

Hence, the theorem

Corollary (A1) : If (X, T) is a locally finite space and V, a non-empty b-closed set, then there exists at least one finite minimal b-closed set U such that $U \subset V$.

Proof : Let (X, T) be a locally finite space and V, a non-empty b-closed set.

Let $x \in V$. Since, (X, T) is a locally finite space, hence there exists a finite open set V_x such that $x \in V_x$.

Now, $V \cap V_x$ is a finite b-closed set and by theorem (3.3) (A) there exists atleast one finite minimal b-closed set U such that $U \subset V \cap V_x$. But $V \cap V_x \subset V$. Consequently, there exist at least one finite minimal b-closed set U such that $U \subset V$.

Corollary (A2) : If V is a finite minimal closed set, then there exists at least one finite minimal b-closed set U such that $U \subset V$.

Proof : Since, every finite closed set is a finite b-closed set, hence, V is also a finite b-closed set. Consequently, with the help of theorem (3.3) (A) there exists at least one finite minimal b-closed set U such that $U \subset V$.

Theorem (3.3) (B) : If V is a non-empty finite b-closed set, then there exists at least one finite maximal b-closed set U such that $V \subset U$.

Proof : Let V be a non-empty finite b-closed set. If V is a maximal b-closed set, it may be set $U = V$.

If V is not a maximal b-closed set, then there exists finite b-closed set V_1 such that $V \subset V_1$. If V_1 is a maximal b-closed set, then $U = V_1$. If V_1 is not a maximal b-closed set, then there exists a finite b-closed set V_2 such that $V_1 \subset V_2$.

Continuing this process, a sequence of b-closed sets $V \subset V_1 \subset V_2 \subset V_3 \subset \ldots \subset V_k \subset \ldots\ldots$ is obtained. Since, V is a finite set, this process repeats finitely. Hence, we get a maximal b-closed set $U = V_n$ for some positive integer n. *i.e.* $V \subset U$.

Corollary (B1) : If (X, T) is a locally finite space and V, a non-empty b-closed set, then there exits at least one finite maximal b-closed set U such that $V \subset U$.

Proof : Let (X, T) be a locally finite space and V, a non-empty b-closed set.

Let $x \in V$. Since, (X, T) is a locally finite space, hence, there exists a finite closed set V_x such that $x \in V_x$. Now, $V \cup V_x$ is a finite b-closed set and by theorem (3.3) (B) there exist at least one finite maximal b-closed set U such that $V \subset V \cup V_x \subset U$.

Consequently, there exists atleast one finite maximal b-closed set U in the manner that $V \subset U$.

Corollary (B2) : If V is a finite maximal closed set, then there exists atleast one finite maximal b-closed set U such that $V \subset U$.

Proof : Since, every closed set is a b-closed set, hence, V is also a b-closed set. Consequently, with the help of theorem (3.3) (B), there exists atleast one maximal b-closed set U such that $V \subset U$.

Conclusion

Basic Nature of minimal and maximal b-open/closed sets is on discourse and analyzed in this paper. Characterization of minimal b-open sets and maximal b-open sets has been done using bcl and bint operators respectively. The future scope of the study is to obtain results for minimal/maximal regular b-open/b-closed sets.

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