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ECOLOGICAL AMMENSALISM-A SERIES SOLUTION BY HOMOTOPY PERTURBATION METHOD

DR. K.V.L.N. ACHARYULU

Associate Professor, Department of Mathematics, Bapatla Engineering College, Bapatla-522101, India

AND

DR. N. PHANI KUMAR

Prof. Deptt. of Mathematics, College of Natural and Computational Science, Post Box No: 395 Nekemte, Ethiopia

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The paper intends to find a series solution in an ecological Ammensalism. The Ammensalism species is restricted to have limited resources. The model equations are constructed by a pair of non linear first order differential equations. Homotopy perturbation method is employed for evaluating series solution.

KEYWORDS : Ammensalism, Homotopy Analysis, Stability, Dominance Reversal time.

INTRODUCTION

Abbasbandy, S. [1] utilized this perturbation technique and invented some innovative results in the concept of asymptotic techniques. Later Liao [5-8] modified Homotopy Perturbation Method (HPM) in 1992. Some other methods with independent physical parameters were developed by eminent Mathematicians [2, 4]. In the recent years, the HPM methodology has been utilized in the field of Engineering and Modern Sciences [3, 9].

BASIC IDEA OF HOMOTOPY PERTURBATION METHOD

Step (1) : Let us consider nonlinear differential equation:

$$A(u) - f(r) = 0, \quad r \in \Omega \qquad \ldots \text{(I)}$$

With the the boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \qquad r \in \Gamma$$

where A is a general differential operator, B a boundary operator, $f(r)$ is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal drawn outwards from Ω.

Step (2): In general the operator A, is divided into two parts : a linear part L and a nonlinear part N. Therefore above differential equation (I) is expressed in the form of

$$L(u)-N(u)-f(r)=0 \qquad \text{… (II)}$$

Step (3) : With the help of Homotopy Perturbation Method (HPM), one can constitute a homotopy $v(r,\ p):\Omega\times[0,\ 1]\to R$ which satisfies

$$H(v,\ p)=(1-p)\left[L(v)-L(u_0)\right]+p\left[A(v)-f(r)\right]=0,\ p\in[0,\ 1],\ r\in\Omega \qquad \text{… (III)}$$

It is nothing but

$$H(v,\ p)=L(v)-L(u_0)+pL(u_0)+p\left[A(v)-f(r)\right]=0 \qquad \text{… (IV)}$$

where $p\in[0,\ 1]$ is named as an embedding parameter, and u_0 is an initial approximation of equation (1), which satisfies the boundary conditions.

Step (4): Then equations (III), (IV) follow that

$$H(v,0)=L(v)-L(u_0)=0$$

and

$$H(v,\ 1)=A(v)-f(r)=0$$

Thus the changing process of P from zero to unity is just that of $v(r,\ p)$ from $u_0(r)$ to $u(r)$.

Step (5): According to the HPM, we can first use the imbedding parameter p as a 'small parameter' and assume that the solutions of the equations (III) and (IV) can be written as a power series in p :

$$v=v_0+pv_1+p^2v_2+p^3v_3+p^4v_4+----------$$

The approximate solution of equation (I) can be obtained as

$$u=\underset{p\to 1}{Lt}\ v=v_0+v_1+v_2+v_3+v_4+-----------$$

NOTATIONS ADOPTED

N_1 (t) : The population rate of the species S_1 at time t

N_2 (t) : The population rate of the species S_2 at time t

a_i : The natural growth rate of S_i, $i = 1, 2$.

a_{ii} : The rate of decrease of S_i; due to its own insufficient resources, $I = 1, 2$.

a_{12} : The inhibition coefficient of S_1 due to S_2 *i.e.* the Commensal coefficient.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, α, h_1, h_2 are assumed to be non-negative constants.

BASIC EQUATIONS

$$\frac{dN_1}{dt}=a_1N_1-a_{11}N_1^2-a_{12}N_1N_2 \qquad \text{... (1)}$$

$$\frac{dN_2}{dt}=a_2N_2-a_{22}N_2^2 \text{ with initial conditions } N_1(0)=c_1 \text{ and } N_2(0)=C_{12} \qquad \text{... (2)}$$

The following system can be constructed by the concept of homotopy as follows

$$v_1' - N_{10}' + p(N_{10}' - a_1 v_1 + a_{11} v_1^2 - a_{12} v_1 v_2) = 0 \quad \dots (3)$$

$$v_2' - N_{20}' + p(N_{20}' - a_2 v_2 + a_{22} v_2^2) = 0 \quad \dots (4)$$

The initial approximations are considered as

$$v_{1,0}(t) = N_{10}(t) = v_1(0) = c_1 \quad \dots (5)$$

$$v_{2,0}(t) = N_{20}(t) = v_2(0) = c_2 \quad \dots (6)$$

and $v_1(t) = v_{1,0}(t) + pv_{1,1}(t) + p^2 v_{1,2}(t) + p^3 v_{1,3}(t) + p^4 v_{1,4}(t) + p^5 v_{1,5}(t) + \cdots$ …(7)

$$v_2(t) = v_{2,0}(t) + pv_{2,1}(t) + p^2 v_{2,2}(t) + p^3 v_{2,3}(t) + p^4 v_{2,4}(t) + p^5 v_{2,5}(t) + \cdots \quad \dots (8)$$

where $v_{i,J}(i = 1,2, J = 1,2,3 \dots)$ are to be computed by substituting (5), (6), (7), (8) in (3), (4)

We get

$$v_{1,0}'(t) + pv_{1,1}'(t) + p^2 v_{1,2}'(t) + p^3 v_{1,3}'(t) + p^4 v_{1,4}'(t) + p^5 v_{1,5}'(t) + \cdots - N_{10}' +$$
$$p[N_{10}' - a_1(v_{1,0}(t) + pv_{1,1}(t) + p^2 v_{1,2}(t) + p^3 v_{1,3}(t) + p^4 v_{1,4}(t) + p^5 v_{1,5}(t) + \cdots)$$
$$+a_{11}\big(v_{1,0}(t) + pv_{1,1}(t) + p^2 v_{1,2}(t) + p^3 v_{1,3}(t) + p^4 v_{1,4}(t) + p^5 v_{1,5}(t) + \cdots\big)(v_{1,0}(t)$$
$$+ pv_{1,1}(t) + p^2 v_{1,2}(t) + p^3 v_{1,3}(t) + p^4 v_{1,4}(t) + p^5 v_{1,5}(t) + \cdots + a_{12}(v_{1,0}(t) + pv_{1,1}(t)$$
$$+ p^2 v_{1,2}(t) + p^3 v_{1,3}\big(t) + p^4 v_{1,4}(t) + p^5 v_{1,5}(t) + \cdots\big)(v_{2,0}(t) + pv_{2,1}(t) + p^2 v_{2,2}(t)$$
$$+p^3 v_{2,3}(t) + p^4 v_{2,4}(t) + p^5 v_{2,5}(t) + \cdots)] = 0 \quad \dots (9)$$

From equation (4)

$$v_{2,0}'(t) + pv_{2,1}'(t) + p^2 v_{2,2}'(t) + p^3 v_{2,3}'(t) + p^4 v_{2,4}'(t) + p^5 v_{2,5}'(t) + \cdots - N_{20}'$$
$$+p[N_{20}' - a_2(v_{2,0}(t) + pv_{2,1}(t) + p^2 v_{2,2}(t) + p^3 v_{2,3}(t) + p^4 v_{2,4}(t) + p^5 v_{2,5}(t) + \cdots$$
$$+a_{22}\big(v_{2,0}(t) + pv_{2,1}(t) + p^2 v_{2,2}(t) + p^3 v_{2,3}(t) + p^4 v_{2,4}(t) + p^5 v_{2,5}(t) + \cdots\big)$$
$$(v_{2,0}(t) + pv_{2,1}(t) + p^2 v_{2,2}(t) + p^3 v_{2,3}(t) + p^4 v_{2,4}(t) + p^5 v_{2,5}(t) + \cdots)] = 0 \quad \dots (10)$$

From (9),

$$0 + pv_{1,1}'(t) + p^2 v_{1,2}'(t) + p^3 v_{1,3}'(t) + p^4 v_{1,4}'(t) + p^5 v_{1,5}'(t) + \cdots - 0$$
$$+p[0 - a_1 v_{1,0}(t) - a_1 p v_{1,1}(t) - a_1 p^2 v_{1,2}(t) - a_1 p^3 v_{1,3}(t) - a_1 p^4 v_{1,4}(t) - a_1 p^5 v_{1,5}(t)$$
$$- \dots + a_{11}\, v_{1,0}^2(t) + a_{11} p v_{1,0}(t) v_{1,1}(t) + a_{11} p^2 v_{1,0}(t) v_{1,2}(t) + a_{11} p^3 v_{1,0}(t) v_{1,3}(t)$$
$$+ a_{11} p^4 v_{1,0}(t) v_{1,4}(t) + \cdots + a_{11} p v_{1,0}(t) v_{1,1}(t) + a_{11} p^2 v_{1,1}^2(t) + a_{11} p^3 v_{1,1}(t) v_{1,2}(t)$$
$$+ a_{11} p^4 v_{1,1}(t) v_{1,3}(t) + a_{11} p^5 v_{1,1}(t) v_{1,4}(t) + \cdots + a_{11} p^2 v_{1,0}(t) v_{1,2}(t)$$
$$+ a_{11} p^3 v_{1,1}(t) v_{1,2}(t) + a_{11} p^4 v_{1,2}^2(t) + a_{11} p^5 v_{1,2}(t) v_{1,3}(t) + \cdots + a_{11} p^3 v_{1,0}(t) v_{1,3}(t)$$
$$+ a_{11} p^4 v_{1,1}(t) v_{1,3}(t) + a_{11} p^5 v_{1,2}(t) v_{1,3}(t) + \cdots + a_{11} p^4 v_{1,0}(t) v_{1,4}(t)$$
$$+ a_{11} p^5 v_{1,1}(t) v_{1,4}(t) + \cdots + a_{11} p^5 v_{1,0}(t) v_{1,5}(t) + \cdots + a_{12}\, v_{1,0}(t) v_{2,0}(t)$$
$$+a_{12} p v_{1,0}(t) v_{2,1}(t) + a_{12} p^2 v_{1,0}(t) v_{2,2}(t) + a_{12} p^3 v_{1,0}(t) v_{2,3}(t)$$
$$+a_{12} p^4 v_{10}(t) v_{2,4}(t) \dots\dots + a_{12}\, p v_{1,1}(t) v_{2,0}(t) + a_{12} p^2 v_{1,1}(t) v_{2,1}(t)$$
$$+a_{12} p^3 v_{1,1}(t) v_{2,2}(t) + a_{12} p^4 v_{1,1}(t) v_{2,3}(t). \dots + a_{12} p^2 v_{2,0}(t) v_{1,2}(t) + a_{12} p^3 v_{1,2}(t) v_{2,1}(t)$$
$$+a_{12} p^4 v_{1,2}(t) v_{2,2}(t) \dots + a_{12} p^3 v_{1,3}(t) v_{2,0}(t) + a_{12} p^4 v_{1,3}(t) v_{2,1}(t). \dots$$
$$+a_{12} p^4 v_{1,4}(t) v_{2,0}(t) \; \dots.] = 0 \quad \dots (11)$$

From (10),

$$0 + pv'_{2,1}(t) + p^2v'_{2,2}(t) + p^3v'_{2,3}(t) + p^4v'_{2,4}(t) + p^5v'_{2,5}(t) + \cdots - 0 + p[0 - a_2v_{2,0}(t)$$
$$-a_2pv_{2,1}(t) - a_2p^2v_{2,2}(t) - a_2p^3v_{2,3}(t) - a_2p^4v_{2,4}(t) - a_2p^5v_{2,5}(t) - \cdots + a_{22}\, v_{2,0}^2(t)$$
$$+a_{22}pv_{2,0}(t)v_{2,1}(t) + a_{22}p^2v_{2,0}(t)v_{2,2}(t) + a_{22}p^3v_{2,0}(t)v_{2,3}(t) + a_{22}p^4v_{2,0}(t)v_{2,4}(t) + \cdots$$
$$+a_{22}\, pv_{2,1}(t)v_{2,0}(t) + a_{22}p^2v_{2,1}^2(t) + a_{22}p^3v_{2,1}(t)v_{2,2}(t) + a_{22}p^4v_{2,1}(t)v_{2,3}(t)$$
$$+a_{22}p^5v_{2,1}(t)v_{2,4}(t) + \cdots + a_{22}p^2v_{2,0}(t)v_{2,2}(t) + a_{22}p^3v_{2,2}(t)v_{2,1}(t) + a_{22}p^4v_{2,2}^2(t)$$
$$+a_{22}p^5v_{2,2}(t)v_{2,3}(t) + \cdots + a_{22}p^3v_{2,0}(t)v_{2,3}(t) + a_{22}p^4v_{2,1}(t)v_{2,3}(t) + a_{22}$$
$$p^5v_{2,2}(t)v_{2,3}(t) + \cdots + a_{22}p^4v_{2,0}(t)v_{2,4}(t) + a_{22}p^5v_{2,1}(t)v_{2,4}(t) + \cdots$$
$$+a_{22}p^5v_{2,0}(t)v_{2,5}(t) + \cdots\,] = 0 \qquad \ldots (12)$$

Now comparing the coefficient of various powers of *p* in (11) and (12),we obtain

The coefficient of P^1 :

$$v'_{1,1}(t) - a_1v_{1,0}(t) + a_{11}v_{1,0}^2(t) + a_{12}v_{1,0}(t)v_{2,0}(t) = 0$$
$$v'_{2,1}(t) - a_2v_{2,0}(t) + a_{22}v_{2,0}^2(t) = 0$$

The coefficient of P^2:

$$v'_{1,2}(t) - a_1v_{1,1}(t) + a_{11}v_{1,0}(t)v_{1,1}(t) + a_{11}v_{1,0}(t)v_{1,1}(t) + a_{12}v_{1,0}(t)v_{2,1}(t)$$
$$+a_{12}v_{1,1}(t)v_{2,0}(t) = 0$$
$$v'_{2,2}(t) - a_2v_{2,1}(t) + a_{22}v_{2,0}(t)v_{2,1}(t) + a_{22}v_{2,0}(t)v_{2,1}(t) = 0$$

The coefficient of P^3:

$$v'_{1,3}(t) - a_1v_{1,2}(t) + a_{11}v_{1,0}(t)v_{1,2}(t) + a_{11}v_{1,1}^2(t) + a_{11}v_{1,0}(t)v_{1,2}(t) +$$
$$a_{12}v_{1,0}(t)v_{2,2}(t) - a_{12}v_{1,1}(t)v_{2,1}(t) + a_{12}v_{2,0}(t)v_{1,2}(t) = 0$$
$$v'_{2,3}(t) - a_2v_{2,2}(t) + a_{22}v_{2,0}(t)v_{2,2}(t) + a_{22}v_{2,1}^2(t) + a_{22}v_{2,0}(t)v_{2,2}(t) = 0$$

The coefficient of P^4:

$$v'_{1,4}(t) - a_1v_{1,3}(t) + a_{11}v_{1,0}(t)v_{1,3}(t) + a_{11}v_{1,1}(t)v_{1,2}(t) + a_{11}v_{1,1}(t)v_{1,2}(t)$$
$$+ a_{11}v_{1,0}(t)v_{1,3}(t) + a_{12}v_{1,0}(t)v_{2,3}(t) + a_{12}v_{1,1}(t)v_{2,2}(t) + a_{12}v_{2,1}(t)v_{1,2}(t)$$
$$+ a_{12}v_{2,0}(t)v_{1,3}(t) = 0$$
$$v'_{2,4}(t) - a_2v_{2,3}(t) + a_{22}v_{2,0}(t)v_{2,3}(t) + a_{22}v_{2,1}(t)v_{2,2}(t) + a_{22}v_{2,1}(t)v_{2,2}(t)$$
$$+a_{22}v_{2,0}(t)v_{2,3}(t) = 0$$

Now $v_1(0) = c_{1,}v_2(0) = c_2$

$$v_{1,1}(t) = a_1\int_0^t v_{1,0}(t)dt - a_{11}\int_0^t v_{1,0}^2(t)dt - a_{12}\int_0^t v_{1,0}(t)v_{2,0}(t)dt$$
$$= c_1a_1t - a_{11}c_1^2t + a_{12}c_1c_2t$$
$$\therefore \quad v_{1,1}(t) = (a_1 - a_{11}c_1 - a_{12}c_2)c_1t$$
$$v_{2,1}(t) = a_2\int_0^t v_{2,0}(t)dt - a_{22}\int_0^t v_{2,0}^2(t)dt = a_2c_2t - a_{22}c_2^2t$$

$$\therefore \quad v_{2,1}(t) = (a_2 - a_{22}c_2)c_2 t$$

$$v_{1,2}(t) = a_1 \int_0^t v_{1,1}(t)dt - 2a_{11}\int_0^t v_{1,0}(t)v_{1,1}(t)dt - a_{12}\int_0^t v_{1,0}(t)v_{2,1}(t)dt - a_{12}\int_0^t v_{1,1}(t)v_{2,0}(t)dt$$

$$= a_1(a_1 - a_{11}c_1 - a_{12}c_2)c_1\frac{t^2}{2} - 2a_{11}c_1(a_1 - a_{11}c_1 - a_{12}c_2)c_1\frac{t^2}{2} - a_{12}c_1(a_2 - a_{22}c_2)c_2\frac{t^2}{2} - a_{12}c_2(a_1 - a_{11}c_1 - a_{12}c_2)c_1\frac{t^2}{2}$$

$$\therefore \quad v_{1,2}(t) = [(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 - a_{12}c_1(a_2 - a_{22}c_2)c_2]\frac{t^2}{2}$$

$$v_{2,2}(t) = a_2\int_0^t v_{2,1}(t)dt - 2a_{22}\int_0^t v_{2,0}(t)v_{2,1}(t)dt$$

$$= [a_2(a_2 - a_{22}c_2)c_2 - 2a_{22}c_2(a_2 - a_{22}c_2)c_2]\frac{t^2}{2}$$

$$\therefore \quad v_{2,2}(t) = [(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)c_2]\frac{t^2}{2}$$

$$v_{1,3}(t) = a_1\int_0^t v_{1,2}(t)dt - 2a_{11}c_1\int_0^t v_{1,2}(t)dt - a_{11}\int_0^t v_{1,1}^2(t)dt - a_{12}c_1\int_0^t v_{2,2}(t)dt - a_{12}c_2\int_0^t v_{1,2}(t)dt - a_{12}\int_0^t v_{1,1}(t)v_{2,1}(t)dt$$

$$= (a_1 - 2a_{11}c_1 - a_{12}c_2)\{(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 - a_{12}c_1c_2(a_2 - a_{22}c_2)\}\frac{t^3}{6} - a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)\, c_1^2\frac{t^3}{3}$$
$$- a_{12}c_1\{(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)c_2\}\frac{t^3}{6} - a_{12}c_2(a_2 - a_{22}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)\, c_1\frac{t^3}{3}$$

$$\therefore \quad v_{1,3}(t) = [(a_1 - 2a_{11}c_1 - a_{12}c_2)\{(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 - a_{12}c_1c_2(a_2 - a_{22}c_2)\} + (a_1 - a_{11}c_1 - a_{12}c_2)c_1\{2a_{12}c_2(a_2 - a_{22}c_2) - 2a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1\} - a_{12}c_1c_2\{(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)\}]\frac{t^3}{6}$$

$$v_{2,3}(t) = a_2\int_0^t v_{2,2}(t)dt - 2a_{22}\int_0^t v_{2,0}(t)v_{2,2}(t)dt - a_{22}\int_0^t v_{2,1}^2(t)dt$$

$$= (a_2 - 2a_{22}c_2)\{(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)c_2\}\frac{t^3}{6} - a_{22}(a_2 - a_{22}c_2)^2c_2^2\frac{t^3}{3}$$

$$\therefore \quad v_{2,3}(t) = [(a_2 - a_{22}c_2)c_2\{(a_2 - 2a_{22}c_2)(a_2 - 2a_{22}c_2) - 2a_{22}(a_2 - a_{22}c_2)c_2\}\frac{t^3}{6}$$

$$v_{1,4}(t) = (a_1 - 2a_{11}c_1 - a_{12}c_2)\int_0^t v_{1,3}(t)dt - 2a_{11}\int_0^t v_{1,1}(t)v_{1,2}(t)dt$$

$$-a_{12}\int_0^t v_{1,1}(t)v_{2,2}(t)dt - a_{12}\int_0^t v_{1,2}(t)v_{2,1}(t)dt - a_{12}c_1\int_0^t v_{2,3}(t)dt$$

$$= [(a_1 - 2a_{11}c_1 - a_{12}c_2)\{ (a_1 - 2a_{11}c_1 - c_2)\{ (a_1 - 2a_{11}c_1 - a_{12}c_2)$$
$$(a_1 - a_{11}c_1 - a_{12}c_2)\, c_1 - a_{12}c_1c_2(a_2 - a_{22}c_2)\}+(a_1 - a_{11}c_1 - a_{12}c_2)$$
$$c_1\{2a_{12}c_2(a_2 - a_{22}c_2) - 2a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1\} - a_{12}c_1c_2\{(a_2 - a_{22}c_2)$$
$$(a_2 - 2a_{22}c_2\,)\}]\frac{t^4}{24} - 2a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1\{(a_1 - 2a_{11}c_1 - a_{12}c_2)$$
$$(a_1 - a_{11}c_1 - a_{12}c_2)c_1 - a_{12}c_1(a_2 - a_{22}c_2)c_2\}\frac{t^4}{8} - a_{12}c_1\{(a_2 - 2a_{22}c_2)$$
$$[(a_2 - a_{22}c_2)c_2(a_2 - 2a_{22}c_2)] - 2(a_2 - a_{22}c_2)c_2a_{22}(a_2 - a_{22}c_2)c_2\}\frac{t^4}{24}$$
$$-a_{12}c_1(a_1 - a_{11}c_1 - a_{12}c_2)[(a_2 - a_{22}c_2)c_2(a_2 - 2a_{22}c_2)]\frac{t^4}{8}$$
$$+a_{12}(a_2 - a_{22}c_2)c_2[(a_1 - 2a_{11}c_1 - a_{12}c_2)\,(a_1 - a_{11}c_1 - a_{12}c_2)c_1$$
$$- a_{12}c_1c_2(a_2 - 2a_{22}c_2)]\,\frac{t^4}{8}$$

$$\therefore \quad v_{1,4}(t) = \{[(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 + a_{12}c_1c_2(a_2 - a_{22}c_2))$$
$$[(a_1 - 2a_{11}c_1 - a_{12}c_2)^2 - c_16a_{11}\,(a_1 - a_{11}c_1 - a_{12}c_2)]$$
$$+(a_1 - 2a_{11}c_1 - a_{12}c_2)\{2(a_1 - a_{11}c_1 - a_{12}c_1)c_1[a_{12}c_2(a_{22}c_2 - a_2)$$
$$-a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1] - a_{12}c_1c_2[(a_2 - 2a_{22}c_2)(a_2 - a_{22}c_2)]\}$$
$$+[(a_2 - a_{22}c_2)c_2(a_2 - 2a_{22}c_2)][a_{12}c_1(a_2 - 2a_{22}c_2) - 3a_{12}c_1(a_1 - a_{11}c_1 - a_{12}c_2)]$$
$$+(a_2 - a_{22}c_2)c_2\{[(a_1 - a_{11}c_1 - a_{12}c_2)c_1[3a_{12}(2a_{11}c_1 - a_1 - a_{12}c_2)]$$
$$+ (a_2 - a_{22}c_2)c_2[3a_{12}^2c_1 + 2a_{22}a_{12}c_1]\}\frac{t^4}{24}$$

$$v_{2,4}(t) = a_2\int_0^t v_{2,3}(t)dt - 2a_{22}c_2\int_0^t v_{2,3}(t)dt - 2a_{22}\int_0^t v_{2,1}(t)v_{2,2}(t)dt$$

$$= (a_2 - 2a_{22}c_2)\,(a_2 - a_{22}c_2)c_2\{(a_2 - 2a_{22}c_2)^2 - 2a_{22}c_2(a_2 - a_{22}c_2)\}\frac{t^4}{24}$$
$$-2a_{22}(a_2 - a_{22}c_2)c_2\{(a_2 - a_{22}c_2)c_2(a_2 - 2a_{22}c_2)\}\frac{t^4}{8}$$

$$\therefore\ v_{2,4}(t) = (a_2 - a_{22}c_2)c_2(a_2 - 2a_{22}c_2)\{(a_2 - 2a_{22}c_2)^2 - 8a_{22}c_2(a_2 - a_{22}c_2)\}\,\frac{t^4}{24}$$

Up to the terms which contain maximum the power of four, we obtain

$$N_1(t) = \lim_{p\to 1} v_1(t) = \sum_{x=0}^{4} v_{1,x}(t) = v_{1,0}(t) + v_{1,1}(t) + v_{1,2}(t) + v_{1,3}(t) + v_{1,4}(t)$$

$$N_1(t) = \lim_{p\to 1} v_2(t) = \sum_{x=0}^{4} v_{2,x}(t) = v_{2,0}(t) + v_{2,1}(t) + v_{2,2}(t) + v_{2,3}(t) + v_{2,4}(t)$$

The solutions by Homotopy Perturbation Method are derived as

$$\begin{aligned} N_1(t) = {} & c_1 + [(a_1 - a_{11}c_1 - a_{12}c_2)c_1]t \\ & + [(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 - a_{12}c_1(a_2 - a_{22}c_2)c_2]\frac{t^2}{2} \\ & + \{(a_1 - 2a_{11}c_1 - a_{12}c_2)[(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 \\ & - a_{12}c_1c_2(a_2 - a_{22}c_2)] + (a_1 - a_{11}c_1 - a_{12}c_2) \\ & c_1[2a_{12}c_2(a_{22}c_2 - a_2) - 2a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1] - a_{12}c_1c_2 \\ & [(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)]\frac{t^3}{6} + \{[(a_1 - 2a_{11}c_1 - a_{12}c_2)(a_1 - a_{11}c_1 - a_{12}c_2)c_1 \\ & - a_{12}c_1c_2(a_2 - a_{22}c_2)][(a_1 - 2a_{11}c_1 - a_{12}c_2)^2 - 6a_{11}c_1(a_1 - a_{11}c_1 - a_{12}c_2)] \\ & + (a_1 - 2a_{11}c_1 - a_{12}c_2)\{2(a_1 - a_{11}c_1 - a_{12}c_2)c_1a_{12}[a_{22}c_2 - a_2) \\ & - a_{11}(a_1 - a_{11}c_1 - a_{12}c_2)c_1] - a_{12}c_1c_2[(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)]\} \\ & + [(a_2 - a_{22}c_2)c_2(a_2 - 2a_{22}c_2)][a_{12}c_1(2a_{22}c_2 - a_2) - 3a_{12}c_1(a_1 - a_{11}c_1 - a_{12}c_2)] \\ & + (a_2 - a_{22}c_2)c_2\{(a_1 - a_{11}c_1 - a_{12}c_2)c_1[3(a_1 - 2a_{11}c_1 - a_{12}c_2)a_{12}] \\ & + (a_2 - a_{22}c_2)c_2(3a_{12}^2c_1 + 2a_{22}a_{12}c_1)\}\frac{t^4}{24} \end{aligned}$$

$$\begin{aligned} \therefore\ N_2(t) = {} & c_2 + [(a_2 - a_{22}c_2)c_2]t + [(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)c_2]\frac{t^2}{2} \\ & + [(a_2 - a_{22}c_2)c_2\{(a_2 - 2a_{22}c_2)(a_2 - 2a_{22}c_2) - 2(a_{22}c_2)(a_2 - a_{22}c_2)\}]\frac{t^3}{6} \\ & + [(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)c_2\{(a_2 - 2a_{22}c_2)(a_2 - 2a_{22}c_2) - 8a_{22}c_2(a_2 - a_{22}c_2)\}]\frac{t^4}{24} \end{aligned}$$

NUMERICAL ILLUSTRATIONS

The nature of the ecological Ammensalism is to be established with a set of numerical solutions which can be illustrated in a period of time.

The fixed parameters are assumed as

$a_{11} = 0.6$, $a_{12} = 0.4$, $a_2 = 1.5$, $a_{22} = 0.5$, $c_1 = 1$ and $c_2 = 1$

The varying variable is a_1, *i.e.* a_1 = from 0.6 to 2.4 with difference 0.2 and then t^* is derived (2.5, 1.3, 0.8, 0.6 and 0.5).

The obtained solutions are illustrated from Fig. (1) to Fig. (10).

Case (1): **In the case where natural growth rate of Ammensal Species is less than the growth rate of enemy species ($a_1 < a_2$)-From Fig. (1) to Fig. (5)**

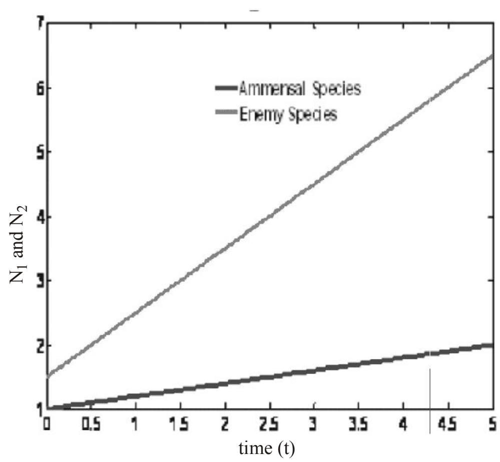

Fig. 1.

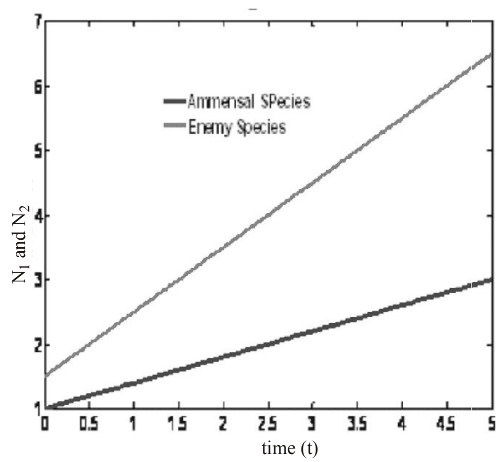

Fig. 2

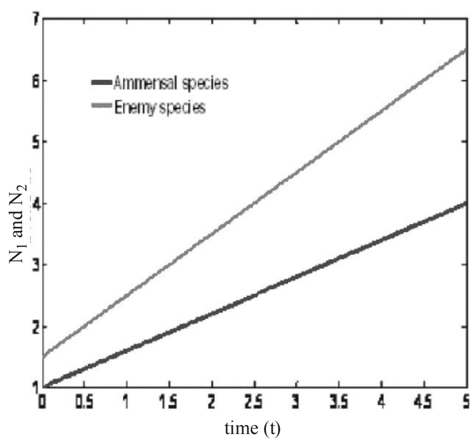

Fig. 3.

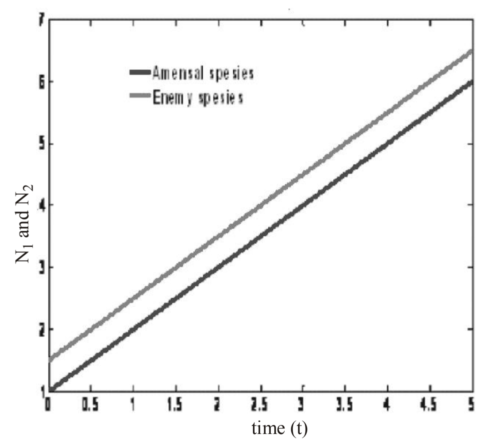

Fig. 4

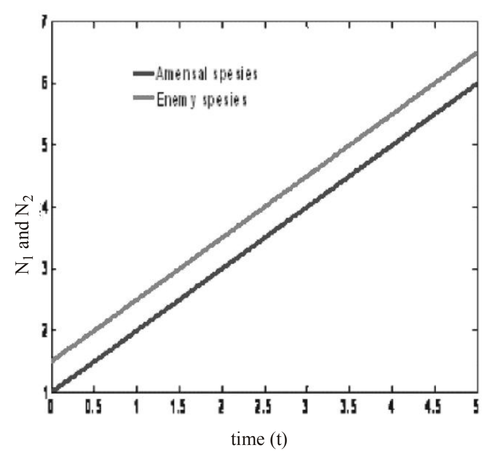

Fig. 5

Case (2) : **In the case where natural growth rate of Ammensal Species is greater than to the growth rate of Enemy species ($a_1 > a_2$)-From Fig. (6) to Fig. (10)**

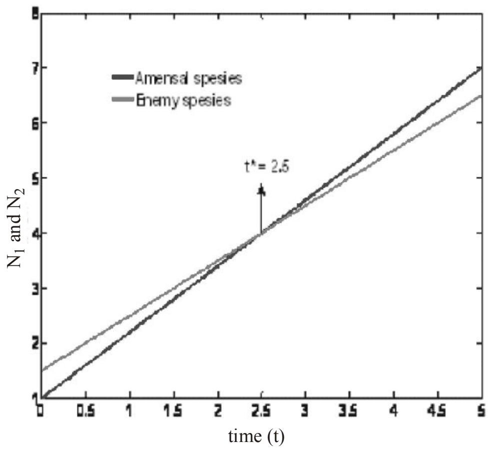

Fig. 6

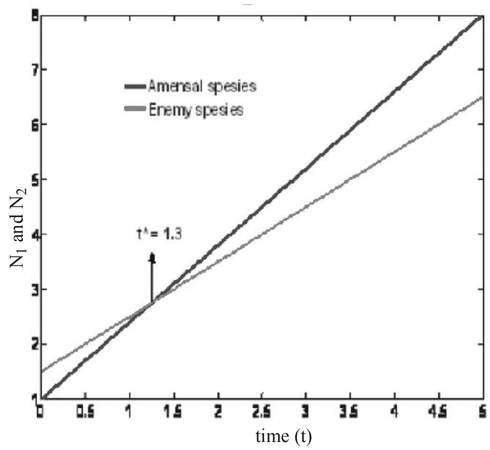

Fig. 7

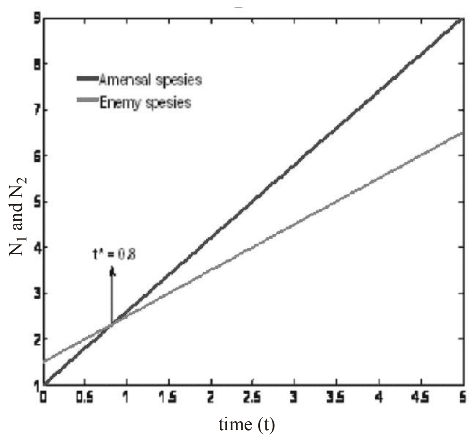

Fig. 8

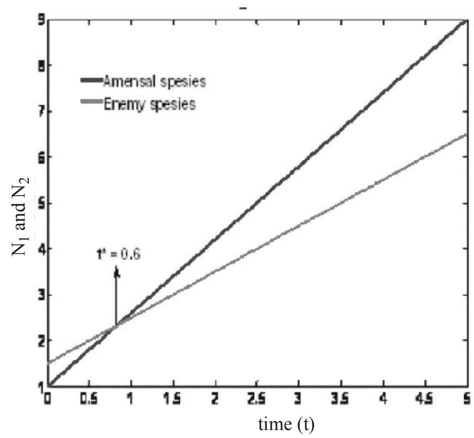

Fig. 9

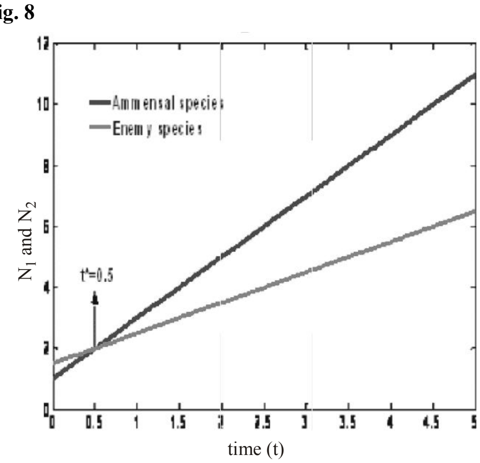

Fig. 10

Conclusions

Case (1) : From Fig. (1) to Fig. (5)

The following results are identified

(i) At initial stage, Enemy species dominates over Ammensal species and there is no interaction between the two species. The Ammensal species has no observable growth rate. Afterwards, Ammensal species gradually strengthens in it's growth.

(ii) Enemy species has an exponential growth rate than the growth rate of Ammensal species. In the course of time, it is also noticed that the Ammensal species has obtained a steady growth rate.

Case (2) : From Fig. (6) to Fig. (10)

(i) Enemy Species flourishes over Ammensal species up to time distinct (t^*). After the dominance reversal time, the Ammensal species out numbers Enemy species throughout the interval with a constant growth rate.

(ii) In the starting stage, Enemy species has a minimum growth rate than Ammensal Species. It dominates over the Ammensal species up to t^*, then after Ammensal species reins over Enemy species in the rest of the interval.

Overall conclusions

A mathematical model of Ammensalism with limited resources is formed by a couple of first order nonlinear differential equations. A series solution of ecological Ammensalism is successfully derived by Homotopy Perturbation Method.

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