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MHD FLOW OF A VISCOELASTIC FLUID BETWEEN TWO PARALLEL POROUS VERTICAL PLATES MOVING IN OPPOSITE DIRECTION

N. P. SINGH, A.K. SINGH, R. R. SINGH

Department of Mathematics, C. L. Jain (PG) College, Firozabad-283203 (INDIA)

AND

KHILANAND GUPTA

Department of Physical Sciences, MGCG, Vishwavidyalaya, Chitrakoot-485780 (INDIA)

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Heat and mass transfer of unsteady incompressible electrically conducting viscoelastic fluid flow between two infinite vertical parallel porous plates moving in opposite direction with thermal radiation and chemical reaction is investigated. A uniform magnetic field is assumed to act perpendicular to the porous plates and the permeability of porous medium is supposed to be dependent on time. A closed form solutions of the equations governing the flow are obtained for the velocity, temperature and concentration profiles considering the hall effect. The velocity, temperature and concentration profiles are evaluated numerically and shown graphically for different values of flow parameters.

KEYWORDS : Viscoelastic fluid, Thermal radiation, Chemical reaction, Hall current.

NOMENCLATURE

B_0 : magnetic field component along y-axis,

C' : concentration in dimensional form ,

C : concentration in non-dimensional form,

C'_d : concentration in the equilibrium state,

C'_e, C'_0 : concentration at the walls,

C_p : specific heat at constant pressure,

D_T : chemical molecular diffusivity,

d : half width of the channel,

Gm : modified Grashoff number,

Gr : the Grashoff number,

g : acceleration due to gravity,

H_0 : magnetic induction,

K'_p : the viscoelasticity,

K'_0 : the permeability parameter in dimensional form,

K_0 : the permeability parameter in non-dimensional form,

K'_1 : chemical reaction,

K_1 : chemical reaction parameter,

K_T : thermal conductivity of the fluid,

m : Hall parameter,

M : the Hartmann number,

N : the radiation parameter,

Pr : Prandtl number,

Q' : heat generation/absorption coefficient,

Q : heat generation/absorption parameter,

q' : dimensional radiative heat transfer,

q : non- dimensional radiative heat transfer,

Sc : Schmidt number,

T' : dimensional temperature,

T : non- dimensional temperature,

T'_e, T'_0 : temperature of the walls,

T'_d : temperature in the equilibrium state,

t' : dimensional time,

t : non-dimensional time,

u', w' : velocity components along x', z' direction,

u, w : velocity components along x, z direction,

U : mean velocity,

ν_0 : suction/injection velocity,

W_C : walls concentration ratio parameter,

W_T : walls temperature ratio parameter,

x', y' : dimensional coordinates,

X, y : non-dimensional coordinates,

Greek symbols

α' : the viscoelastic parameter,

β' : coefficient of volume expansion for mass transfer,

β : coefficient of volume expansion for heat transfer,

λ : the suction parameter,

ρ : density of the fluid,

σ : electrical conductivity of the fluid,

μ_e : the magnetic permeability parameter,

υ : kinematic viscosity of the fluid,

ω : frequency of the oscillation,

ω_e : the cyclotron frequency,

τ_e : the electron collision time,

ε : real number,

$\omega' t'$: dimensional phase-angle,

ωt : non- dimensional phase-angle.

Introduction

In recent years free convective flow in a porous medium have received much attention due to its wide application in geothermal and oil reservoir engineering as well as in other geophysical, astrophysical and biological studies. Convective flow in porous media is important in environmental studies involving air and water pollution. Moreover, considerable interest has been shown in radiation interaction with convection for heat transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly in free convection problems involving absorbing-emitting fluids. The effects of transversely applied magnetic field on the flow of electrically conducting viscous and viscoelastic fluids have been discussed widely owing to their engineering applications. It is applied to study the stellar and solar structure, interstellar matter and radio propagation through the ionosphere. Cogley *et al*. [1] discussed differential approximation for radiative transfer. Das *et al*. [2] discussed transient free convection flow past and infinite vertical plate with periodic temperature variation. Li *et al*. [3] investigated natural convection from a vertical flat plate with a surface temperature oscillation. Muthucumaraswamy and Ganesan [4] discussed natural convection on a moving isothermal vertical plate with chemical reaction. Singh *et al*. [5] analysed heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Cookey *et al*. [6] discussed influence of viscous dissipation and radiation on unsteady MHD free-convection flow past and infinite heated vertical plate in a porous medium with time dependent suction. Das *et al*. [7] analysed magnetohydrodynamics three dimensional flow and heat transfer past a vertical porous plate through a porous medium with periodic suction. Singh *et al*. [8] discussed effects of periodic permeability and suction velocity on three-dimensional free convection flow past a vertical porous plate embedded in highly porous medium. Manglesh *et* *al*. [9] discussed MHD flow of viscoelastic fluid with variable suction and permeability two infinite porous plates moving in opposite direction. In the present paper it is proposed to study MHD flow of a viscoelastic fluid between two parallel porous vertical plates and is thus advancement to the study of Manglesh *et al*. [0].

FORMULATION OF THE PROBLEM

Consider an unsteady MHD flow of a viscoelastic incompressible electrically conducting fluid past infinitely long vertical flat plates located at the $y' = -d$ and $y' = d$ planes and extend form $x' \to -\infty$ to ∞ and $z' \to -\infty$ to ∞ embedded in a porous medium on taking hall current into account. A cartesian coordinate system is introduced such that x'-axis lies vertically upward and y'-axis is perpendicular to it. The plates are moving in opposite direction with velocity $U(1+\varepsilon\cos\omega' t')$. The fluid is considered to be gray, absorbing-emitting radiation but non-scattering medium. The flow field is exposed to the influence of injection and suction velocity, thermal and mass buoyancy effect, thermal radiation and chemically reactive species. Darcy's resistance term is taken into account with variable permeability for the medium. Further due to the infinite plane surface assumption, the flow variables are function of y' and t' only.

Since the Hall current term is retained and if (j'_x, j'_y, j'_z) are the components of electric current density, j'. The equation of conservation of electric charge, $\nabla . j' = 0$, gives $j'_y =$ constant. Since the plates are electrically non-conducting, $j'_y = 0$ and is zero everywhere in the flow. When the magnetic field is large, generalized Ohm's law, in the absence of electric field, neglecting the ion slips and thermo electric effect yields:

$$j'_x - \omega_e \tau_e j'_z = -\sigma\mu_e H_0 w', \qquad \dots (1)$$

$$j'_z + \omega_e \tau_e j'_x = \sigma\mu_e H_0 u'. \qquad \dots (2)$$

The solution of equations (1) and (2) are :

$$j'_x = \frac{\sigma\mu_e H_0}{1+m^2}(mu' - w'), \qquad \dots (3)$$

$$j'_z = \frac{\sigma\mu_e H_0}{1+m^2}(u' + mw'). \qquad \dots (4)$$

where $m = \omega_e \tau_e$.

The chemical reactions are taking place in the flow and all thermo physical properties are assumed to be constant. Under usual Boussinesq's approximation and in the absence of pressure gradient, the unsteady equations governing the MHD flow of the viscoelastic fluid are :

$$\frac{\partial u'}{\partial t'}+v_0(1+\varepsilon e^{i\omega' t'})\frac{\partial u'}{\partial y'}=\upsilon\frac{\partial^2 u'}{\partial y'^2}-K'_p\ \frac{\partial^3 u'}{\partial t'\partial y'^2}$$

$$-\frac{\sigma B_0^2}{\rho(1+m^2)}(u'+mw')+g_T\beta(T'-T'_d\ )+g_C\beta'(C'-C'_d\ )-\frac{\upsilon u'}{K'_0\ (1+\varepsilon e^{i\omega' t'})},\quad \ldots (5)$$

$$\frac{\partial w'}{\partial t'}+v_0(1+\varepsilon e^{i\omega' t'})\frac{\partial w'}{\partial y'}=\upsilon\frac{\partial^2 w'}{\partial y'^2}-K'_p\ \frac{\partial^3 w'}{\partial t'\partial y'^2}$$

$$+\frac{\sigma B_0^2}{\rho(1+m^2)}(mu'-w')-\frac{\upsilon w'}{K'_0(1+\varepsilon e^{i\omega' t'})},\quad \ldots (6)$$

$$\frac{\partial T'}{\partial t'}+v_0(1+\varepsilon e^{i\omega' t'})\ \frac{\partial T'}{\partial y'}=\frac{K_T}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}\ -\frac{1}{\rho C_p}\frac{\partial q'}{\partial y'}\ -\frac{1}{\rho C_p}Q'(T'-T'_d),\quad \ldots (7)$$

$$\frac{\partial C'}{\partial t'}+v_0(1+\varepsilon e^{i\omega' t'})\frac{\partial C'}{\partial y'}=D_T\ \frac{\partial^2 C'}{\partial y'^2}-K'_1(C'-C'_d)\quad \ldots (8)$$

The boundary conditions of the problem are:

$$u'=U(1+\varepsilon\cos\omega' t'),\qquad w'=0\,,$$
$$T'=T'_0+\varepsilon(T'_0-T'_d)\ \cos\omega' t'\,,$$
$$C'=C'_0+\varepsilon(C'_0-C'_d)\cos\omega' t' \text{ at } y'=-d,$$
$$u'=-U(1+\varepsilon\cos\omega' t'),\qquad w'=0,$$
$$T'=T'_e+\varepsilon(T'_e-T'_d)\ \cos\omega' t',$$
$$C'=C'_e+\varepsilon(C'_e-C'_d)\cos\omega' t' \text{ at } y=d.\quad \ldots (9)$$

In the spirit of Cogley *et al* [1]. the radiative heat flux for the present problem becomes:

$$\frac{\partial q'}{\partial y'}=4\alpha^2(T'-T'_d),\quad \ldots (10)$$

where α is the mean radiation absorption coefficient.

We introduce the following non-dimensional variables and parameters:

$$u=\frac{u'}{U},\ w=\frac{w'}{U},\ x=\frac{x'}{d},\ y=\frac{y'}{d},\ T=\frac{T'-T'_d}{T'_0-T'_d},\ C=\frac{C'-C'_d}{C'_0-C'_d},\ t=\frac{t'\upsilon}{d^2},\qquad \omega=\frac{\omega' d^2}{\upsilon}$$

$$\lambda=\frac{v_0 d}{\upsilon},\ \alpha'=\frac{K'_p}{d^2},\ M=B_0 d\sqrt{\frac{\sigma}{\mu}},\quad K_0=\frac{K'_0}{d^2},\quad Pr=\frac{\mu C_p}{K_T},\quad G_m=\frac{g_C\beta'(C'_0-C'_d)\ d^2}{\upsilon U},$$

$$G_r=\frac{g_T\beta(T'_0-T'_d)d^2}{\upsilon U},\qquad Sc=\frac{\upsilon}{D_T},\quad K_1=\frac{K'_1 d^2}{\upsilon},\quad N=\frac{2\alpha d}{\sqrt{K_T}},\quad Q=\frac{Q' d^2}{K_T}.$$

Introducing above mentioned variables and parameters in equations (5)-(8), we obtain:

$$\frac{\partial u}{\partial t}+\lambda(1+\varepsilon e^{i\omega t})\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-\alpha'\frac{\partial^3 u}{\partial t\partial y^2}-\frac{M^2}{1+m^2}$$

$$\left(u+mw\right)+Gr\,T+Gm\,C-\frac{u}{K_0(1+\varepsilon e^{i\omega t})}, \quad \ldots (11)$$

$$\frac{\partial w}{\partial t}+\lambda(1+\varepsilon e^{i\omega t})\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial y^2}-\alpha'\frac{\partial^3 w}{\partial t\partial y^2}-\frac{M^2}{(1+m^2)}(mu-w)-\frac{w}{K_0(1+\varepsilon e^{i\omega t})}, \quad \ldots (12)$$

$$\frac{\partial T}{\partial t}+\lambda(1+\varepsilon e^{i\omega t})\frac{\partial T}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2}-\frac{1}{Pr}(N^2+Q)T, \quad \ldots (13)$$

$$\frac{\partial C}{\partial t}+\lambda(1+\varepsilon e^{i\omega t})\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-K_1 C. \quad \ldots (14)$$

The boundary conditions (9) become:

$$u=1+\varepsilon\cos\omega t,\ \ w=0,\ T=1+\varepsilon\cos\omega t,\qquad C=1+\varepsilon\cos\omega t\ \text{ at }\ y=-1,$$

$$u=-(1+\varepsilon\cos\omega t),\ w=0,\ T=W_T(1+\varepsilon\cos\omega t),$$

$$C=W_C(1+\varepsilon\cos\omega t),\ \text{ at }\ y=1, \quad \ldots (15)$$

where $W_T=\dfrac{T'_e-T'_d}{T'_0-T'_d},\quad W_C=\dfrac{C'_e-C'_d}{C'_0-C'_d}$.

Using complex velocity $q=u+iw$ equations (11) and (12) can be combined to give :

$$\frac{\partial q}{\partial t}+\lambda(1+\varepsilon e^{i\omega t})\frac{\partial q}{\partial y}=\frac{\partial^2 q}{\partial y^2}-\alpha'\frac{\partial^3 q}{\partial t\partial y^2}-\frac{M^2}{(1+m^2)}(1-im)q+GrT+Gm\,C-\frac{q}{K_0(1+\varepsilon e^{i\omega t})}. \quad \ldots (16)$$

Using complex velocity $q=u+iw$ the boundary conditions (15) can be combined to give :

$$q=\left\{1+\frac{\varepsilon}{2}(e^{i\omega t}+e^{-i\omega t})\right\},\quad T=\left\{1+\frac{\varepsilon}{2}(e^{i\omega t}+e^{-i\omega t})\right\},$$

$$C=\left\{1+\frac{\varepsilon}{2}(e^{i\omega t}+e^{-i\omega t})\right\}\ \text{ at }\ y=-1,$$

$$q=-\left\{1+\frac{\varepsilon}{2}(e^{i\omega t}+e^{-i\omega t})\right\},\quad T=W_T\left\{1+\frac{\varepsilon}{2}(e^{i\omega t}+e^{-i\omega t})\right\},$$

$$C=W_C\left\{1+\frac{\varepsilon}{2}(e^{i\omega t}+e^{-i\omega t})\right\}\ \text{ at }\ y=1. \quad \ldots (17)$$

In order to solve the system of equations (13), (14) and (16) subject to the boundary conditions (17), we assume the solution is of the form :

$$q(y,t)) = q_0(y) + \frac{\varepsilon}{2}\{q_1(y)e^{i\omega t} + q_2(y)e^{-i\omega t}\},$$

$$T(y,t) = T_0(y) + \frac{\varepsilon}{2}\{T_1(y)e^{i\omega t} + T_2(y)e^{-i\omega t}\},$$

$$C(y,t) = C_0(y) + \frac{\varepsilon}{2}\{C_1(y)e^{i\omega t} + C_2(y,t)\ e^{-i\omega t}\}. \qquad \ldots (18)$$

Method of Solution

Substituting (18) in equations (13), (14) and (16), we obtain:

$$\frac{\partial^2 T_0}{\partial y^2} - \lambda Pr(1+\varepsilon e^{i\omega t})\frac{\partial T_0}{\partial y} - (N^2+Q)T_0 = 0, \qquad \ldots (19)$$

$$\frac{\partial^2 T_1}{\partial y^2} - \lambda Pr(1+\varepsilon e^{i\omega t})\frac{\partial T_1}{\partial y} - (N^2+Q+i\omega Pr)T_1 = 0, \qquad \ldots (20)$$

$$\frac{\partial^2 T_2}{\partial y^2} - \lambda Pr(1+\varepsilon e^{i\omega t})\frac{\partial T_2}{\partial y} - (N^2+Q-i\omega Pr)T_2 = 0, \qquad \ldots (21)$$

$$\frac{\partial^2 C_0}{\partial y^2} - \lambda Sc(1+\varepsilon e^{i\omega t})\frac{\partial C_0}{\partial y} - K_1 ScC_0 = 0, \qquad \ldots (22)$$

$$\frac{\partial^2 C_1}{\partial y^2} - \lambda Sc(1+\varepsilon e^{i\omega t})\frac{\partial C_1}{\partial y} - (K_1+i\omega)\,ScC_1 = 0, \qquad \ldots (23)$$

$$\frac{\partial^2 C_2}{\partial y^2} - \lambda Sc(1+\varepsilon e^{i\omega t})\frac{\partial C_2}{\partial y} - (K_1-i\omega)\,ScC_2 = 0, \qquad \ldots (24)$$

$$\frac{\partial^2 q_0}{\partial y^2} - \lambda(1+\varepsilon e^{i\omega t})\frac{\partial q_0}{\partial y} - \left\{\frac{M^2}{1+m^2}(1-im) + \frac{1}{K_0(1+\varepsilon e^{i\omega t})}\right\}q_0 = -GrT_0 - GmC_0, \qquad \ldots (25)$$

$$(1-\alpha' i\omega)\frac{\partial^2 q_1}{\partial y^2} - \lambda(1+\varepsilon e^{i\omega t})\frac{\partial q_1}{\partial y}$$
$$- \left\{\frac{M^2}{1+m^2}(1-im) + \frac{1}{K_0(1+\varepsilon e^{i\omega t})} + i\omega\right\}q_1 = -GrT_1 - GmC_1, \qquad \ldots (26)$$

$$(1+\alpha' i\omega)\frac{\partial^2 q_2}{\partial y^2} - \lambda(1+\varepsilon e^{i\omega t})\frac{\partial q_2}{\partial y}$$
$$- \left\{\frac{M^2}{1+m^2}(1-im) + \frac{1}{K_0(1+\varepsilon e^{i\omega t})} - i\omega\right\}q_2 = -GrT_2 - GmC_2 \qquad \ldots (27)$$

Corresponding boundary conditions (17) become

$$q_0 = q_1 = q_2 = 1,\ T_0 = T_1 = T_2 = 1,\ C_0 = C_1 = C_2 = 1,\ \text{at}\ y = -1,,$$

$$q_0 = q_1 = q_2 = -1,\ T_0 = T_1 = T_2 = W_T,\ C_0 = C_1 = C_2 = W_C,\ \text{at}\ y = 1. \qquad \ldots (28)$$

The solutions of equations (19)-(27) satisfying the boundary conditions (28) yield:

$$T_0 = \frac{1}{(e^{(m_1 - m_2)} - e^{-(m_1 - m_2)})}\{e^{m_1 + m_2 y} - e^{m_2 + m_1 y} + W_T(e^{-m_2 + m_1 y} - e^{-m_1 + m_2 y})\} \qquad \ldots (29)$$

$$T_1 = \frac{1}{(e^{(m_3 - m_4)} - e^{-(m_3 - m_4)})}\{e^{m_3 + m_4 y} - e^{m_4 + m_3 y} + W_T(e^{-m_4 + m_3 y} - e^{-m_3 + m_4 y})\} \qquad \ldots (30)$$

$$T_2 = \frac{1}{(e^{(m_5 - m_6)} - e^{-(m_5 - m_6)})}\{e^{m_5 + m_6 y} - e^{m_6 + m_5 y} + W_T(e^{-m_6 + m_5 y} - e^{-m_5 + m_6 y})\} \qquad \ldots (31)$$

$$C_0 = \frac{1}{(e^{(m_7 - m_8)} - e^{-(m_7 - m_8)})}\{e^{m_7 + m_8 y} - e^{m_8 + m_7 y} + W_C(e^{-m_8 + m_7 y} - e^{-m_7 + m_8 y})\} \qquad \ldots (32)$$

$$C_1 = \frac{1}{(e^{(m_9 - m_{10})} - e^{-(m_9 - m_{10})})}\{e^{m_9 + m_{10} y} - e^{m_{10} + m_9 y} + W_C(e^{-m_{10} + m_9 y} - e^{-m_9 + m_{10} y})\} \qquad \ldots (33)$$

$$C_2 = \frac{1}{(e^{(m_{11} - m_{12})} - e^{-(m_{11} - m_{12})})}\{e^{m_{11} + m_{12} y} - e^{m_{12} + m_{11} y} + W_C(e^{-m_{12} + m_{11} y} - e^{-m_{11} + m_{12} y})\} \qquad \ldots (34)$$

$$q_0 = A_5\, e^{m_{13} y} + A_6\, e^{m_{14} y} + A_1(e^{m_1 + m_2 y} - W_T e^{-m_1 + m_2 y}) + A_2(e^{m_2 + m_1 y} - W_T e^{-m_2 + m_1 y})$$
$$+ A_3(e^{m_7 + m_8 y} - W_C\, e^{-m_7 + m_8 y}) + A_4(e^{m_8 + m_7 y} - W_C\, e^{-m_8 + m_7 y}), \qquad \ldots (35)$$

$$q_1 = A_{11}\, e^{m_{15} y} + A_{12} e^{m_{16} y} + A_7(e^{m_3 + m_4 y} - W_T\, e^{-m_3 + m_4 y}) + A_8(e^{m_4 + m_3 y} - W_T\, e^{-m_4 + m_3 y})$$
$$+ A_9(e^{m_9 + m_{10} y} - W_C\, e^{-m_9 + m_{10} y}) + A_{10}(e^{m_{10} + m_9 y} - W_C\, e^{-m_{10} + m_9 y}), \qquad \ldots (36)$$

$$q_2 = A_{17}\, e^{m_{17} y} + A_{18}\, e^{m_{18} y} + A_{13}(e^{m_5 + m_6 y} - W_T\, e^{-m_5 + m_6 y}) + A_{14}(e^{m_6 + m_5 y} - W_T\, e^{-m_6 + m_5 y})$$
$$+ A_{15}(e^{m_{11} + m_{12} y} - W_C\, e^{-m_{11} + m_{12} y}) + A_{16}(e^{m_{12} + m_{11} y} - W_C\, e^{-m_{12} + m_{11} y}) \qquad \ldots (37)$$

The constant are defined in the appendix.

The Shear stress, Nusselt number and Sherwood number

The Shear stress, Nusselt number and Sherwood number at the plates is given by:

$$\tau = \left(\frac{\partial q}{\partial y}\right)_{y = \mp 1}$$
$$= \left[q_0(y) + \frac{\varepsilon}{2}\{q_1(y)\, e^{i\omega t} + q_2(y) e^{-i\omega t}\}\right]_{y = \mp 1}, \qquad \ldots (38)$$

$$Nu = \left(\frac{\partial T}{\partial y}\right)_{y=\mp 1}$$

$$= \left[T_0(y) + \frac{\varepsilon}{2}\{T_1(y)e^{i\omega t} + T_2(y)e^{-i\omega t}\}\right]_{y=\mp 1}, \qquad \ldots (39)$$

$$Sh = \left(\frac{\partial C}{\partial y}\right)_{y=\mp 1}$$

$$= \left[C_0(y) + \frac{\varepsilon}{2}\{C_1(y)e^{i\omega t} + C_2(y)e^{-i\omega t}\}\right]_{y=\mp 1}. \qquad \ldots (40)$$

Verification of the problem

When the heat generation/absorption parameter (Q) is not considered in the problem, the results of the present study are exactly the same as obtained by Manglesh *et al*. [9] except notations.

RESULTS AND DISCUSSION

Graphical representation of results is very useful to demonstrate the effects of different parameters. Here we have examined the nature of variation of various physical quantities associated with the problem under consideration.

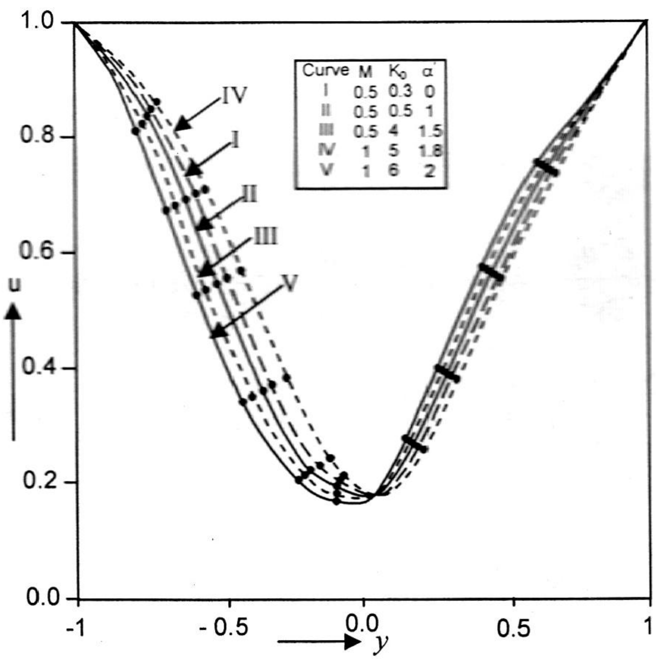

Fig. 1. The velocity distribution versus y for different values of M, K_0 and α' ($N = 1$, $Pr = 7.0$, $\lambda = 0.3$, $Sc = 0.40$, $K_1 = 1$, $Q = 5$).

Fig. 1 shows the velocity profiles for the different parameters of M, K_0 and α' against y. M due to the effect of transverse magnetic field on electrically conducting fluid which gives rise to a resistive type of force called Lorentz force similar to drag force which slows down the motion of fluid. The permeability parameter K_0 increases the velocity because increase in permeability of medium implies less resistance due to the porous matrix present in the medium. Besides, an increase in viscoelastic parameter adheres hydrodynamic boundary layer strongly, which in tern retards the flow in the left half of the channel in the vicinity of upward moving plate and the reverse effect is seen on the downward moving plate due to its opposite direction.

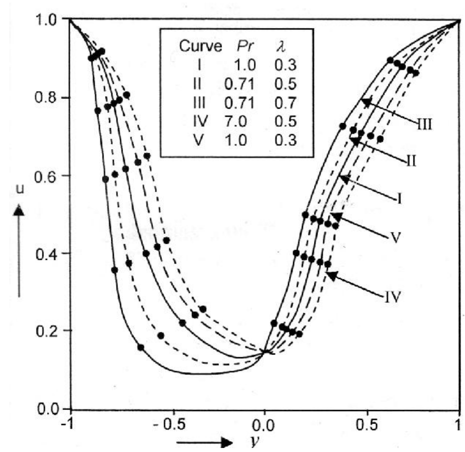

Fig. 2. The velocity distribution versus *y* for different values of *Pr* and λ ($M = 0.5, N = 1, \alpha' = 1, Sc = 0.40, K_1 = 1, K_0 = 0.3, Q = 5$).

Fig. 2 shows the velocity profiles for the different values of Pr and λ against y. The velocity increases with increase of Prandtl number. It can be interpreted that velocity increases in the left half of channel due to injection at left upward moving plate and decreases in right half with suction on right plate as suction stabilize the boundary layer growth.

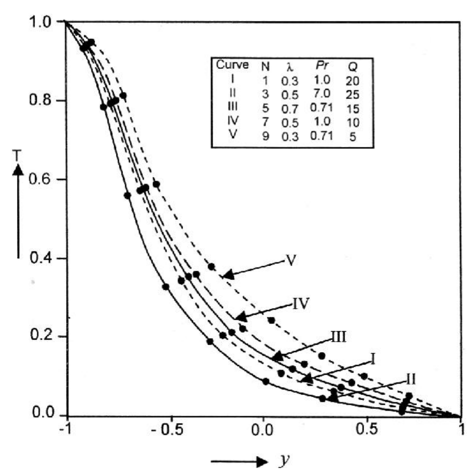

Fig. 3. The temperature distribution versus y for different values of N, λ, *Pr* and *Q* ($M = 0.5, K_0 = 0.3, \alpha' = 1, Sc = 0.40, K_1 = 1$).

Fig. 3 shows the temperature profiles for the different values of N, λ, Pr and Q against y. It is observed that fluid temperature decreases with an increase in radiation parameter. Since the effect of radiation decrease the rate of energy transport to the fluid, thereby decreasing the fluid temperature. It is also noted that the fluid temperature increases with the decrease of suction parameter. Besides, the fluid temperature decreases with an increase of Prandtl number. We note that increase in heat generation parameter increases the temperature, whereas an increase in the heat absorption parameter decreases the temperature. As stated earlier, the positive and negative values of Q represent heat absorption and heat generation respectively.

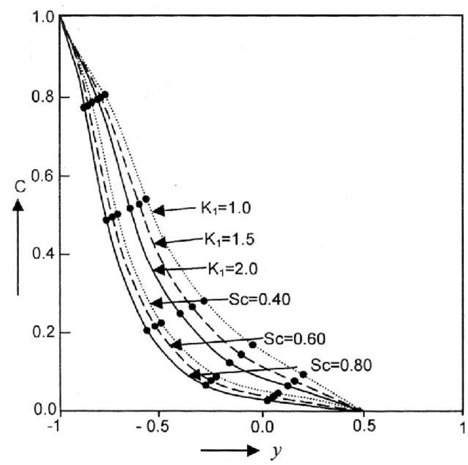

Fig. 4. The concentration distribution versus *y* for different values of K_1 and Sc (M = 0.5, Pr = 7, λ = 0.3, N = 1, α' =1, K_0 = 0.3, Q = 5).

Fig. 4 shows the concentration profiles for the different values of Sc and K_1 against y. It is observed that the concentration decreases for increasing chemical reaction parameter which indicates that the diffusion rates can be tremendously changed by chemical reaction. It is also noted that the concentration field increases with decrease in Schmidt number.

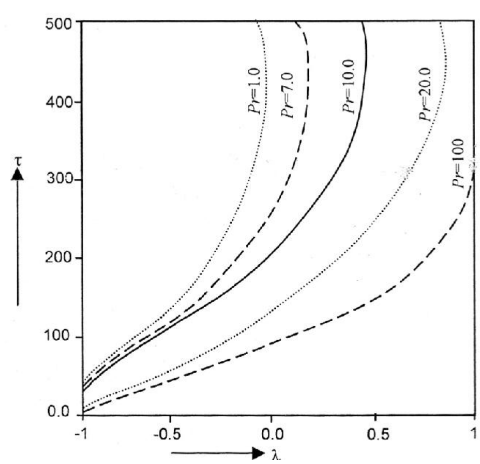

Fig. 5. Shear stress versus λ for different value of *Pr* (N = 1, K_0 = 0.3, α' = 1, Sc = 0.40, K_1 = 1, M = 0.5, Q = 5).

Fig. 5 represents variations in the Shear stress with respect to the suction parameter for different numerical values of Prandtl number. It is observed that an increase in suction parameter increases the Shear stress whereas an increase in Prandtl number decreases the Shear stress.

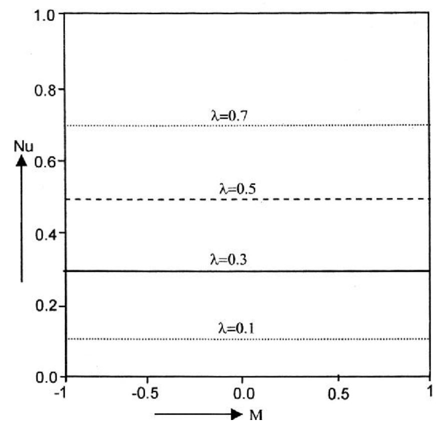

Fig. 6. Nusselt number versus M for different value of λ (N = 1, K_0 = 0.3, α' = 1, Sc = 0.40, K_1 = 1, Pr = 7.0, Q = 5).

Fig. 6 illustrates the variation of rate of heat transfer (Nu) versus the magnetic parameter (M) corresponding to the different values of λ. It is clearly show that the Nusselt number increases with an increase in the value of λ whereas the magnetic parameter has no effect on Nusselt number.

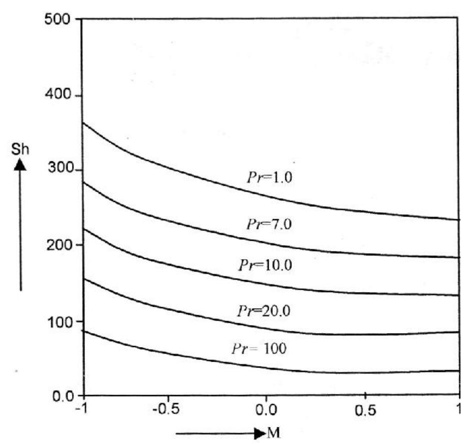

Fig. 7. Sherwood number versus *M* for different value of *Pr* (N = 1, K_0 = 0.3, α' = 1, Sc = 0.40, K_1=1, λ = 0.3, Q = 5)

Fig. 7 illustrates the variation of Sherwood number versus the magnetic parameter for different values of Prandtl number. It is observed that Sherwood number decreases with increase in Prandtl number for increasing magnetic parameter.

Conclusions

The conclusions of the study are as follows :

(i) The velocity decreases with increase in M.

(ii) The velocity increases with increase of Prandtl number.

(iii) The fluid temperature decreases with an increase in radiation parameter.

(iv) The fluid temperature increases with decrease of suction parameter.

(v) The fluid temperature decreases with increase of Prandtl number.

(vi) The fluid temperature decreases with increase in the heat absorption parameter, but increases with increase in the heat generation parameter.

(vii) The concentration decreases an increase of chemical reaction parameter.

(viii) The concentration field increases with decrease in Schmidt number.

(ix) An increase in suction parameter increases the Shear stress whereas an increase in Prandtl number decreases the Shear stress.

(x) The Nusselt number increases with the increase the suction parameter whereas the magnetic parameter has no effect on Nusselt number.

(xi) The Sherwood number decreases with an increase in Prandtl number but increasing the magnetic parameter.

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Appendix

$$a_1 = \lambda Pr(1+\varepsilon e^{i\omega t}), \qquad a_2 = N^2 + Q + i\omega Pr,$$

$$a_3 = N^2 + Q - i\omega Pr, \qquad a_4 = \lambda Sc(1+\varepsilon e^{i\omega t}),$$

$$a_5 = K_1 Sc, \qquad a_6 = (K_1 + i\omega)Sc,$$

$$a_7 = (K_1 - i\omega)Sc, \qquad a_8 = \lambda(1+\varepsilon e^{i\omega t}),$$

$$a_9 = \frac{M^2}{1+m^2}(1-im) + \frac{1}{K_0(1+\varepsilon e^{i\omega t})}, \qquad a_{10} = (1-\alpha' i\omega),$$

$$a_{11} = \frac{M^2}{1+m^2}(1-im) + \frac{1}{K_0(1+\varepsilon e^{i\omega t})} + i\omega, \quad a_{12} = (1+\alpha' i\omega),$$

$$a_{13} = \frac{M^2}{1+m^2}(1-im) + \frac{1}{K_0(1+\varepsilon e^{i\omega t})} - i\omega,$$

$$m_1 = \frac{a_1 + \sqrt{a_1^2 + 4(N^2+Q)}}{2}, \quad m_2 = \frac{a_1 - \sqrt{a_1^2 + 4(N^2+Q)}}{2},$$

$$m_3 = \frac{a_1 + \sqrt{a_1^2 + 4a_2}}{2}, \qquad m_4 = \frac{a_1 - \sqrt{a_1^2 + 4a_2}}{2},$$

$$m_5 = \frac{a_1 + \sqrt{a_1^2 + 4a_3}}{2}, \qquad m_6 = \frac{a_1 - \sqrt{a_1^2 + 4a_3}}{2},$$

$$m_7 = \frac{a_4 + \sqrt{a_4^2 + 4a_5}}{2}, \qquad m_8 = \frac{a_4 - \sqrt{a_4^2 + 4a_5}}{2},$$

$$m_9 = \frac{a_4 + \sqrt{a_4^2 + 4a_6}}{2}, \qquad m_{10} = \frac{a_4 - \sqrt{a_4^2 + 4a_6}}{2},$$

$$m_{11} = \frac{a_4 + \sqrt{a_4^2 + 4a_7}}{2}, \qquad m_{12} = \frac{a_4 - \sqrt{a_4^2 + 4a_7}}{2},$$

$$m_{13} = \frac{a_8 + \sqrt{a_8^2 + 4a_9}}{2}, \qquad m_{14} = \frac{a_8 - \sqrt{a_8^2 + 4a_9}}{2},$$

$$m_{15} = \frac{a_8 + \sqrt{a_8^2 + 4a_{10}a_{11}}}{2a_{10}}, \qquad m_{16} = \frac{a_8 - \sqrt{a_8^2 + 4a_{10}a_{11}}}{2a_{10}},$$

$$m_{17} = \frac{a_8 + \sqrt{a_8^2 + 4a_{12}a_{13}}}{2a_{12}}, \qquad m_{18} = \frac{a_8 - \sqrt{a_8^2 + 4a_{12}a_{13}}}{2a_{12}},$$

$$A_1 = -\frac{Gr}{(m_2^2 - a_8 m_2 - a_9)(e^{(m_1-m_2)} - e^{-(m_1-m_2)})},$$

$$A_2 = \frac{Gr}{(m_1^2 - a_8 m_1 - a_9)(e^{(m_1-m_2)} - e^{-(m_1-m_2)})},$$

$$A_3 = -\frac{Gm}{(m_8^2 - a_8 m_8 - a_9)(e^{(m_7-m_8)} - e^{-(m_7-m_8)})},$$

$$A_4 = \frac{Gm}{(m_7^2 - a_8 m_7 - a_9)(e^{(m_7 - m_8)} - e^{-(m_7 - m_8)})},$$

$$A_5 = -\frac{1}{(e^{(m_{13} - m_{14})} - e^{-(m_{13} - m_{14})})} [(e^{m_{14}} + e^{-m_{14}}) - A_1(e^{-m_2 + m_{14}} - e^{m_2 - m_{14}})$$

$$\times (e^{m_1} - W_T e^{-m_1}) - A_2(e^{-m_1 + m_{14}} - e^{m_1 - m_{14}})$$

$$\times (e^{m_2} - W_T e^{-m_2}) - A_3(e^{-m_8 + m_{14}} - e^{m_8 - m_{14}})$$

$$\times (e^{m_7} - W_C e^{-m_7}) - A_4(e^{-m_7 + m_{14}} - e^{m_7 - m_{14}})\ (e^{m_8} - W_C e^{-m_8})],$$

$$A_6 = \frac{1}{(e^{(m_{13} - m_{14})} - e^{-(m_{13} - m_{14})})} [(e^{m_{13}} + e^{-m_{13}}) - A_1(e^{-m_2 + m_{13}} - e^{m_2 - m_{13}})$$

$$\times (e^{m_1} - W_T e^{-m_1}) - A_2(e^{-m_1 + m_{13}} - e^{m_1 - m_{13}})$$

$$\times (e^{m_2} - W_T e^{-m_2}) - A_3(e^{-m_8 + m_{13}} - e^{m_8 - m_{13}})$$

$$\times (e^{m_7} - W_C e^{-m_7}) - A_4(e^{-m_7 + m_{13}} - e^{m_7 - m_{13}})\ (e^{m_8} - W_C e^{-m_8})],$$

$$A_7 = -\frac{Gr}{(a_{10} m_4^2 - a_8 m_4 - a_{11})(e^{(m_3 - m_4)} - e^{-(m_3 - m_4)})},$$

$$A_8 = \frac{Gr}{(a_{10} m_3^2 - a_8 m_3 - a_{11})(e^{(m_3 - m_4)} - e^{-(m_3 - m_4)})},$$

$$A_9 = -\frac{Gm}{(a_{10} m_{10}^2 - a_8 m_{10} - a_{11})(e^{(m_9 - m_{10})} - e^{-(m_9 - m_{10})})},$$

$$A_{10} = \frac{Gm}{(a_{10} m_9^2 - a_8 m_9 - a_{11})(e^{(m_9 - m_{10})} - e^{-(m_9 - m_{10})})},$$

$$A_{11} = -\frac{1}{(e^{(m_{15} - m_{16})} - e^{-(m_{15} - m_{16})})}$$

$$\times [(e^{m_{16}} + e^{-m_{16}}) - A_7(e^{-m_4 + m_{16}} - e^{m_4 - m_{16}})$$

$$\times (e^{m_3} - W_T e^{-m_3}) - A_8(e^{-m_3 + m_{16}} - e^{m_3 - m_{16}})$$

$$\times (e^{m_4} - W_T e^{-m_4}) - A_9(e^{-m_{10} + m_{16}} - e^{m_{10} - m_{16}})$$

$$\times (e^{m_9} - W_C e^{-m_9}) - A_{10}(e^{-m_9 + m_{16}} - e^{m_9 - m_{16}})\ (e^{m_{10}} - W_C e^{-m_{10}})],$$

$$A_{12} = \frac{1}{(e^{(m_{15}-m_{16})} - e^{-(m_{15}-m_{16})})} [(e^{m_{15}} + e^{-m_{15}}) - A_7(e^{-m_4+m_{15}} - e^{m_4-m_{14}})$$

$$\times (e^{m_3} - W_T e^{-m_3}) - A_8(e^{-m_3+m_{15}} - e^{m_3-m_{15}})$$

$$\times (e^{m_4} - W_T e^{-m_4}) - A_9(e^{-m_{10}+m_{15}} - e^{m_{10}-m_{15}})$$

$$\times (e^{m_9} - W_C e^{-m_9}) - A_{10}(e^{-m_9+m_{15}} - e^{m_9-m_{15}})\,(e^{m_{10}} - W_C e^{-m_{10}})],$$

$$A_{13} = -\frac{Gr}{(a_{12}m_6^2 - a_8 m_6 - a_{13})(e^{(m_5-m_6)} - e^{-(m_5-m_6)})},$$

$$A_{14} = \frac{Gr}{(a_{12}m_5^2 - a_8 m_5 - a_{13})(e^{(m_5-m_6)} - e^{-(m_5-m_6)})},$$

$$A_{15} = -\frac{Gm}{(a_{12}m_{12}^2 - a_8 m_{12} - a_{13})(e^{(m_{11}-m_{12})} - e^{-(m_{11}-m_{12})})},$$

$$A_{16} = \frac{Gm}{(a_{12}m_{11}^2 - a_8 m_{11} - a_{13})(e^{(m_{11}-m_{12})} - e^{-(m_{11}-m_{12})})},$$

$$A_{17} = -\frac{1}{(e^{(m_{17}-m_{18})} - e^{-(m_{17}-m_{18})})}$$

$$\times [(e^{m_{18}} + e^{-m_{18}}) - A_{13}(e^{-m_6+m_{18}} - e^{m_6-m_{18}})$$

$$\times (e^{m_5} - W_T e^{-m_5}) - A_{14}(e^{-m_5+m_{18}} - e^{m_5-m_{18}})$$

$$\times (e^{m_6} - W_T e^{-m_6}) - A_{15}(e^{-m_{12}+m_{18}} - e^{m_{12}-m_{18}})$$

$$\times (e^{m_{11}} - W_C e^{-m_{11}}) - A_{16}(e^{-m_{11}+m_{18}} - e^{m_{11}-m_{18}})\,(e^{m_{12}} - W_C e^{-m_{12}})],$$

$$A_{18} = \frac{1}{(e^{(m_{17}-m_{18})} - e^{-(m_{17}-m_{18})})} [(e^{m_{17}} + e^{-m_{17}}) - A_{13}(e^{-m_5+m_{17}} - e^{m_6-m_{17}})$$

$$\times (e^{m_5} - W_T e^{-m_5}) - A_{14}(e^{-m_5+m_{17}} - e^{m_5-m_{17}})$$

$$\times (e^{m_6} - W_T e^{-m_6}) - A_{15}(e^{-m_{12}+m_{17}} - e^{m_{12}-m_{17}})$$

$$\times (e^{m_{11}} - W_C e^{-m_{11}}) - A_{16}(e^{-m_{11}+m_{17}} - e^{m_{11}-m_{17}})\,(e^{m_{12}} - W_C e^{-m_{12}})].$$

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