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# BINARY-$T_{1/2}$ SPACES

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Recently the authors introduced the concept of binary topology between two sets and investigate its basic properties where a binary topology from $X$ to $Y$ is a binary structure satisfying certain axioms that are analogous to the axioms of topology. In this paper we introduce and study Binary-$T_{1/2}$ spaces.

**KEYWORDS :** Binary topology, Binary-$T_{1/2}$ spaces, generalized binary continuous functions, binary irresolute functions and generalized binary irresolute functions.

## INTRODUCTION

The authors [4] introduced the concept of binary topology and discussed some of its basic properties. The purpose of this paper is to introduce binary $T_{1/2}$ spaces and characterize its basic properties. Section 2 deals with basic concepts. Binary $T_{1/2}$ spaces in binary topological spaces are discussed in section 3. Section 4 deals with generalized binary continuous functions, binary irresolute functions and generalized binary irresolute functions in binary topological spaces. Throughout the paper, $\wp(X)$ denotes the power set of $X$.

## PRELIMINARIES

Let $X$ and $Y$ be any two nonempty sets. A binary topology [4] from $X$ to $Y$ is a binary structure $\mathscr{M} \subseteq \wp(X) \times \wp(Y)$ that satisfies the axioms namely (i) $(\varnothing, \varnothing)$ and $(X, Y) \in \mathscr{M}$, (ii) $(A_1 \cap A_2, B_1 \cap B_2) \in \mathscr{M}$ whenever $(A_1, B_1) \in \mathscr{M}$ and $(A_2, B_2) \in \mathscr{M}$, and (iii) If $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$ is a family of members of $\mathscr{M}$, then $\left(\bigcup_{\alpha\in\Delta} A_\alpha , \bigcup_{\alpha\in\Delta} B_\alpha\right) \in \mathscr{M}$. If $\mathscr{M}$ is a binary topology from $X$ to $Y$ then the triplet $(X, Y, \mathscr{M})$ is called a binary topological space and the members of $\mathscr{M}$ are called the binary open subsets of the binary topological space $(X, Y, \mathscr{M})$. The elements of $X \times Y$ are called the binary points of the binary topological space $(X, Y, \mathscr{M})$. If $Y = X$ then $\mathscr{M}$ is called a binary topology on $X$ in which case we write $(X, \mathscr{M})$ as a binary topological space. The examples of binary topological spaces are given in [4].

**Definition 2.1.[4]** Let $X$ and $Y$ be any two nonempty sets and let $(A, B)$ and $(C, D) \in \wp(X) \times \wp(Y)$. We say that $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.

**Definition 2.2.[4]** Let $(X, Y, \mathscr{M})$ be a binary topological space and $A \subseteq X$, $B \subseteq Y$. Then $(A, B)$ is called binary closed in $(X, Y, \mathscr{M})$ if $(X \backslash A, Y \backslash B) \in \mathscr{M}$.

**Proposition 2.3.** [4] Let $(X, Y, \mathscr{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$.

Let $(A, B)^{1*} = \cap \{A_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \cap \{B_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$. Then $((A, B)^{1*}, (A, B)^{2*})$ is binary closed and $(A, B) \subseteq ((A, B)^{1*}, (A, B)^{2*})$.

**Definition 2.4.** [4] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ is called the binary closure of $(A, B)$, denoted by $b\text{-}cl\,(A, B)$ in the binary space $(X, Y, \mathscr{M})$ where $(A, B) \subseteq (X, Y)$.

**Definition 2.5.** [6] Let $(X, Y, \mathscr{M})$ be a binary topological space. Let $(A, B) \subseteq (X, Y)$.

Define $\mathscr{M}_{(A, B)} = \{(A \cap U, B \cap V) : (U, V) \in \mathscr{M}\}$. Then $\mathscr{M}_{(A, B)}$ is a binary topology from $A$ to $B$. The binary topological space $(A, B, \mathscr{M}_{(A, B)})$ is called a binary sub-space of $(X, Y, \mathscr{M})$.

**Definition 2.6.** Let $X$ and $Y$ be any two nonempty sets and let $(A, B)$ and $(C, D) \in \wp(X) \times \wp(Y)$. We say that $(A, B) \not\subset (C, D)$ if one of the following holds :

(i) $A \subseteq C$ and $B \not\subset D$ (ii) $A \not\subset C$ and $B \subseteq D$ (iii) $A \not\subset C$ and $B \not\subset D$.

**Definition 2.7. [8]** Let $(X, Y, \mathscr{M})$ be a binary topological space. Let $(A, B) \in \wp(X) \times \wp(Y)$. Then $(A, B)$ is called generalized binary closed if $b\text{-}cl\,(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and $(U, V)$ is binary open in $(X, Y, \mathscr{M})$.

**Definition 2.8. [4]** Let $(X, Y, \mathscr{M})$ be a binary topological space and let $(Z, \tau)$ be a topological space. Let $f : Z \to X \times Y$ be a function. Then $f$ is called binary continuous if $f^{-1}(A, B)$ is open in $Z$ for every binary open set $(A, B)$ in $(X, Y, \mathscr{M})$.

**Definition 2.9. [6]** A binary topological space $(X, Y, \mathscr{M})$ is called a binary-$T_0$ if for any two distinct binary points $(x_1, y_1), (x_2, y_2) \in X \times Y$, there exists $(A, B) \in \mathscr{M}$ such that exactly one of the following holds.

- $(x_1, y_1) \in (A, B)$, $(x_2, y_2) \in (X \backslash A, Y \backslash B)$
- $(x_1, y_1) \in (X \backslash A, Y \backslash B)$, $(x_2, y_2) \in (A, B)$.

## BINARY-$T_{1/2}$ SPACES

In this section we introduce binary -$T_{1/2}$ spaces and study its basic properties. Now we start with the definition of binary-$T_{1/2}$ space.

**Definition 3.1.** A binary topological space $(X, Y, \mathscr{M})$ is called a binary-$T_{1/2}$ space if every generalized binary closed set is binary closed.

**Example 3.2.** Consider $X = \{a\}$, $Y = \{1\}$. Then $\wp(X) = \{\varnothing, X\}$ and $\wp(Y) = \{\varnothing, Y\}$.

Now, $\wp(X) \times \wp(Y) = \{(\varnothing, \varnothing), (\varnothing, Y), (X, \varnothing), (X, Y)\}$.

Consider $\mathscr{M} = \{(\varnothing, \varnothing), (X, \varnothing), (X, Y)\}$. Clearly $\mathscr{M}$ is a binary topology from $X$ to $Y$. Also the binary closed sets are $(\varnothing, \varnothing), (\varnothing, Y), (X, Y)$. Therefore, $(\varnothing, \varnothing), (\varnothing, Y), (X, Y)$ are generalized binary closed. Consider $(X, \varnothing)$. Clearly $(X, \varnothing) \subseteq (X, \varnothing)$ which is binary open.

Now, $b\text{-}cl\,(X, \varnothing) = (X, Y) \not\subset (X, \varnothing)$. Therefore $(X, \varnothing)$ is not generalized binary closed. This implies that every generalized binary closed set in $(X, Y, \mathscr{M})$ is binary closed. Hence, $(X, Y, \mathscr{M})$ is binary-$T_{1/2}$.

In a topological space, for each $x \in X$, either $\{x\}$ is closed or complement of $\{x\}$ is generalized closed. This result is proved by Dunham [3]. The analogous result is given in the following Proposition.

**Proposition 3.3.** Let $(X, Y, \mathcal{M})$ be a binary topological space. For each $(x, y) \in X \times Y$, either $(\{x\}, \{y\})$ is binary closed or $(X\backslash\{x\}, Y\backslash\{y\})$ is generalized binary closed.

**Proof.** Suppose $(\{x\}, \{y\})$ is not binary closed. Then $(X, Y)$ is the only binary open set which contains $(X\backslash\{x\}, Y\backslash\{y\})$. Thus $b\text{-}cl\ (X\backslash\{x\}, Y\backslash\{y\})$ is contained in each of its binary neighbourhoods and $(X\backslash\{x\}, Y\backslash\{y\})$ is generalized binary closed.

In [2] Dunham establishes the following characterization of $T_{1/2}$ spaces; A topological space $(X, \tau)$ is $T_{1/2}$ if and only if every singleton in $X$ is either open or closed. The analogous result is given in the following Proposition.

**Proposition 3.4.** Let $(X, Y, \mathcal{M})$ be a binary topological space. $(X, Y, \mathcal{M})$ is binary $T_{1/2}$ if and only if for each $(x, y) \in X \times Y$, $(\{x\}, \{y\})$ is either binary open or binary closed.

**Proof.** Assume that $(X, Y, \mathcal{M})$ is binary $T_{1/2}$. Let $(x, y) \in X \times Y$. Suppose $(\{x\}, \{y\})$ is not binary closed. Therefore, $(X, Y)$ is the only binary open set which contains $(X\backslash\{x\}, Y\backslash\{y\})$.

Hence, $b\text{- }cl\ (X\backslash\{x\}, Y\backslash\{y\}) \subseteq (X, Y)$. This shows that $(X\backslash\{x\}, Y\backslash\{y\})$ is generalized binary closed. Therefore, $(X\backslash\{x\}, Y\backslash\{y\})$ binary closed. Thus, $(\{x\}, \{y\})$ is binary open.

Conversely, assume that for each $(x, y) \in X \times Y$, $(\{x\}, \{y\})$ is either binary open or binary closed . We shall show that $(X, Y, \mathcal{M})$ is binary $T_{1/2}$. Let $(A, B) \in \wp(X) \times \wp\ (Y)$ be generalized binary closed. Let $(x, y) \in b\text{-}cl\ (A, \mathrm{B})$. If $(\{x\}, \{y\})$ is binary open, we have $(\varnothing, \varnothing) \neq (\{x\} \cap A, \{y\} \cap B)$. Otherwise $(\{x\}, \{y\})$ is binary closed and $(\varnothing, \varnothing) \neq (\{x\}^{1*} \cap A, \{y\}^{2*} \cap B) = (\{x\} \cap A, \{y\} \cap B)$. In either case $(x, y) \in (A, B)$ and so $(A, B)$ is binary closed.

**Proposition 3.5**. If $(X, Y, \mathcal{M})$ is binary $T_{1\backslash 2}$ and $(A, B, \mathcal{M}_{(A,\ B)})$ is a binary subspace of $(X, Y, \mathcal{M})$, then $(A, B, \mathcal{M}_{(A,\ B)})$ is binary $T_{1\backslash 2}$.

**Proof.** For $(a, b) \in (A\ B)$, $(\{a\}, \{b\})$ is either binary open in $(X, Y, \mathcal{M})$ or binary closed in $(X, Y, \mathcal{M})$ and thus $(\{a\}, \{b\})$ is either binary open in $(A, B, \mathcal{M}_{(A,\ B)})$ or binary closed in $(A, B, \mathcal{M}_{(A,\ B)})$. Hence, $(A, B, \mathcal{M}_{(A,\ B)})$ is binary $T_{1/2}$.

**Proposition 3.6.** Let $(X, Y, \mathcal{M})$ be a binary topological space. If $(X, Y, \mathcal{M})$ is binary $T_1$, then $(X,Y, \mathcal{M})$ binary-$T_{1\backslash 2}$.

**Proof.** Suppose $(A, B)$ is not binary closed in $(X, Y, \mathcal{M})$. Take $(x, y) \in ((A, B)^{1*}\backslash A, (A, B)^{2*}\backslash B)$.

Then $(\{x\}, \{y\}) \subseteq ((A, B)^{1*}\backslash A, (A, B)^{2*}\backslash B)$ and $(\{x\}, \{y\})$ is binary closed since we are in binary $T_1$ space. By Proposition 3.5 and 3.6[8], $(A, B)$ is not generalized binary closed in $(X, Y, \mathcal{M})$.

# Generalized binary continuous functions

**Definition 4.1.** Let $(X, Y, \mathcal{M})$ be a binary topological space. Let $(A, B) \in \wp\ (X) \times \wp\ (Y)$. Then $(A, B)$ is called generalized binary open if $(X\backslash A, Y\backslash B)$ is generalized binary closed in $(X, Y, \mathcal{M})$.

**Definition 4.2.** Let $(X, Y, \mathscr{M})$ be a binary topological space and let $(Z, \tau)$ be a topological space. Let $f : Z \to X \times Y$ be a function. Then f is called binary closed if $f(A)$ is binary closed whenever $A$ is closed.

**Proposition 4.3.** If $A$ is a generalized closed set in $Z$ and if $f : Z \to X \times Y$ is binary continuous and binary closed, then $f(A)$ is generalized binary closed.

**Proof .** Let $f(A) \subseteq (U, V)$ where $(U, V)$ is binary open in $(X, Y, \mathscr{M})$. We shall show that $b\text{-}cl\,(f(A)) \subseteq (U, V)$. Now, $f(A) \subseteq (U, V)$ implies $A \subseteq f^{-1}(U, V)$. Since $A$ is a generalized closed and $f^{-1}(U, V)$ is open in $Z$, we have $cl\,(A) \subseteq f^{-1}(U, V)$. Thus, $f(cl\,(A)) \subseteq (U, V)$. Since $f$ is binary closed and $cl\,(A)$ is a closed set, we have $f(cl\,(A))$ is binary closed. Now $b\text{-}cl\,(f(A)) \subseteq (b\text{-}cl\,(f(cl\,(A)) = f(cl\,(A)) \subseteq (U, V)$. Therefore, $b\text{-}cl\,(f(A)) \subseteq (U, V)$. Hence $f(A)$ is generalized binary closed.

**Definition 4.4.** Let $(Z, \tau)$ be a topological space and $(X, Y, \mathscr{M})$ be a binary topological space. Then the map $f : Z \to X \times Y$ is called generalized binary continuous if $f^{-1}(A, B)$ is generalized open in $Z$ for every binary open set $(A, B)$ in $(X, Y, \mathscr{M})$.

**Definition 4.5.** Let $(Z, \tau)$ be a topological space and $(X, Y, \mathscr{M})$ be a binary topological space. Then the map $f : Z \to X \times Y$ is called binary irresolute if $f^{-1}(A, B)$ is open in $Z$ for every generalized binary open set $(A, B)$ in $(X, Y, \mathscr{M})$.

**Definition 4.6.** Let $(Z, \tau)$ be a topological space and $(X, Y, \mathscr{M})$ be a binary topological space. Then the map $f : Z \to X \times Y$ is called generalized binary irresolute if $f^{-1}(A, B)$ is generalized open in $Z$ for every generalized binary open set $(A, B)$ in $(X, Y, \mathscr{M})$.

**Proposition 4.7.** Let $f : Z \to X \times Y$ be binary continuous. Then $f$ is generalized binary continuous.

**Proof.** Let $(A, B)$ be binary open in $(X, Y, \mathscr{M})$. Since f is binary continuous, we have $f^{-1}(A, B)$ is open in $Z$. Therefore, $Z \backslash f^{-1}(A, B)$ is closed in $Z$. Since every closed set in $Z$ is generalized closed, we have $Z \backslash f^{-1}(A, B)$ is generalized closed in $Z$. This implies $f^{-1}(A, B)$ is generalized open in $Z$. Hence, $f$ is generalized binary continuous.

The converse of the Proposition 4.7 need not be true as shown in Example 4.9.

**Example 4.8.** Consider $Z = \{a, b, c\}$, $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$. Let $\tau = \{\varnothing, Z, \{a\}\}$ and $\mathscr{M} = \{(\varnothing, \varnothing), (X, Y), (\{x_1\}, \{y_1\})\}$. Clearly $\tau$ is a topology on $Z$ and $\mathscr{M}$ is a binary topology from $X$ to $Y$. Define $f : Z \to X \times Y$ by $f(a) = (x_1, y_1)$ and $f(b) = (x_2, y_2) = f(c)$. Then $f$ is binary continuous. For, $f^{-1}(\varnothing, \varnothing) = \{z \in Z : f(z) \in (\varnothing, \varnothing)\} = \varnothing$, $f^{-1}(X, Y) = \{a, b, c\}$ and $f^{-1}(\{x_1\}, \{y_1\}) = \{a\}$. Thus inverse image of every binary open set is open in $Z$. Also $f$ is generalized binary continuous.

**Example 4. 9.** Consider $Z = \{a, b, c\}$, $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$. Let $\tau = \{\varnothing, Z, \{b, c\}\}$ and $\mathscr{M} = \{(\varnothing, \varnothing), (X, Y), (\{x_1\}, \{y_1\})\}$. Clearly $\tau$ is a topology on $Z$ and $\mathscr{M}$ is a binary topology from $X$ to $Y$. Define $f : Z \to X \times Y$ by $f(a) = (x_1, y_1)$ and $f(b) = (x_2, y_2) = f(c)$. Then $f$ is not binary continuous, since $f^{-1}(\{x_1\}, \{y_1\}) = \{a\}$ which is not open in $Z$.

**Example 4.10.** Consider $Z = \{a, b, c\}$, $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$. Let $\tau = \{\varnothing, Z, \{b\}, \{a, b\}\}$ and $\mathscr{M} = \{(\varnothing, \varnothing), (X, Y), (\{x_1\}, \{y_1\})\}$. Clearly $\tau$ is a topology on $Z$ and $\mathscr{M}$ is a binary topology from $X$ to $Y$. Define $f : Z \to X \times Y$ by $f(a) = (x_1, y_1)$ and $f(b) = (x_2, y_2) = f(c)$. Also the set of all closed sets on $Z$ is $\tau^c = \{\varnothing, Z, \{a, c\}, \{c\}\}$. Then $f$ is generalized binary continuous.

For, $f^{-1}(\varnothing, \varnothing) = \{z \in Z : f(z) \in (\varnothing, \varnothing)\} = \varnothing$ which is open in $Z$, hence $f^{-1}(\varnothing, \varnothing)$ is generalized open in $Z$. Also, $f^{-1}(X, Y) = \{a, b, c\} = Z$ which is open in $Z$, hence $f^{-1}(X, Y)$ is generalized open and $f^{-1}(x_1, y_1) = \{a\}$. Now, $\{a\}^c = \{b, c\} \subseteq Z$, $cl\{b, c\} = Z$. Therefore, $\{b, c\}$ is generalized closed set in $Z$. Hence, $\{a\}^c$ is generalized closed set in $Z$. This implies $\{a\}$ is generalized open in $Z$. Thus inverse image of every binary open set is generalized open in $Z$. But $f$ is not binary continuous, since $f^{-1}(x_1, y_1) = \{a\}$ is not open in $Z$.

**Proposition 4.11.** Let $f : Z \to X \times Y$ be generalized binary continuous. Then $f$ is binary continuous if $Z$ is $T_{1/2}$.

**Proof.** Let $(A, B)$ be binary open in $(X, Y, \mathscr{M})$. Since $f$ is generalized binary continuous, we have $f^{-1}(A, B)$ is generalized open in $Z$. Therefore, $Z \backslash f^{-1}(A, B)$ is generalized closed in $Z$. Since $Z$ is $T_{1/2}$, we have $Z \backslash f^{-1}(A, B)$ is closed in $Z$. This implies $f^{-1}(A, B)$ is open in $Z$. Hence, $f$ is binary continuous.

**Proposition 4.12.** Let $f : Z \to X \times Y$ be binary irresolute. Then $f$ is generalized binary irresolute.

**Proof.** Let $(A, B)$ be generalized binary open in $(X, Y, \mathscr{M})$. Since f is binary irresolute, we have $f^{-1}(A, B)$ is open in $Z$. Therefore, $f^{-1}(A, B)$ is generalized open in $Z$. Hence, $f$ is generalized binary irresolute.

The converse of the Proposition 4.12 need not be true which is shown in the following Example.

**Example 4.13.** Consider $Z = \{a, b, c\}$, $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$. Let $\tau = \{\varnothing, Z, \{b, c\}\}$ and $\mathscr{M} = \{(\varnothing, \varnothing), (X, Y), (\{x_1\}, \{y_2\})\}$. Clearly $\tau$ is a topology on $Z$ and $\mathscr{M}$ is a binary topology from $X$ to $Y$. Define $f : Z \to X \times Y$ by $f(a) = (x_1, y_1)$ and $f(b) = (x_2, y_2) = f(c)$.

$$\begin{aligned}\wp(X) \times \wp(Y) = \{&(\varnothing, \varnothing), (\varnothing, \{y_1\}), (\varnothing, \{y_2\}), (\varnothing, Y),\\ &(\{x_1\}, \varnothing), (\{x_1\}, \{y_1\}), (\{x_1\}, \{y_2\}), (\{x_1\}, Y),\\ &(\{x_2\}, \varnothing), (\{x_2\}, \{y_1\}), (\{x_2\}, \{y_2\}), (\{x_2\}, Y),\\ &(X, \varnothing), (X, \{y_1\}), (X, \{y_2\}), (X, Y)\}.\end{aligned}$$

The binary sets $(\{x_2\}, \{y_1\})$, $(\{x_2\}, Y)$ and $(X, \{y_1\})$ are not generalized binary open.

For, the complement of $(\{x_2\}, \{y_1\})$ is $(\{x_1\}, \{y_2\})$ and $b\text{-}cl(\{x_1\}, \{y_2\}) = (X, Y)$. $(\{x_1\}, \{y_2\}) \subseteq (\{x_1\}, \{y_2\})$ where $(\{x_1\}, \{y_2\})$ is binary open in $(X, Y, \mathscr{M})$, but $b\text{-}cl(\{x_1\}, \{y_2\}) = (X, Y) \not\subset (\{x_1\}, \{y_2\})$. Hence $(\{x_1\}, \{y_2\})$ is not generalized binary closed. Therefore, $(\{x_2\}, \{y_1\})$ is not generalized binary open.

Now, the complement of $(\{x_2\}, Y)$ is $(\{x_1\}, \varnothing)$ and $b\text{-}cl(\{x_1\}, \varnothing) = (X, Y)$.

Also, $(\{x_1\}, \varnothing) \subseteq (\{x_1\}, \{y_2\})$ where $(\{x_1\}, \{y_2\})$ is binary open in $(X, Y, \mathscr{M})$, but $b\text{-}cl(\{x_1\}, \varnothing) = (X, Y) \not\subset (\{x_1\}, \{y_2\})$. Hence $(\{x_1\}, \varnothing)$ is not generalized binary closed. Therefore, $(\{x_2\}, Y)$ is not generalized binary open. Now, the complement of $(X, \{y_1\})$ is $(\varnothing, \{y_2\})$ and $b\text{-}cl(\{\varnothing, \{y_2\}) = (X, Y)$. Also, $(\varnothing, \{y_2\}) \subseteq (\{x_1\}, \{y_2\})$ where $(\{x_1\}, \{y_2\})$ is binary open in $(X, Y, \mathscr{M})$, but $b\text{-}cl(\varnothing, \{y_2\}) = (X, Y) \not\subset (\{x_1\}, \{y_2\})$. Hence $(\varnothing, \{y_2\})$ is not generalized binary closed. Therefore, $(X, \{y_1\})$ is not generalized binary open.

Now, consider the binary set $(\{x_1\}, \{y_1\})$. The complement of $(\{x_1\}, \{y_1\})$ is $(\{x_2\}, \{y_2\})$ and $b\text{-}cl(\{x_2\}, \{y_2\}) = (X, Y)$. Also $(\{x_2\}, \{y_2\}) \subseteq (\{X, Y)$ where $(X, Y)$ is binary open in $(X, Y, \mathscr{M})$ and $b\text{-}cl(\{x_2\}, \{y_2\}) = (X, Y) \subseteq (X, Y)$. Hence $(\{x_2\}, \{y_2\})$ is generalized binary closed. Therefore, $(\{x_1\}, \{y_1\})$ is generalized binary open. Similarly, we can check the remaining binary sets are generalized binary open.

Now, $f^{-1}(\varnothing, \varnothing) = \{z \in Z : f(z) \in (\varnothing, \varnothing)\} = \varnothing$, $f^{-1}(\varnothing, \{y_1\}) = \varnothing$, $f^{-1}(\varnothing, \{y_2\}) = \varnothing$, $f^{-1}(\varnothing, Y) = \varnothing$, $f^{-1}(\{x_1\}, \varnothing) = \varnothing$, $f^{-1}(\{x_2\}, \varnothing) = \varnothing$, $f^{-1}(X, \varnothing) = \varnothing$, $f^{-1}(\{x_1\}, \{y_1\}) = \{a\}$, $f^{-1}(\{x_1\}, \{y_2\}) = \varnothing$, $f^{-1}(\{x_1\}, Y) = \{a\}$, $f^{-1}(\{x_2\}, \{y_2\}) = \{b, c\}$, $f^{-1}(X, \{y_2\}) = \{b, c\}$ and $f^{-1}(X, Y) = Z$. This show that inverse image of every generalized binary open set is generalized open in $Z$. Hence, $f$ is generalized binary irresolute. But $f$ is not binary irresolute, since the generalized open set $\{a\}$ is not open in $Z$.

**Example 4.14.** Consider $Z = \{a, b, c\}$, $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$. Let $\tau = \{\varnothing, Z, \{a\}\}$ and $\mathscr{M} = \{(\varnothing, \varnothing), (X, Y), (\{x_1\}, \{y_1\})\}$. Clearly $\tau$ is a topology on $Z$ and $\mathscr{M}$ is a binary topology from $X$ to $Y$. Define $f : Z \to X \times Y$ by $f(a) = (x_1, y_1)$ and $f(b) = (x_2, y_2) = f(c)$. The generalized binary open sets are $(\varnothing, \varnothing)$, $(\varnothing, \{y_1\})$, $(\varnothing, \{y_2\})$, $(\varnothing, Y)$, $(\{x_1\}, \varnothing)$, $(\{x_1\}, \{y_1\})$, $(\{x_1\}, \{y_2\})$, $(\{x_1\}, Y)$, $(\{x_2\}, \varnothing)$, $(\{x_2\}, \{y_1\})$, $(X, \varnothing)$, $(X, \{y_1\})$, $(X, Y)\}$. The binary sets $(\{x_2\}, \{y_2\})$, $(\{x_2\}, Y)$ and $(X, \{y_2\})$ are not generalized binary open.

Now, $f^{-1}(\varnothing, \varnothing) = \{z \in Z : f(z) \in (\varnothing, \varnothing)\} = \varnothing$, $f^{-1}(\varnothing, \{y_1\}) = \varnothing$, $f^{-1}(\varnothing, \{y_2\}) = \varnothing$, $f^{-1}(\varnothing, Y) = \varnothing$, $f^{-1}(\{x_1\}, \varnothing) = \varnothing$, $f^{-1}(\{x_2\}, \varnothing) = \varnothing$, $f^{-1}(X, \varnothing) = \varnothing$, $f^{-1}(\{x_1\}, \{y_1\}) = \{a\}$, $f^{-1}(\{x_1\}, \{y_2\}) = \varnothing$, $f^{-1}(\{x_1\}, Y) = \{a\}$, $f^{-1}(X, Y) = Z$, $f^{-1}(\{x_2\}, \{y_1\}) = \varnothing$, $f^{-1}(X, \{y_1\}) = \{a\}$. This show that every generalized binary open set is open in $Z$. Hence, $f$ is binary irresolute.

**Proposition 4.15.** Let $f : Z \to X \times Y$ be generalized binary irresolute. Then $f$ is binary irresolute if $Z$ is $T_{1/2}$.

**Proof.** Let $(A, B)$ be generalized binary open in $(X, Y, \mathscr{M})$. Since $f$ is generalized binary irresolute, we have $f^{-1}(A, B)$ is generalized open in $Z$. Therefore, $Z \backslash f^{-1}(A, B)$ is generalized closed in $Z$. Since $Z$ is $T_{1\backslash 2}$ we have $Z \backslash f^{-1}(A, B)$ is closed in $Z$. This implies $f^{-1}(A, B)$ is open in $Z$. Hence, $f$ is binary irresolute.

**Proposition 4.16.** Let $f : Z \to X \times Y$ be binary irresolute. Then $f$ is binary continuous.

**Proof.** Let $(A, B)$ be binary open in $(X, Y, \mathscr{M})$. Then $(A, B)$ is generalized binary open in $(X, Y, \mathscr{M})$. Since $f$ is binary irresolute, we have $f^{-1}(A, B)$ is open in $Z$. Hence, $f$ is binary continuous.

Converse of the above Proposition is not true which is shown in the following example.

**Example 4.17.** Consider $Z = \{a, b, c\}$, $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$. Let $\tau = \{\varnothing, Z, \{a\}, \{b\}, \{a, b\}\}$ and $\mathscr{M} = \{(\varnothing, \varnothing), (X, Y), (\{x_1\}, \{y_1\})\}$. Clearly $\tau$ is a topology on $Z$ and $\mathscr{M}$ is a binary topology from $X$ to $Y$. Define $f : Z \to X \times Y$ by $f(a) = (x_1, y_1)$ and $f(b) = (x_2, y_1)$ and $f(c) = (x_2, y_2)$. Then $f$ is binary continuous. For, $f^{-1}(\varnothing, \varnothing) = \{z \in Z : f(z) \in (\varnothing, \varnothing)\} = \varnothing$, $f^{-1}(X, Y) = \{a, b, c\} = Z$ and $f^{-1}(x_1, y_1) = \{a\}$. Thus inverse image of every binary open set is open in $Z$. The generalized binary open sets are $(\varnothing, \varnothing)$, $(\varnothing, \{y_1\})$, $(\varnothing, \{y_2\})$, $(\varnothing, Y)$, $(\{x_1\}, \varnothing)$, $(\{x_1\}, \{y_1\})$, $(\{x_1\}, \{y_2\})$, $(\{x_1\}, Y)$, $(\{x_2\}, \varnothing)$, $(\{x_2\}, \{y_1\})$, $(X, \varnothing)$, $(X, \{y_1\})$, $(X, Y)\}$.

Now, $f^{-1}(X, \{y_2\}) = \{z \in Z : f(z) \in (X, \{y_2\})\} = \{c\}$ which is not open in $Z$. Hence, $f$ is not binary irresolute.

**Proposition 4.18.** Let $f : Z \to X \times Y$ be generalized binary irresolute. Then $f$ is generalized binary continuous.

**Proof .** Let $(A, B)$ be binary open in $(X, Y, \mathscr{M})$. Then $(A, B)$ is generalized binary open in $(X, Y, \mathscr{M})$. Since $f$ is generalized binary irresolute, we have $f^{-1}(A, B)$ is generalized open in $Z$. Hence, $f$ is generalized binary continuous.

**Proposition 4.19.** Let $f : Z \rightarrow X \times Y$ be generalized binary continuous. Then $f$ is generalized binary irresolute if $(X, Y, \mathscr{M})$ is binary $T_{1/2}$.

**Proof.** Let $(A, B)$ be generalized binary open in $(X, Y, \mathscr{M})$. Since $(X, Y, \mathscr{M})$ is binary $T_{1/2,}$ we have $(A, B)$ is binary open in $(X, Y, \mathscr{M})$. Also, since $f$ is generalized binary continuous, we have $f^{-1}(A, B)$ is generalized open in $Z$. Hence, $f$ is generalized binary irresolute.

## Conclusion

Binary $T_{1/2}$ spaces are introduced and its basic properties are discussed. Also generalized continuous functions, irresolute functions and generalized irresolute functions in topological spaces are extended to binary topological spaces. Further the relations between these functions are discussed.

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