Acta Ciencia Indica, Vol. XLI M, No. 3, 261 (2015)

EFFECT ON OSCILLATORY FLOW OF NON-NEWTONIAN BLOOD (RIVLIN-ERICKSEN) FLOW THROUGH POROUS MEDIUM IN A STENOSIS ARTERY

VARUN KUMAR TANWAR, N.K. VARSHNEY

Deptt. of Mathematics, S. V. College, Aligarh

AND

RAJA AGARWAL

Deptt. of Mathematics, Accurate Institute of Management and Technology, Gr. Noida

RECEIVED : 23 June, 2015

REVISED : 3 August, 2015

In this investigation, A mathematical model of oscillatory flow of blood conducting Visco-elastic (Rivlin-Ericksen) fluid through porous medium with mild stenosis has been developed. Analytical expressions for velocity profile, volumetric flow rate, wall shear stress and resistive impedance have been obtained. The computational results are presented graphically. It is noticed that the axial velocity decreases as visco-elastic coefficient increases and axial velocity profile increases with increasing the pressure gradient.

KEYWORDS : Blood flow, Non-Newtonian fluid, Stenosed artery, Porous medium.

INTRODUCTION

The Blood flow is significantly altered and fluid dynamical factors play important roles as the stenosis continues to enlarge leading to the development of cardiovascular diseases such as heart attack etc. Due to stenosis in the human artery the flow of blood is disturbed and resistance to the flow become higher than that of normal one. Various mathematical models have been investigated by several researchers to explore the effect on oscillatory flow of non-Newtonian blood flow through porous medium in a stenosis artery. Womersley [23] discussed oscillatory motions of a viscous liquids in a thin-walled elastic tube. Barnes *et al* [3] studied on pulsatile flow of non-Newtonian liquids. Daly [6] considered a numerical study of pulsatile flow through stenosed canine femoral arteries. Back *et al* [4] studied pulsatile, viscous blood flow through diseased coronary arteries of man. Newman *et al* [14] studied the oscillatory flow numerically in a rigid tube with stenosis. Haldar and Ghosh [7] discussed effect of magnetic field on blood flow through an indented tube in the presence of erythrocytes. Rathod and Shrikanth [16] studied MHD flow of Rivlin-Ericksen fluid through an inclined channel. Bhardwaj and Kanodia [5] studied oscillatory arterial blood flow with mild stenosis. Singh and Mishra [18] studied the flow of visco-elastic fluid through porous medium under the influence of magnetic field. Jain and Sharma [9] discussed mathematical analysis of MHD flow of blood in very narrow capillaries. Rathod and Tanveer [15] discussed pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and magnetic field. Jain and Sharma [10] studied mathematical modeling of blood flow in a stenosed artery under MHD effect through porous medium. Sanyal and Biswas [17] analysed pulsatile motion of blood through an axisymmetric artery in presence of magnetic field. Mathur and Jain [11] analyzed pulsatile flow of blood through a stenosed tube : effect of periodic body acceleration and a magnetic field. Tripathi and Kumar [21] considered a study of oscillatory flow of blood through porous medium in a stenosed artery in the presence of magnetic field. Tanwar and Varshney [22] analysed magnetic field effect on Oscillatory Arterial blood flow with mild stenosis. Mishra and Singh [12] considered a study of oscillatory blood flow through porous medium in a stenosed artery. Agarwal and Varshney [1] considered the effect of magnetic field of pulsatile inclined two layered blood flow with periodic body acceleration. Mohan and Prashad [13] Considered MHD oscillatory flow of elastico viscous blood through porous medium in a stenosed artery. Agarwal and Varshney [2] analyzed Slip velocity effect on MHD oscillatory blood flow through stenosed artery. Guojie *et. al* [8] considered unsteady non-newtonian solver on unstructured grid for the simulation of blood flow. Sinha *et. al* [19] discussed Slip effect on pulsatile flow of blood through a stenosed arterial segment under periodic body acceleration. Singh *et. al* [20] considered MHD flow of blood through radially non-symmetric stenosed artery. In the present paper we consider the problem of Mishra *et. al*. [9] with Elastico-Viscous (Rivlin-Ericksen) fluid under the same conditions.

MATHEMATICAL MODEL

Let us consider the oscillatory blood flow through a uniform rigid circular tube in the presence of porous medium with mild stenosed. We considered the flow is axially symmetric, laminar and incompressible, where the flowing blood is modeled as a non-Newtonian (Rivlin-Ericksen) fluid. The geometry of the stenosis is given by

$$\frac{R(x)}{R_0} = 1 - \frac{\varepsilon}{2R_0}\left[1 + \cos\frac{\pi x}{d}\right] \qquad \text{... (1)}$$

where R_0 is the radius of the normal artery, $R\ (x)$ is the radius of the artery in the stenotic region, $2d$ is the length of stenosis and ε is the maximum height of the stenosis such that $\varepsilon / R_0 << 1$.

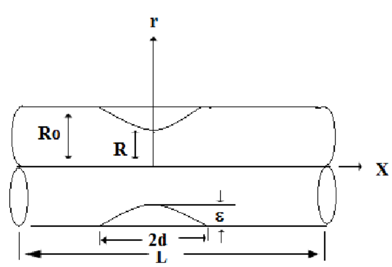

Fig. 1. Geometry of Stenosed Artery

The equation of motion governing the flow field in the tube is

$$\rho\frac{\partial w}{\partial t} = -\frac{\partial p}{\partial x} + \left(\mu + \lambda\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial t^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \frac{\mu}{k}w \qquad \text{... (2)}$$

where p is the fluid pressure, ρ is the density and w is the velocity in the axial direction, λ is Visco-elastic coefficient, μ *is* viscosity, k is the permeability of porous medium.

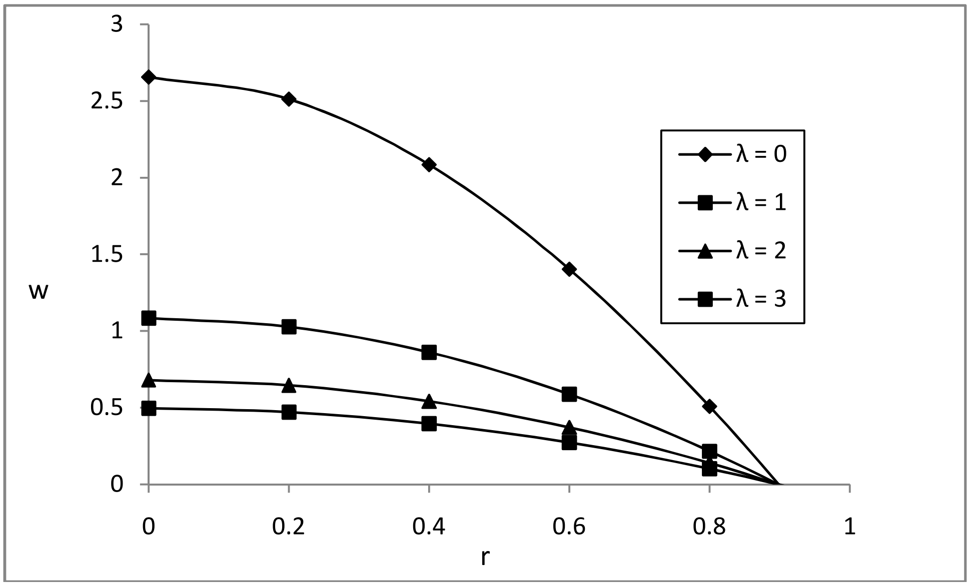

Fig. 1. Variation of axial velocity with radial distance for different values of visco-elastic coefficient with K = 100, H = 2, and t = 0.5, ε = 0.2.

The boundary conditions are provided by no-slip velocity at the wall and axially symmetry of the flow

$$w = 0 \text{ on } r = R \quad \text{... (3)}$$

$$\frac{\partial w}{\partial r} = 0 \text{ on } r = 0 \quad \text{...(4)}$$

Solution of the problem

The simple solution of the motion of a viscous fluid will be obtained in the section under pressure gradient which varies with time. Before proceeding with the solution, transformation is defined by $y = r / R_0$ is introduced.

The basic equation (2) becomes on using the boundary conditions (3) and (4).

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial y^2}+\frac{1}{y}\frac{\partial w}{\partial y}\right)-\frac{R_0^2}{\mu}\frac{\partial w}{\partial t}-\frac{wR_0^2}{k}=\frac{R_0^2}{\mu}\frac{\partial p}{\partial x} \quad \text{... (5)}$$

$$w = 0 \text{ on } y = \frac{R}{R_0}$$

$$\frac{\partial w}{\partial y} = 0 \text{ on } y = 0 \quad \text{... (6)}$$

Let the solution for P and W be set in the for

$$w(y,\, t) = W(y)\; e^{int}$$

$$-\frac{\partial p}{\partial x} = Pe^{int} \qquad \text{... (7)}$$

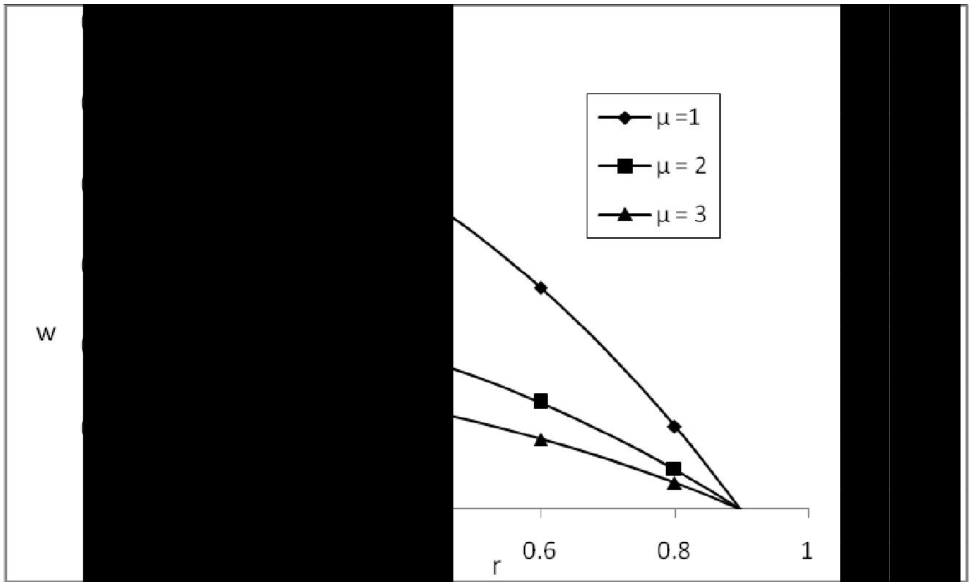

Fig. 2. Variation of axial velocity with radial distance for different values of viscosity with *K* = 100, *H* = 2, and *t* = 0.5, ε = 0.2.

Substituting (7) into equation (5) we get

$$\frac{d^2W}{dy^2} + \frac{1}{y}\frac{dW}{dy} - k_1^2 W = -\frac{R_0{}^2\, p}{\mu\left(1 + in\lambda\right)} \qquad \text{... (8)}$$

where

$$k_1{}^2 = \frac{\left[\dfrac{in\rho R_0^2}{\mu} + \dfrac{R_0^2}{k}\right]}{\left[1 + in\lambda\right]} \qquad \text{... (9)}$$

The solution of equation (8) subject to the boundary condition (6) is

$$W(y) \;=\; \frac{PR^2{}_0}{\mu[1 + in\lambda]k_1^2}\left[1 - \frac{J_0(\mathrm{k_1 y})}{J_0\left(\mathrm{k}_1\dfrac{R}{R_0}\right)}\right] \qquad \text{... (10)}$$

where J_0 is the Bessel function of order zero.

Then the resulting expression for the axial velocity in the tube is given by

$$w(r,t) = \frac{PR_0{}^2}{\mu[1 + in\lambda]k_1^2}\left[1 - \frac{J_0\left(k_1\dfrac{r}{R_0}\right)}{J_0\left(k_1\dfrac{R}{R_0}\right)}\right]e^{int} \qquad \text{... (11)}$$

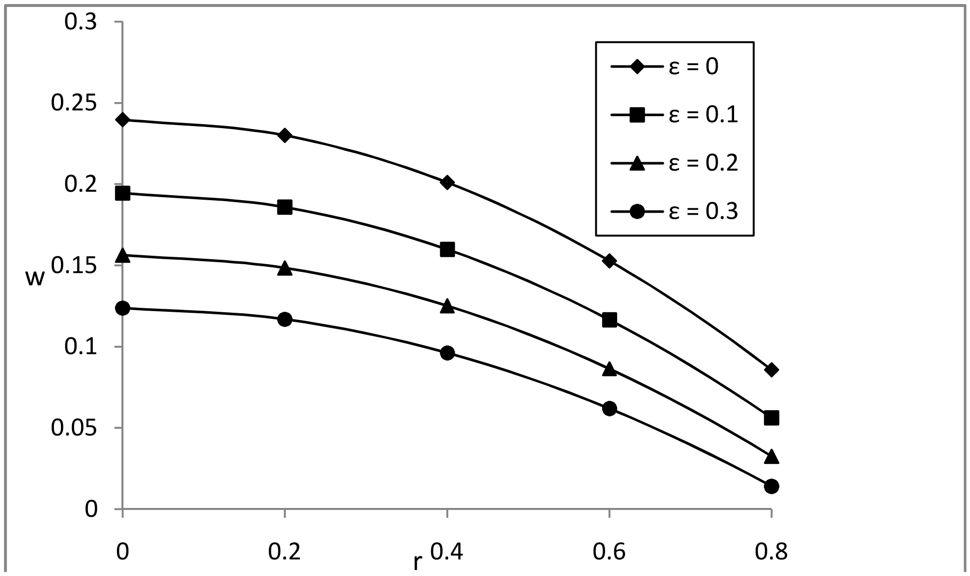

Fig. 3. Variation of axial velocity with radial distance for different values of stenotic height with $K = 100$, $R_0 = 1$, $H = 2$, $P = 4$ and $t = 0.5$.

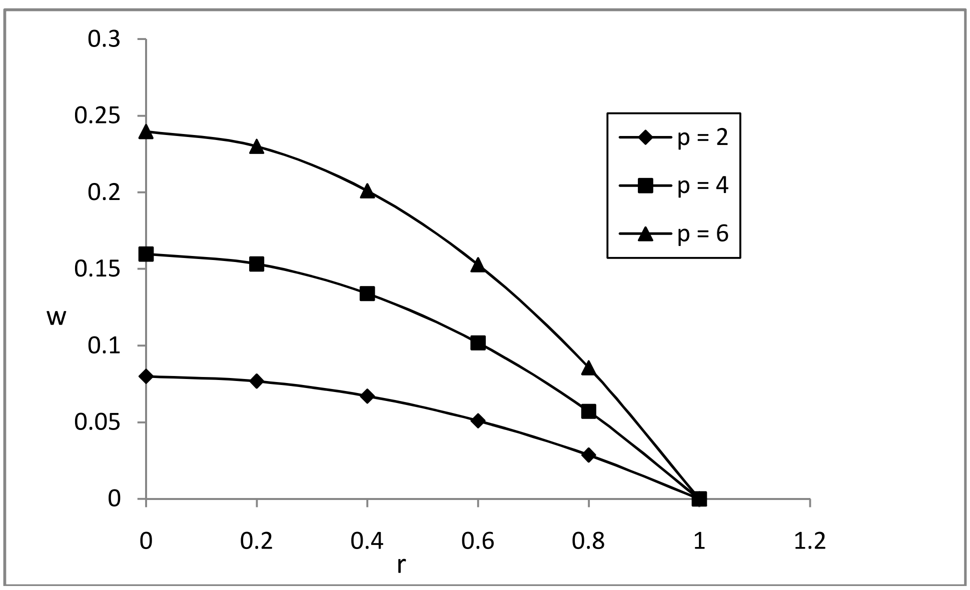

Fig. 4. Variation of axial velocity with radial distance for different values of pressure gradient with $K = 100$, $H = 2$, and $t = 0.5$, $\varepsilon = 0.2$.

The volumetric flow rate Q is given by

$$Q = 2\pi \int_0^R w\, r\, dr \qquad \text{...... (12)}$$

Which gives on integration,

$$Q = \frac{\pi P R_0{}^4}{\mu[1+in\lambda]k_1^2}\left(\frac{R}{R_0}\right)\left[\frac{R}{R_0} - \frac{2J_1\left(k_1\frac{R}{R_0}\right)}{k_1 J_0\left(k_1\frac{R}{R_0}\right)}\right]e^{int} \qquad \text{... (13)}$$

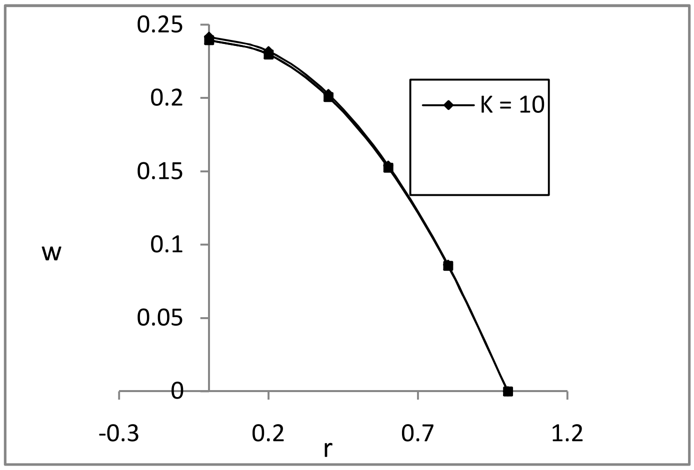

Fig. 5. Variation of axial velocity with radial distance for different values of permeability porous medium with $\mu = 3$, $H = 2$, and $t = 0.5$, $\varepsilon = 0.2$.

The shear stress at the wall $r = R$ is defined by

$$\tau_R = \mu\left(\frac{\partial w}{\partial r}\right)_{r=R} \qquad \text{... (14)}$$

$$\tau_R = \frac{PR_0}{[1+in\lambda]k_1}\left[\frac{J_1\left(k_1\frac{R}{R_0}\right)}{J_0\left(k_1\frac{R}{R_0}\right)}\right]e^{\text{int}} \qquad \text{... (15)}$$

Substituting expression (11) for w into above equation and using the relation (13) for Q, one obtains τ_R as

$$\frac{\tau_R}{Q} = \frac{\mu k_1^2 J_1\left(k_1\frac{R}{R_0}\right)}{\pi R_0{}^3\left[k_1\left(\frac{R}{R_0}\right)^2 J_0\left(k_1\frac{R}{R_0}\right) - 2\left(\frac{R}{R_0}\right)J_1\left(k_1\frac{R}{R_0}\right)\right]} \qquad \text{... (16)}$$

If τ_R is normalized with steady flow solution given by

$$\tau_N = -\frac{R_0}{2}\left(\frac{\partial p}{\partial x}\right)_{t=0} \qquad \text{... (17)}$$

Then the expression for wall shear stress is

$$\left|\bar{\tau}\right| = \left|\frac{\tau_R}{\tau_n}\right| = \frac{\dfrac{PR_0 e^{\text{int}}}{[1+in\lambda]k_1}\left[\dfrac{J_1\left(k_1 \dfrac{R}{R_0}\right)}{J_0\left(k_1 \dfrac{R}{R_0}\right)}\right]}{\dfrac{R_0}{2}\left(\dfrac{\partial p}{\partial x}\right)_{t=0}} \quad \ldots (18)$$

The resistance impedance to the flow is defined by

$$Z = -\left(\frac{\partial p / \partial x}{Q}\right) \quad \ldots (19)$$

$$Z = -\left(\frac{\partial p / \partial x}{Q}\right) = \frac{-[1+in\lambda]\mu k_1^2}{\pi R_0{}^4\left(\dfrac{R}{R_0}\right)}\left[\frac{R}{R_0} - \frac{2J_1\left(k_1 \dfrac{R}{R_0}\right)}{k_1 J_0\left(k_1 \dfrac{R}{R_0}\right)}\right]^{-1} \quad \ldots (20)$$

Deduction : If λ is taken as zero then results agree with Anil Tripathi and K.K. Singh (2012).

Numerical results and discussions

The present model has been developed to study the oscillatory flow of blood through a stenosed artery considering blood as to behave like a non-Newtonian fluid. Analytical expressions are obtained for axial velocity, flow rate, wall shear stress and resistive impedance. Since velocity profiles provide a detailed description of the flow field, it is of interest to study their pattern. A comparison of velocity profile using equation (11) for cases of visco-elastic coefficient in figure 1. It is observed that the axial velocity decreases with increasing the visco-elastic coefficient. It is notice that the axial velocity decreases with the increases of viscosity (Fig. 2). It can be noticed that the axial velocity decreases with increasing stenosis height at the axis of tube in stenotic region (Fig. 3). Figure 4 is plotted for velocity profile increases with increasing the pressure gradient. it can be notice that the axial velocity slightly decreases with increasing the permeability of porous medium (Fig. 5).

Conclusion

Rivlin-Ericksen fluid is one of special type fluid which can not be characterized by maxwell's constitutive relation and Oldroyd's constitutive relation. According to all the theoretical and experimental evidence we can consider oscillatory flow of blood (Rivlin-Ericksen) through a porous medium. which is assumed to be a Non-newtonian fluid. Here assumed that the surface roughness is cosine-shaped and the maximum height of the roughness is very small compared with the radius of the unconstricted tube. Numerical solutions are presented for the volumetric flow rate, wall shear stress and resistive impedence. it is clear that blood velocity is going to decreases as the radius of artery is going to increase it is observed that instantaneous flow rate is decrease as λ (visco-elastic cofficient) is increase. It appears that the non-Newtonian behaviour of the blood is helpful in the functioning of diseased arterial circulation.

References

1. Agarwal, R. and Varshney, N. K., The effect of magnetic field of pulsatile inclined two layered blood flow with periodic body acceleration, *International Journal of Mechanical Engineering,* Vol. **3 (3),** 187-196 (2013).
2. Agarwal, R., Varshney, N.K., Slip velocity effect on MHD oscillatory blood flow through stenosed artery, *Advances in Applied Science Research*, **5 (1),** 84-90 (2014).
3. Barnes, H.A., Townsend, P. and Walters, K., On pulsatile flow of non-Newtonian liquids, *Rheo. Acta*., **10**, 517 (1971).
4. Back, L.D., Radbill, J.R. and Crawford, D.W., Analysis of pulsatile, viscous blood flow through diseased coronary arteries of man, *J. Biomechanics,* **10**, 339 (1977).
5. Bhardwaj, Kavita and Kanodia, K.K., Oscillatory arterial blood flow with mild stenosis, *Acta Ciencia Indica*, XXXIII, No. **1**, 71 (2007).
6. Daly, B.J., A numerical study of pulsatile flow through stenosed canine femoral arteries, *J. Biomechanics,* **9**, 465 (1976).
7. Haldar, K. and Ghosh, S.N., Effect of magnetic field on blood flow through an indented tube in the presence of erythrocytes, *Indian. J. Pure Appl. Math.* **25(3)** 345-352 (1994).
8. Guojie Li, Bin Chen, Gaoling Zhou., Unsteady non-Newtonian Solver on Unstructured grid for the Simulation of Blood Flow, *Advances in Machanical Engineering*, 1-13 (2013).
9. Jain, M., Sharma, G.C. and Singh, A., Mathematical analysis of MHD flow of blood in very narrow capillaries, *IJE Transactions B Applications*, Vol. **22,** No. 3 , 307-315 (2009).
10. Jain, M., Sharma, G.C. and Singh, R., Mathematical modeling of blood flow in a stenosed artery under MHD effect through porous medium, *IJE Transactions B Applications*, Vol. **23**, No. 3 & 4, 243-251 (2010).
11. Mathur, P. and Jain, S., Pulsatile flow of blood through a stenosed tube : effect of periodic body acceleration and a magnetic field, *J. Biorheol* , Vol. **25**, **1(2),** pp 71-72, (2011).
12. Mishra, Gaurav, Kumar, Ravindra and Singh K.K., A study of oscillatory blood flow through porous medium in a stenosed artery, *Ultra Scientist*, Vol**. 24(2)** A, 369-373 (2012).
13. Mohan, V., Prashad V., Varshney, N.K., MHD Oscillatory Flow of elastico viscous Blood through Porous Medium in a Stenosed Artery, *IOSR Journal of Mathematics (IOSR-JM),* **7(3),** 68-71 (2013).
14. Newman, D.L., Westerhof, N. and Sipkema, P., The oscillatory flow numerically in a rigid tube with stenosis, J. Biomechanics, **12**, 229 (1979).
15. Rathod, V.P. and Tanveer, Shakera, Pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and magnetic field, Bull. Malays. Math. Sci. Soc., **32(2),** 245-259 (2009).
16. Rathod V. P. and Hosurker Shrikanth, G., MHD flow of Rivlin-ericksen fluid through an inclined channel, Bull. of Pure and Appl. Sci., **17E(1),** 125-134 (1998).
17. Sanyal, Dulal, Chandra and Biswas, Ananda., Pulsatile motion of blood through an axi-symmetric artery in presence of magnetic field, Assam Univ. Journal of Science and Technology , Physical Sciences and Technology, Vol. **5** Number II, 12-20 (2010).
18. Singh, K.K., Mishra G. and Sharma A.K., The flow of visco-elastic fluid through porous medium under the influence of magnetic field. Acta Ciencia Indica, Vol. XXXIV M No.**1**, p.-421 (2008).
19. Sinha A., Shit G.C., Kundu P.K.,Slip effect on pulsatile flow of blood through a stenosed arterial segment under periodic body acceleration , ISRN Biomedical Engineering , 1-10 ,**2013**(2013).
20. Singh A.K., and Singh D.P., MHD flow of blood through radially non-symmetric stenosed artery, International Journal of Engineering Transactions B: Applications, Vol. 26, No. 8, pp. 859- 864. (2013)
21. Tripathi A., Kumar R. and Singh K. K., A study of oscillatory flow of blood through porous medium in a stenosed artery in the presence of magnetic field. Ultra Scientist Vol. **24(3)** B, 439-444 (2012).
22. Tanwar V., Agarwal R. and Varshney N.K., Magnetic field effect on oscillatory arterial blood flow with mild stenosis, Applied Mathematical Sciences, Vol. **6**, No.120, 5959-5966 (2012).
23. Womersley, J.R., Oscillatory motions of a viscous liquids in a thin-walled elastic tube, Phil. Mag., **46**, 199 (1955).

❑