Acta Ciencia Indica, Vol. XLI M, No. 3, 249 (2015)

ON EDGE REGULAR INTERVAL-VALUED FUZZY GRAPHS

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RECEIVED : 30 May, 2015

In this paper, degree of an edge, total degree of an edge, edge regular interval-valued fuzzy graphs and totally edge regular interval-valued fuzzy graphs are introduced. A relation between edge regular and totally edge regular interval-valued fuzzy graph is studied. A necessary and sufficient condition under which they are equivalent is provided. Some properties of an edge regular interval-valued fuzzy graphs are studied and they are examined for totally edge regular interval-valued fuzzy graphs.

KEYWORDS : Degree of a vertex in interval-valued fuzzy graph, total degree, regular interval-valued fuzzy graph, totally regular interval-valued fuzzy graph, edge degree in fuzzy graph, total edge degree in fuzzy graph.

AMS subject classification : 05C12, 03E72, 05C72.

INTRODUCTION

In 1736, Euler first introduced the concept of graph theory. Graph theory is a very useful tool for solving combinatorial problems in different areas such as operations research, optimization, topology, geometry, number theory and computer science. Fuzzy set theory was first introduced by Zadeh in 1965. Fuzzy set theory has emerged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest. The first definition of fuzzy graph was introduced by Haufmann in 1973 based on Zadeh's fuzzy relations in 1971. In 1975, Rosenfeld introduced the concept of fuzzy graphs.

Now, fuzzy graphs have been witnessing a tremendous growth and finds application in many branches of engineering and technology. A. Nagoorgani and K. Radha introduced the concept of regular fuzzy graphs in 2008 [5]. K. Radha and N. Kumaravel introduced the concept of edge degree, total edge degree and discussed about the degree of an edge in some fuzzy graphs [8]. These motivates us to introduce an edge regular interval-valued fuzzy graphs and totally edge regular interval-valued fuzzy graphs and discussed some of its properties. Throughout this paper, the vertices take the membership value $A = (\mu_A^-, \mu_A^+)$ and edges take the membership value $B = (\mu_B^-, \mu_B^+)$.

Preliminaries

We present some known definitions related to fuzzy graphs and interval-valued fuzzy graphs for ready reference to go through the work presented in this paper.

Definition 2.1 : A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions (σ, μ), where $\sigma : V \to [0, 1]$ is a fuzzy subset of a non empty set V and $\mu : V X V \to [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V, the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ is satisfied.

Definition 2.2: An interval-valued fuzzy graph with an underlying set V is defined to be the pair (A, B), where $A = (\mu_A^-, \mu_A^+)$ is an interval-valued fuzzy set on V and $B = (\mu_B^-, \mu_B^+)$ is an interval-valued fuzzy set on E such that $\mu_B^-(x, y) \leq \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(x, y) \leq \max\{\mu_A^+(x), \mu_A^+(y)\}$, for all $(x, y) \in E$. Here, A is called interval-valued fuzzy vertex set on V and B is called interval-valued fuzzy edge set on E.

Definition 2.3 : The strength of connectedness between two vertices u and v is defined as $\mu^\infty(u, v) = \sup\{\mu^k(u, v) : k = 1, 2, ...\}$, where $\mu^k(u, v) = \sup\{\mu(u, u_1) \wedge \mu(u_1, u_2) \wedge ... \wedge \mu(u_{k-1}, v) : u, u_1, u_2, ..., u_{k-1}, v$ is a path connecting u and v of length $k\}$.

Definition 2.4 : Let $G : (A, B)$ be an interval-valued fuzzy graph . The positive degree of a vertex $u \in G$ is defined as $d^+(u) = \sum \mu_B^+(u, v)$, for $uv \in E$. The negative degree of a vertex $u \in G$ is defined as $d^-(u) = \sum \mu_B^-(u, v)$, for $uv \in E$ and $\mu_B^+(uv) = \mu_B^-(uv) = 0$ if uv not in E. The degree of a vertex u is defined as $d(u) = (d^-(u), d^+(u))$.

Definition 2.5 : Let $G : (A, B)$ be an interval-valued fuzzy graph, where $A = (\mu_A^-, \mu_A^+)$ and $B = (\mu_B^-, \mu_B^+)$ be two interval-valued fuzzy sets on a non empty set V. Then G is said to be regular interval-valued fuzzy graph if all the vertices of G has same degree (c_1, c_2).

Definition 2.6 : Let $G : (A, B)$ be an interval-valued fuzzy graph . The total degree of a vertex $u \in V$ is denoted by $td(u)$ and is defined as $td(u) = (td^-(u), td^+(u))$, where $td^+(u) = \sum\mu_B^+(u, v) + (\mu_A^+(u))$ and $td^-(u) = \sum\mu_B^-(u, v) + (\mu_A^-(u))$.

Definition 2.7 : Let $G : (A, B)$ be an interval-valued fuzzy graph, where $A = (\mu_A^-, \mu_A^+)$ and $B = (\mu_B^-, \mu_B^+)$ be two interval-valued fuzzy sets on a non empty set V. Then, G is said to be totally regular interval-valued fuzzy graph if all the vertices of G has same total degree (c_1, c_2).

Definition 2.8 : Let $G : (\sigma, \mu)$ be a fuzzy graph. The degree of an edge uv is defined as $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv)$.

Definition 2.9 : Let $G : (\sigma, \mu)$ be a fuzzy graph. The total degree of an edge uv is defined as $td_G(uv) = d_G(u) + d_G(v) - \mu(uv)$.

Definition 2.10 : Let $G : (\sigma, \mu)$ be a fuzzy graph. If the underlying graph G^* is regular then, G is said to be partially regular fuzzy graph.

Definition 2.11 : Let $G : (A, B)$ be an interval-valued fuzzy graph, where $A = (\mu_A^-, \mu_A^+)$ and $B = (\mu_B^-, \mu_B^+)$ be two interval-valued fuzzy sets on a non empty set V. The order of G is denoted by $O(G)$ and is defined as $O(G) = (O^-(G), O^+(G))$, where $O^+(G) = \sum\mu_A^+(u)$ and $O^-(G) = \sum\mu_A^-(u)$, for all $u \in V$.

Definition 2.12 : Let $G : (A, B)$ be an interval-valued fuzzy graph, where $A = (\mu_A^-, \mu_A^+)$ and $B = (\mu_B^-, \mu_B^+)$ be two interval-valued fuzzy sets on a non empty set V. The size of G is denoted by $S(G)$ and is defined as $S(G) = (S^-(G), S^+(G))$, where $S^+(G) = \sum\mu_B^+(uv)$ and $S^-(G) = \sum \mu_B^-(uv)$, for all $uv \in E$.

EDGE REGULAR INTERVAL-VALUED FUZZY GRAPHS

Definition 3.1: Let G : (A, B) be a interval-valued fuzzy graph on G^* : (V, E). The positive degree of an edge is defined as $d_G^+(uv) = d_G^+(u) + d_G^+(v) - 2\mu_B^+(uv)$. The negative degree of an edge is defined as $d_G^-(uv) = d_G^-(u) + d_G^-(v) - 2\mu_B^-(uv)$. The degree of an edge is defined as $d_G(uv) = (d_G^-(uv), d_G^+(uv))$.

The minimum degree of an edge is $\delta_E(G) = \wedge\{d_G(uv) : uv \in E\}$.

The maximum degree of an edge is $\Delta_E(G) = V\{d_G(uv) : uv \in E\}$

Example 3.2: Consider an interval-valued fuzzy graph on $G^*(V, E)$.

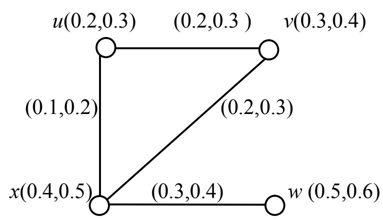

Fig. 1. Interval-valued fuzzy graph $G = (A, B)$

Here, $d(u) = (0.3, 0.5)$, $d(v) = (0.4, 0.6)$, $d(w) = (0.6, 0.9)$, $d(x) = (0.3, 04)$.

$$d_G^-(uv) = d_G^-(u) + d_G^-(v) - 2\mu_B^-(uv) = 0.3 + 0.4 - 2\ (0.2) = 0.3.$$
$$d_G^+(uv) = d_G^+(u) + d_G^+(v) - 2\mu_B^+(uv) = 0.5 + 0.6 - 2\ (0.3) = 0.5.$$
$$d_G(uv) = (0.3, 0.5).$$

$$d_G^-(vw) = d_G^-(v) + d_G^-(w) - 2\mu_B^-(vw) = 0.4 + 0.6 - 2\ (0.2) = 0.6.$$
$$d_G^+(vw) = d_G^+(v) + d_G^+(w) - 2\mu_B^+(vw) = -0.6 + 0.9 - 2\ (0.3) = 0.9.$$
$$d_G(vw) = (0.6, 0.9).$$

$$d_G^-(wx) = d_G^-(w) + d_G^-(x) - 2\mu_B^-(wx) = 0.6 + 0.3 - 2\ (0.3) = 0.3.$$
$$d_G^+(wx) = d_G^+(w) + d_G^+(x) - 2\mu_B^+(wx) = 0.9 + 0.4 - 2\ (0.4) = 0.5.$$
$$d_G(wx) = (0.3, 0.5).$$

$$d_G^-(uw) = d_G^-(u) + d_G^-(w) - 2\mu_B^-(uw) = 0.3 + 0.6 - 2\ (0.1) = 0.7.$$
$$d_G^+(uw) = d_G^+(u) + d_G^+(w) - 2\mu_B^+(uw) = 0.5 + 0.9 - 2\ (0.2) = 1.$$
$$d_G(uw) = (0.7, 1).$$

Definition 3.3 : Let G : (A, B) be a interval-valued fuzzy graph on $G^*(V, E)$. The total positive degree of an edge is defined as $td_G^+(uv) = d_G^+(u) + d_G^+(v) - \mu_B^+(uv)$. The total negative degree of an edge is defined as $td_G^-(uv) = d_G^-(u) + d_G^-(v) - \mu_B^-(uv)$. The total edge degree is defined as $td_G(uv) = (td_G^-(uv), td_G^+(uv))$. It can also be defined as

$$td_G(uv) = d_G(uv) + B(uv), \text{ where } B(uv) = (\mu_B^-(uv), \mu_B^+(uv)).$$

The minimum total degree of an edge is $\delta_{Te}(G) = \wedge\ \{td_G(uv) : uv \in E\}$.

The maximum total degree of an edge is $\Delta_{tE}(G) = V\{td_G(uv) : uv \in E\}$.

Example 3.4: Consider an interval-valued fuzzy graph on $G^*(V, E)$, a cycle of length 5.

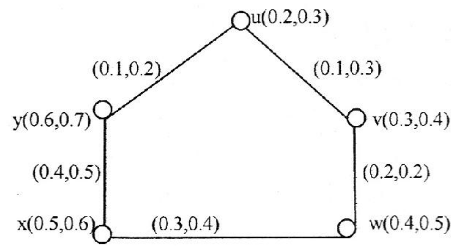

Fig. 2: Interval-valued fuzzy graph G = (A, B)

Here, $d_G(u) = (0.2, 0.5)$, $d_G(v) = (0.3, 0.5)$, $d_G(w) = (0.5, 0.6)$, $d_G(x) = (0.7, 0.9)$ and $d_G(y) = (0.5, 0.7)$

$$td_G^-(uv) = d_G^-(u) + d_G^-(v) - \mu_B^-(uv) = 0.2 + 0.3 - 0.1 = 0.4$$
$$td_G^+(uv) = d_G^+(u) + d_G^+(v) - \mu_B^+(uv) = 0.5 + 0.5 - (0.3) = 0.7$$
$$td_G(uv) = (0.4, 0.7).$$
$$td_G^-(vw) = d_G^-(v) + d_G^-(w) - \mu_B^-(vw) = 0.3 + 0.5 - 0.2 = 0.6$$
$$td_G^+(vw) = d_G^+(v) + d_G^+(w) - \mu_B^+(vw) = -0.5 + 0.6 - (0.2) = 0.9$$
$$td_G(vw) = (0.6, 0.9).$$
$$td_G^-(wx) = d_G^-(w) + d_G^-(x) - \mu_B^-(wx) = 0.5 + 0.7 - 0.3 = 0.9$$
$$td_G^+(wx) = d_G^+(w) + d_G^+(x) - \mu_B^+(wx) = 0.6 + 0.9 - (0.4) = 1.1$$
$$td_G(wx) = (0.9, 1.1).$$
$$td_G^-(xy) = d_G^-(x) + d_G^-(y) - \mu_B^-(xy) = 0.7 + 0.5 - 0.4 = 0.8$$
$$td_G^+(xy) = d_G^+(x) + d_G^+(y) - \mu_B^+(xy) = 0.9 + 0.7 - (0.5) = 1.1$$
$$td_G(xy) = (0.8, 1.1).$$
$$td_G^-(yu) = d_G^-(y) + d_G^-(u) - \mu_B^-(yu) = 0.5 + 0.2 - 0.1 = 0.6$$
$$td_G^+(yu) = d_G^+(y) + d_G^+(u) - \mu_B^+(yu) = 0.7 + 0.5 - (0.2) = 1$$
$$td_G(yu) = (0.6, 1).$$

Definition 3.5 : Let $G : (A, B)$ be a interval-valued fuzzy graph on $G^*(V, E)$. If each edge in G has the same degree (k_1, k_2), then G is said to be an (k_1, k_2)-edge regular interval-valued fuzzy graph.

Definition 3.6 : Let $G : (A, B)$ be a interval-valued fuzzy graph on $G^*(V, E)$. If each edge in G has the same total degree (k_1, k_2), then G is said to *e* totally (k_1, k_2) edge regular interval-valued fuzzy graph.

Remark 3.7:

1. G is (k_1, k_2)-edge regular interval-valued fuzzy graph if and only if $\delta_E(G) = \Delta_E(G) = (k_1, k_2)$.

2. G is totally (c_1, c_2)-edge regular interval-valued fuzzy graph if and only if $\delta_{tE}(G) = \Delta_{tE}(G) = (c_1, c_2)$.

Example 3.8 : A (k_1, k_2)-edge regular interval-valued fuzzy graph need not be totally (c_1, c_2)-edge regular interval-valued fuzzy graph.

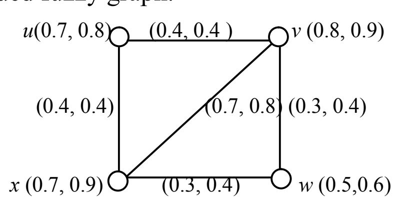

Fig. 3

Here, $d_G\,(uv) = (1.4,\ 1.6)$, for all $uv \in E$. Hence G is (1.4, 1.6)-edge regular interval-valued fuzzy graph. But G is not totally edge regular interval-valued fuzzy graph since, $td_G\,(uv) \neq td_G\,(vx)$.

Example 3.9 : A totally $(c_1,\ c_2)$-edge regular interval-valued fuzzy graph need not be $(k_1,\ k_2)$-edge regular interval-valued fuzzy graph.

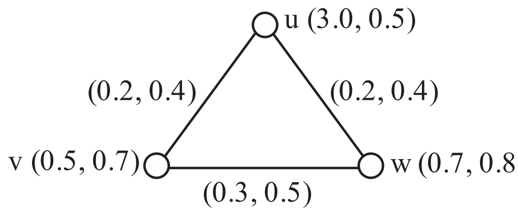

Fig. 4

Here, $td_G\,(uv) = (1.3,\ 0.7)$, for all $uv \in E$. Hence G is totally (1.3, 0.7)-regular interval-valued fuzzy graph. But G is not edge regular interval-valued fuzzy graph since, $d_G\,(uv) \neq d_G\,(vw)$.

Example 3.10: Consider an interval-valued fuzzy graph on $G^*\,(V, E)$.

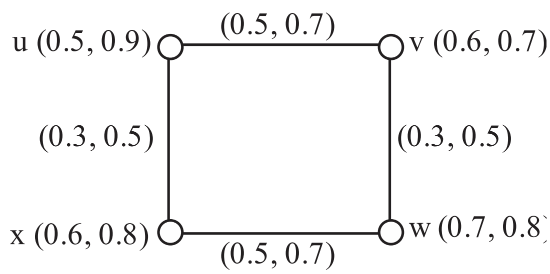

Fig. 5

Here G is neither an edge regular interval-valued fuzzy graph nor totally edge regular interval-valued fuzzy graph. But G is (0.8, 1.2)-regular interval-valued fuzzy graph.

Example 3.11: Consider an interval-valued fuzzy graph on $G^*(V, E)$.

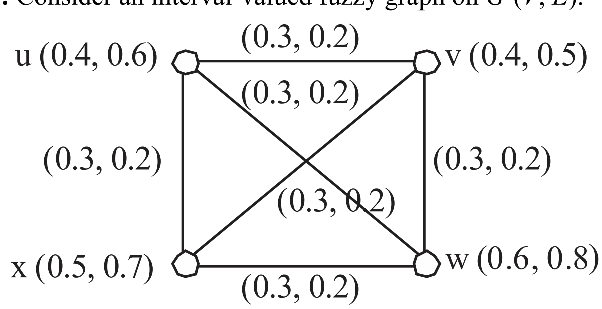

Fig. 6

Here, G is (1.2, 0.8)- edge regular interval-valued fuzzy graph and totally (1.5, 1)-edge regular interval-valued fuzzy graph. Also, it is (0.9, 0.6)-regular interval-valued fuzzy graph.

Example 3.12: Consider an interval-valued fuzzy graph on $G^*(V, E)$.

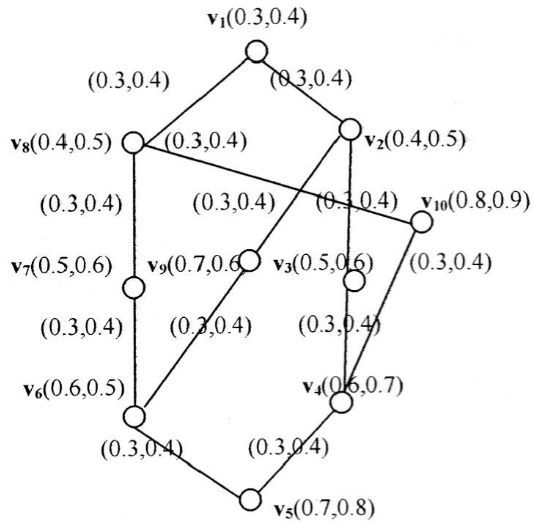

Fig. 7

Here, G is (0.9, 1.2)- edge regular interval-valued fuzzy graph and totally (1.2, 1.6)-edge regular interval- valued fuzzy graph. But G is not regular interval-valued fuzzy graph.

Theorem 3.13 : Let G be an interval-valued fuzzy graph on G^* (V, E). Then B is constant function if and only if the following are equivalent.

(i) G is an (k_1, k_2)-edge regular interval-valued fuzzy graph.

(ii) G is a totally $(k_1 + c_1, k_2 + c_2)$-edge regular interval-valued fuzzy graph.

Proof: Suppose that B is a constant function, let $B(uv) = (c_1, c_2)$ for all $uv \in E$. Assume that G is (k_1, k_2)-edge regular interval-valued fuzzy graph. Then $d_G(uv) = (k_1, k_2)$, for all $uv \in E$.

Now, $\quad td_G(uv) = d_G(uv) + B(uv)$, for all $uv \in E$

$\Rightarrow \quad td_G(uv) = (k_1, k_2) + (c_1, c_2)$, for all $uv \in E$

$\Rightarrow \quad td_G(uv) = (k_1 + c_1, k_2 + c_2)$, for all $uv \in E$.

Hence G is totally $(k_1 + c_1, k_2 + c_2)$-edge regular interval-valued fuzzy graph. Thus (i) $\rightarrow$ (ii) is proved.

Now, suppose G is totally $(k_1 + c_1, k_2 + c_2)$-edge regular interval-valued fuzzy graph. Then $td_G(uv) = (k_1 + c_1, k_2 + c_2)$, for all $uv \in E \Rightarrow d_G(uv) + B(uv) = (k_1 + c_1, k_2 + c_2)$, for all $uv \in E$ $\Rightarrow d_G(uv) = (k_1 + c_1, k_2 + c_2) - (c_1, c_2)$, for all $uv \in E \Rightarrow d_G(uv) = (k_1, k_2)$, for all $uv \in E$. So, G is (k_1, k_2)-edge regular interval-valued fuzzy graph. Thus (ii) $\rightarrow$ (i) is proved. Hence (i) and (ii) are equivalent.

Conversely, suppose (i) and (ii) are equivalent. Let G be *a* (k_1, k_2)-edge regular interval-valued fuzzy graph and totally $(k_1 + c_1, k_2 + c_2)$-edge regular interval-valued fuzzy graph. Suppose B is not constant function, then $B(uv) \neq B(xy)$ for atleast one pair of edges $uv, xy \in E$. Let G be (k_1, k_2)-edge regular interval-valued fuzzy graph. Then $d_G(uv) = d_G(xy) = (k_1, k_2)$ $\Rightarrow td_G(uv) = d_G(uv) + B(uv) = (k_1, k_2) + B(uv)$ and $td_G(xy) = d_G(xy) + B(xy) = (k_1, k_2) + B(xy)$. Since $B(uv) \neq B(xy)$, $td_G(uv) \neq td_G(xy)$. Hence G is not totally $(k_1 + c_1, k_2 + c_2)$-edge regular interval-valued fuzzy graph. Which is a contradiction. Now, let G be a totally $(k_1 + c_1, k_2 + c_2)$-edge regular interval-valued fuzzy graph. Then $td_G(uv) = td_G(xy) \Rightarrow d_G(uv) + B(uv)$

$= d_G(xy) + B(xy) \Rightarrow d_G(uv) - d_G(xy) = B(xy) - B(uv) \neq 0 \Rightarrow d_G(uv) \neq d_G(xy)$. Thus, G is not (k_1, k_2)-edge regular interval-valued fuzzy graph. Which is a contradiction. Hence, B is constant function.

Theorem 3.14 : Let G be an interval-valued fuzzy graph on $G^*(V, E)$. Then B is constant function if and only if the following are equivalent.

(i) G is (r_1, r_2)-regular interval-valued fuzzy graph.

(ii) G is (k_1, k_2)-edge regular interval-valued fuzzy graph.

(iii) G is a totally $(k_1 + c_1, k_2 + c_2)$-edge regular interval-valued fuzzy graph.

Proof : Proof similar to Theorem 3.13.

Theorem 3.15 : If an interval-valued fuzzy graph G is both edge regular and totally edge regular then B is constant function.

Proof : Let G be a (k_1, k_2)-edge regular and totally (k_3, k_4)- edge regular interval-valued fuzzy graph. Then $d_G(uv) = (k_1, k_2)$ and $td_G(uv) = (k_3, k_4)$, for all $uv \in E$. Now, $td_G(uv) = (k_3, k_4) \Rightarrow d_G(uv) + B(uv) = (k_3, k_4)$, for all $uv \in E \Rightarrow (k_1, k_2) + B(uv) = (k_3, k_4)$, for all $uv \in E$. $B(uv) = (k_3, k_4) - (k_1, k_2) = (k_3 - k_1, k_4 - k_2)$. Thus B is not constant function.

Remark 3.16 : The converse of Theorem 3.15 need not be true. Consider an interval-valued fuzzy graph G on $G^*(V, E)$.

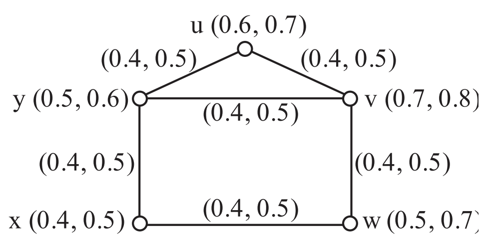

Fig. 8

Here, B is constant function. But G is neither an edge regular nor totally edge regular interval-valued fuzzy graph.

PROPERTIES OF EDGE REGULAR AND TOTALLY EDGE REGULAR INTERVAL-VALUED FUZZY GRAPH

Definition 4.1: Let G be an interval-valued fuzzy graph on $G^*(V, E)$. If the underlying graph G^* is regular, then G is said to be partially regular interval-valued fuzzy graph.

Theorem 4.2 : Let G be an interval-valued fuzzy graph on $G^*(V, E)$ such that B is constant function. Then G is regular interval-valued fuzzy graph if and only if G is partially regular interval-valued fuzzy graph.

Theorem 4.3 : Let G be an interval-valued fuzzy graph on $G^*(V, E)$. If G is both edge regular interval-valued fuzzy graph and totally edge regular interval-valued fuzzy graph, then G is regular interval-valued fuzzy graph if and only if G^* is regular graph.

Proof : Suppose G is both edge regular and totally edge regular interval-valued fuzzy graph then, B is constant function. Hence by Theorem 4.2, the result follows.

Remark 4.4: The converse of Theorem 4.3 need not be true. Consider an interval-valued fuzzy graph G on $G^*(V, E)$.

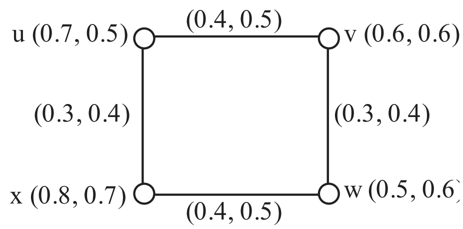

Fig. 9

Here, G is both regular and partially regular interval-valued fuzzy graph. But G is not edge regular and totally edge regular interval-valued fuzzy graph.

Theorem 4.5 : Let $G : (A, B)$ be a regular interval-valued fuzzy graph on $G^* (V, E)$. Then G is edge regular interval-valued fuzzy graph if and only if B is constant function.

Proof : Let $G : (A, B)$ be (k_1, k_2)-regular interval-valued fuzzy graph. Then $d_G(u) = (k_1, k_2)$, for all $u \in V$. Assume that B is constant function. Let $B(uv) = (c_1, c_2)$, for all $uv \in E$.

By definition, $d_G(uv) = d_G(u) + d_G(v) - 2B(uv) = (k_1, k_2) + (k_1, k_2) - 2(c_1, c_2) = (2k_1 - 2c_1, 2k_2 - 2c_2)$, for all $uv \in E$. Hence G is edge regular interval-valued fuzzy graph.

Conversely, assume that G is edge regular interval-valued fuzzy graph. Let $d_G(uv) = (r_1, r_2)$, for all $uv \in E$. By definition, $d_G(uv) = d_G(u) + d_G(v) - 2B(uv)$ $\Rightarrow (r_1, r_2) = (k_1, k_2) + (k_1, k_2) - 2B(uv) \Rightarrow 2B(uv) = (2k_1, 2k_2) - (r_1, r_2) = (2k_1 - r_1, 2k_2 - r_2)$ $\Rightarrow B(uv) = \left(\frac{2k_1 - r_1}{2}, \frac{2k_2 - r_2}{2}\right)$. Hence B is constant function.

Theorem 4.6 : Let G be an interval-valued fuzzy graph on $G^* (V, E)$ such that B is constant function. If G is regular interval-valued fuzzy graph, then G is (k_1, k_2)-edge regular interval-valued fuzzy graph.

Theorem 4.7 : Let G be an interval-valued fuzzy graph on $G^* (V, E)$ such that B is constant function. If G is regular interval-valued fuzzy graph, then G is totally edge regular interval-valued fuzzy graph.

Proof : Let B be constant function say $B(uv) = (c_1, c_2)$, for all $uv \in E$. Assume that G is regular interval-valued fuzzy graph. Then $d_G(u) = (k_1, k_2)$, for all $u \in V$. Now, $td_G(uv) = d_G(u) + d_G(v) - B(uv) = (k_1, k_2) + (k_1, k_2) - (c_1, c_2) = (2k_1, 2k_2) - (c_1, c_2) = (2k_1 - c_1, 2k_2 - c_2)$. Hence G is totally edge regular interval-valued fuzzy graph.

Remark 4.8 : The converse of Theorem 4.7 need not be true.

Theorem 4.9 : Let G be an interval-valued fuzzy graph on $G^* (V, E)$ with G^* is k-regular. Then B is constant function if and only if G is both regular and edge regular interval-valued fuzzy graph.

Theorem 4.10 : Let G be an interval-valued fuzzy graph on $G^* (V, E)$ with G^* is k-regular. Then B is constant function if and only if G is both regular and totally edge regular interval-valued fuzzy graph.

Proof : Let G be an interval-valued fuzzy graph on $G^* (V, E)$ and let G^* be a k-regular graph. Assume that B is constant function. Let $B(uv) = (c_1, c_2)$, for all $uv \in E$. Now, $d_G(u) = \sum B(uv)$, for all $u \in V \Rightarrow d_G(u) = \sum (c_1, c_2)$, for all $u \in V \Rightarrow d_G(u) = (c_1, c_2)\, d_{G^*}(u)$, for all $u \in V \Rightarrow d_G(u) = (c_1, c_2)\, k$, for all $u \in V$. Hence G is regular interval-valued fuzzy graph. Now, $td_G(uv) = \mu_{w \neq v} B(uw) + \mu_{w \neq u} B(wv) + B(uw)$, for all $uv \in E$.

$$= \mu_{w \neq v} (c_1, c_2) + \mu_{w \neq u} (c_1, c_2) + (c_1, c_2), \text{ for all } uv \in E.$$

$= (c_1, c_2)\,(d_G^{*}(u) - 1) + (c_1, c_2)\,(d_G^{*}(v) - 1) + (c_1, c_2)$, for all $uv \in E$.

$= (c_1, c_2)\,(k - 1) + (c_1, c_2)\,(k - 1) + (c_1, c_2)$, for all $uv \in E$.

$= 2\,(c_1, c_2)\,(k - 1) + (c_1, c_2)$, for all $uv \in E$.

Hence, G is totally edge regular interval-valued fuzzy graph.

Conversely, suppose G is both regular and totally edge regular interval-valued fuzzy graph. Since G is regular, $d_G(u) = (k_1, k_2)$, for all $u \in V$. Since G is totally edge regular, $td_G(uv) = (c_1, c_2)$, for all $uv \in E$. $td_G(uv) = d_G(u) + d_G(v) - B(uv)$, for all $uv \in E$. $\Rightarrow (c_1, c_2) = (k_1, k_2) + (k_1, k_2) - B(uv)$, for all $uv \in E$. $B(uv) = (2k_1, 2k_2) - (c_1, c_2)$, for all $uv \in E$. Hence B is constant function.

Definition 4.11 : Let $G^{*} : (V, E)$ be a graph. Then G^{*} is said to be an edge regular graph if each edge in G^{*} has same degree.

Theorem 4.12 : Let G be an interval-valued fuzzy graph on $G^{*}(V, E)$. If B is constant function, then G is an edge regular interval-valued fuzzy graph if and only if G^{*} is an edge regular graph.

Proof : Let G be an interval-valued fuzzy graph on $G^{*}(V, E)$. Assume that B is constant function. Let $B(uv) = (c_1, c_2)$ for all $uv \in E$. Assume that G is an edge regular interval-valued fuzzy graph. To prove G^{*} is an edge regular graph. Suppose G^{*} is not an edge regular graph, then $d_G(uv) \neq d_G(xy)$ for atleast one pair of edges $uv, xy \in E$. By definition,

$d_G(uv) = \mu_{w \neq v} B(uw) + \mu_{w \neq u} B(wv)$, for all $uv \in E$.

$= \mu_{w \neq v}(c_1, c_2) + \mu_{w \neq u}(c_1, c_2)$, for all $uv \in E$.

$= (c_1, c_2)\,(d_G^{*}(u) - 1) + (c_1, c_2)\,(d_G^{*}(v) - 1)$, for all $uv \in E$.

$= (c_1, c_2)\,(d_G^{*}(u) + d_G^{*}(v) - 2)$, for all $uv \in E$.

By definition of an edge degree in G^{*}, $d_G(uv) = c d_G^{*}(uv)$ for all $uv \in E$ and $d_G(xy) = c d_G^{*}(xy)$ for all $uv \in E$. Since $d_G^{*}(uv) \neq d_G^{*}(xy)$, $d_G(uv) \neq d(xy)$. Thus, G is not an edge regular interval-valued fuzzy graph. Which is a contradiction. Hence G^{*} is an edge regular graph.

Conversely, assume that B is constant function and G^{*} is an edge regular graph. To prove G is an edge regular interval-valued fuzzy graph. Suppose G is not an edge regular interval-valued fuzzy graph, then $d_G(uv) \neq d_G(xy)$ for atleast one pair of edges $uv, xy \in E$.

$$\mu_{w \neq v} B(uw) + \mu_{w \neq u} B(wv) \neq \mu_{z \neq y} B(xz) + \mu_{z \neq x} B(zy)$$

$$\mu_{w \neq v}(c_1, c_2) + \mu_{w \neq u}(c_1, c_2) \neq \mu_{z \neq y}(c_1, c_2) + \mu_{z \neq x}(c_1, c_2)$$

$$c\,(d_G^{*}(u) - 1) + c\,(d_G^{*}(v) - 1) \neq c\,(d_G^{*}(x) - 1) + c\,(d_G^{*}(y) - 1)$$

By definition of an edge degree in G^{*}, we have $c\,(d_G^{*}(uv)) \neq c\,(d_G^{*}(xy))$. So, $(d_G^{*}(uv)) \neq (d_G^{*}(xy))$. Thus G^{*} is not an edge regular graph. Which is a contradiction. Hence, G is an edge regular interval-valued fuzzy graph.

Remark 4.13 : The above Theorem 4.12 need not be true when B is constant function.

Theorem 4.14 : Let $G : (A, B)$ be a regular interval-valued fuzzy graph on $G^{*}(V, E)$. Then G is an edge regular interval-valued fuzzy graph if and only if B is a constant function.

Proof : Let G be (k_1, k_2)-regular interval-valued fuzzy graph. Then, $d_G(u) = (k_1, k_2)$, for all $u \in V$. Assume that B is constant function. So, $B(uv) = (c_1, c_2)$, for all $uv \in E$. To prove, G is an edge regular interval-valued fuzzy graph. By definition of an edge degree, $d_G(uv) = d_G(u) + d_G(v) - 2B(uv) = (k_1, k_2) + (k_1, k_2) - 2\,(c_1, c_2)$, for all $uv \in E$. So,

$d_G(uv) = (2k_1 - 2c_1, 2k_2 - 2c_2)$, for all $uv \in E$. Hence G is an edge regular interval-valued fuzzy graph.

Conversely, assume that G is an edge regular interval-valued fuzzy graph. Then, $d_G(uv) = (r_1, r_2)$, for all $uv \in E$. To prove, B is constant function. By definition of edge degree, $d_G(uv) = d_G(u) + d_G(v) - 2B(uv)$, for all $uv \in E$. So, $(r_1, r_2) = (k_1, k_2) + (k_1, k_2) - 2B(uv)$, for all $uv \in E$. Thus, $B(uv) = \left(\frac{2k_1 - r_1}{2}, \frac{2k_2 - r_2}{2}\right)$. Hence B is a constant function.

Definition 4.15 : If G is both an edge regular interval-valued fuzzy graph and partially edge regular interval-valued fuzzy graph, then G is said to be full edge regular interval-valued fuzzy graph.

Theorem 4.16 : Let $G : (A, B)$ be an interval-valued fuzzy graph on $G^*(V, E)$, such that B is constant function. If G is full regular interval-valued fuzzy graph, then G is full edge regular interval valued fuzzy graph.

Proof : Since B is constant function, let $B(uv) = (c_1, c_2)$, for all $uv \in E$. Assume that G is full regular interval-valued fuzzy graph. Then, $d_G(u) = (k_1, k_2)$ and $d_{G^*}(u) = r$, for all $u \in V$. So, $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2 = 2r - 2$. Hence G^* is an edge regular graph.

Now, $d_G(uv) = d_G(u) + d_G(v) - 2B(uv) = (k_1, k_2) + (k_1, k_2) - 2(c_1, c_2) = (2k_1, 2k_2) - (2c_1, 2c_2) = 2(k_1 - c_1, k_2 - c_2)$. Hence G is an edge regular interval-valued fuzzy graph. Thus, G is full edge regular interval-valued fuzzy graph.

Remark 4.17 : The converse of Theorem 4.16 need not be true. Consider an interval-valued fuzzy graph on $G^*(V, E)$.

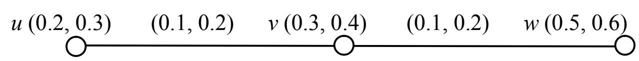

Fig. 10

Here, B is constant function and G is full edge regular interval-valued fuzzy graph.

But G is not full regular interval-valued fuzzy graph.

Theorem 4.18 : Let $G : (A, B)$ be an interval-valued fuzzy graph. Then $\sum td_G(uv) = \sum d_{G^*}(uv)\, B(uv) + S(G)$.

Proof : We have $td_G(uv) = d_G(uv) + B(uv) \Rightarrow \sum td_G(uv) = \sum d_G(uv) + \sum B(uv) = \sum d_{G^*}(uv)\, B(uv) + \sum B(uv)$. Hence $\sum td_G(uv) = \sum d_{G^*}(uv)\, B(uv) + S(G)$.

Theorem 4.19 : The size of (k_1, k_2)- edge regular interval-valued fuzzy graph and c edge regular graph is $\frac{q(k_1, k_2)}{c}$, where $q = |E|$.

Theorem 4.20: Let $G : (A, B)$ be a (r_1, r_2)-totally edge regular interval-valued fuzzy graph and k-partially edge regular interval-valued fuzzy graph. Then $S(G) = \frac{q(r_1, r_2)}{k+1}$, where $q = |E|$.

Proof: Since G is (r_1, r_2)-edge regular interval-valued fuzzy graph and G^* is k-edge regular graph, $td(uv) = (r_1, r_2)$ and $d_{G^*}(uv) = k$, for all $uv \in E$. But $\sum td(uv) = \sum d(uv) + \sum B(uv) = \sum d_{G^*}(\underline{uv})\, B(uv) + S(G)$. $\Rightarrow q(r_1, r_2) = kS(G) + S(G) = S(G)(k+1)$ $\Rightarrow S(G) = \frac{q(r_1, r_2)}{k+1}$.

Theorem 4.21 : If G is (k_1, k_2)-edge regular interval-valued fuzzy graph and totally (r_1, r_2) edge regular interval-valued fuzzy graph then, $S(G) = q(r_1 - k_1, r_2 - k_2)$.

Proof : Let G be a (k_1, k_2)-edge regular interval-valued fuzzy graph and totally (r_1, r_2)-edge regular interval-valued fuzzy graph. Then, $d_G(uv) = (k_1, k_2)$ and $td_G(uv) = (r_1, r_2)$, for all $uv \in E$.

$\sum d_G(uv) = q(k_1, k_2)$ and $\sum td_G(uv) = q(r_1, r_2)$, for all $uv \in E$. Since $td_G(uv) = d_G(uv) + B(uv) \Rightarrow \sum td_G(uv) = \sum d_G(uv) + \sum B(uv) \Rightarrow q(r_1, r_2) = q(k_1, k_2) + S(G) \Rightarrow S(G) = q(r_1, r_2) - q(k_1, k_2) = q(r_1 - k_1, r_2 - k_2)$.

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