Acta Ciencia Indica, Vol. XLI M, No. 2, 131 (2015)

EFFECT OF THE VELOCITY SECOND SLIP BOUNDARY CONDITION ON THE PERISTALTIC PUMPING OF COUPLE-STRESS FLUID FLOW WITH THERMAL PROPERTIES IN AN ASYMMETRIC CHANNEL

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RECEIVED : 20 May, 2015

The heat transfer characteristics of a couple-stress fluid in an asymmetric channel in the presence of the second order slip boundary condition were investigated in this paper. The channel asymmetry is produced by peristaltic wave train on the walls to have different amplitudes and phase. The system governing current flow was found as a set of non-linear PDE, which are solved and the analytical expression for the axial velocity, stream function, pressure gradient and pressure rise are established using long wavelength and low Reynolds number assumptions. The effect of second slip parameter on the present physical parameters was discussed through graphs and the importance of this type of slip is discussed in detail.

KEYWORDS : Couple-stress fluid, peristaltic transport, asymmetric channel, second order slip boundary condition, thermal properties

INTRODUCTION

Peristalsis is well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. Peristalsis is found in the swallowing food through esophagus, transport of urine from kidney to the bladder, vasomotion of the small blood vessels and in many other glandular ducts. The first investigation of Latham [1] is in fluid motion in a peristaltic pump. Various experimental and theoretical studies have been conducted to understand peristaltic flow in asymmetric channel [2-7]. Different models have been proposed to explain the behavior of non-Newtonian fluids. The couple-stress fluid is a special case of a non-Newtonian fluid, which is intended to take into account of the particle size effects. K. S. Mekheimer *et. al*. [8] studied the peristaltic transport of a couple stress fluid in an annulus. Srivastava L. M. [9] investigated peristaltic flow of a couple stress fluid.

T. Hayat *et al*. [10] discussed the peristaltic flow of a nanofluid with slip effects. Y.V.K. Ravi Kumar *et. al*. [11] discussed peristaltic transport of power law fluid in an asymmetric channel. T. Fang *et. al*. [12] studied viscous flow over a shrinking sheet with a second order slip flow model. Y.V.K. Ravi Kumar *et. al*. [13] investigated slip effects on hyperbolic tangent fluid in an inclined asymmetric channel. M. M. Nandeppanavar *et. al*. [14] investigated second order slip flow and heat transfer over a stretching sheet with non-linear Navier boundary condition. Y.V.K. Ravi Kumar *et. al* [15] discussed slip and magnetic field effect on a couple stress fluid in an inclined asymmetric channel. The main feature of previous studies is that less focus was on the second slip effect, while this effect has been recently discussed by many authors. M. Turkyilmazoglu [16] studied heat and mass transfer of MHD second order slip flow. A. V. Rosca [17] investigated the flow and heat transfer over a vertical permeable stretching shrinking sheet with a second order slip. Y. Abd Elmaboud *et al*. [18] investigated thermal properties of couple stress fluid in an asymmetric channel with peristalsis. Emad H. Aly *et. al*. [19] studied the effect of the velocity second slip boundary condition the peristaltic flow of nanofluids in an asymmetric channel.

In view of these works done by various researchers, we propose to study the effect of second slip on the peristaltic pumping of couple stress fluid in an asymmetric channel with thermal properties. Solution is obtained by considering suitable boundary condition governing the flow. Expressions for velocity, stream function, axial pressure gradient and pressure rise are shown. The phenomena of pumping and trapping are also discussed in detail.

Formulation of the problem

2.1 Governing equations

We consider the peristaltic motion of a couple stress fluid confined in a two dimensional infinite asymmetric channel of width $d_1 + d_2$. A rectangular co-ordinate system (*X*, *Y*) is chosen such that *X*-axis lies along the centre line of the channel in the direction of wave propagation and *Y*-axis transverse to it. The asymmetric in the channel is induced by assuming the peristaltic wave train on the walls to have different amplitudes and phases. The wall deformation is given by

$$H_1(\bar{X},\bar{t}) = h_1 = d_1 + a_1 \sin\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right], \text{upper wall} \quad \dots (1)$$

$$H_2(\bar{X},\bar{t}) = h_2 = -d_2 - a_2 \sin\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) + \phi\right], \text{ lower wall} \quad \dots (2)$$

where a_1, a_2 are the amplitudes of the waves, λ is the wavelength, the phase difference ϕ varies in the range $0 \le \phi \le \pi$. $\phi = 0$ corresponds to asymmetric channel with waves out of phase and $\phi = \pi$ the waves are in phase, and further a_1, a_2, d_1, d_2 and ϕ satisfies the condition $a_1^2 + a_2^2 + 2a_1 a_2 \cos\varphi \le (d_1 + d_2)^2$. The walls, $Y = H_1$ and $Y = H_2$ are maintained at temperatures T_1 and T_2 respectively.

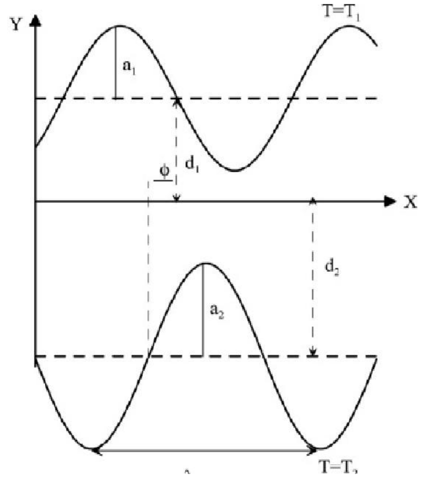

Fig. 1. Schematic diagram of the problem

The governing equations of the flow of an incompressible couple stress fluid in the absence of body force and body couple and the energy equations are

$$\bar{\nabla}.\bar{q} = 0 \qquad \dots (3)$$

$$\rho\frac{D\bar{q}}{Dt} = -\bar{\nabla}\bar{p} - \mu\bar{\nabla}\times\bar{\nabla}\times\bar{q} - \eta\bar{\nabla}\times\bar{\nabla}\times\bar{\nabla}\times\bar{q} \qquad \dots (4)$$

$$\rho\xi\frac{D\bar{T}}{Dt} = k\bar{\nabla}^2\bar{T} + \mu[(\bar{\nabla}\bar{q}):(\bar{\nabla}\bar{q})^T + (\bar{\nabla}\bar{q}):(\bar{\nabla}\bar{q})] + 4\eta[(\bar{\nabla}\bar{w}):(\bar{\nabla}\bar{w})^T] + 4\eta'[(\bar{\nabla}\bar{w}):(\bar{\nabla}\bar{w})] \qquad \dots (5)$$

where ρ is the density, $\boldsymbol{q}$ is the velocity vector, P is the fluid pressure, μ is the fluid viscosity, η and η' are the couple stress fluid parameters, k is the thermal conductivity, ξ is the specific heat at constant temperature, $\boldsymbol{w}$ is the rotation vector, and T is the temperature.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x = \bar{X} - ct,\ y = \bar{Y},\ u = \bar{U} - c,\ v = \bar{V}\ \ p(x) = P(X, t) \qquad \dots (6)$$

where (u, v) are the velocity components in the wave frame (x, y), p and P are pressures in wave and fixed frame respectively. The pressure p remains a constant across any axial station of the channel under the assumption that the wavelength is large and the curvature effects are negligible.

After using these transformations, the equations of motion are

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0, \qquad \dots (7)$$

$$\rho\left(\bar{u}\frac{\partial}{\partial\bar{x}} + \bar{v}\frac{\partial}{\partial\bar{y}}\right)\bar{u} = -\frac{\partial\bar{p}}{\partial\bar{x}} + \mu\left(\frac{\partial^2}{\partial\bar{x}^2} + \frac{\partial^2}{\partial\bar{y}^2}\right)\bar{u} - \eta\left(\frac{\partial^4}{\partial\bar{x}^4} + \frac{\partial^4}{\partial\bar{y}^4} + 2\frac{\partial^4}{\partial\bar{x}^2\partial\bar{y}^2}\right)\bar{u} \qquad \dots (8)$$

$$\rho\left(\bar{u}\frac{\partial}{\partial\bar{x}} + \bar{v}\frac{\partial}{\partial\bar{y}}\right)\bar{v} = -\frac{\partial\bar{p}}{\partial\bar{y}} + \mu\left(\frac{\partial^2}{\partial\bar{x}^2} + \frac{\partial^2}{\partial\bar{y}^2}\right)\bar{v} - \eta\left(\frac{\partial^4}{\partial\bar{x}^4} + \frac{\partial^4}{\partial\bar{y}^4} + 2\frac{\partial^4}{\partial\bar{x}^2\partial\bar{y}^2}\right)\bar{v} \qquad \dots (9)$$

The energy equation

$$\rho\xi\left(\bar{u}\frac{\partial}{\partial\bar{x}}+\bar{v}\frac{\partial}{\partial\bar{y}}\right)\bar{T}=k\left(\frac{\partial^2}{\partial\bar{x}^2}+\frac{\partial^2}{\partial\bar{y}^2}\right)\bar{T}+\mu\left[2\left\{\left(\frac{\partial\bar{u}}{\partial\bar{x}}\right)^2+\left(\frac{\partial\bar{v}}{\partial\bar{x}}\right)^2\right\}+\left(\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{x}}\right)^2\right]$$

$$+\eta\left[\left(\frac{\partial^2\bar{v}}{\partial\bar{x}^2}+\frac{\partial^2\bar{v}}{\partial\bar{y}^2}\right)^2+\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right)^2\right] \quad \ldots (10)$$

Consider the following non dimensional variables and parameters:

$$x=\frac{2\pi\bar{x}}{\lambda},\ y=\frac{\bar{y}}{d_1},\ u=\frac{\bar{u}}{c},\ v=\frac{\bar{v}}{\delta c},\ h_1=\frac{\bar{h}_1}{d_1},\ h_2=\frac{\bar{h}_2}{d_2},\ S=\frac{d_1}{\mu c}\bar{S}(\bar{x}),\ \delta=\frac{2\pi d_1}{\lambda},$$

$$\mathrm{Re}=\frac{\rho c d_1}{\mu},\ t=\frac{2\pi\bar{t}}{\lambda},\ \ p=\frac{d_1^{\,2}}{\lambda\mu c}\bar{p}(\bar{x}), \quad \ldots (11)$$

where T_s is the fluid temperature in static condition, Re is the Reynolds number, δ is the dimensionless wave number, P_r is the Prandtel number, E_c is the Eckert number and α is the couple stress fluid parameter indicating the ratio of the channel width (constant) to material characteristic length.

After non-dimensionalization of the Eqs. (7) - (10), we get

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad \ldots (12)$$

$$\delta\,\mathrm{Re}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)u=-\frac{\partial p}{\partial x}+\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{1}{\alpha^2}\left(\delta^4\frac{\partial^4 u}{\partial x^4}+\frac{\partial^4 u}{\partial x^4}+2\delta^2\frac{\partial^4 u}{\partial x^2\partial y^2}\right) \quad \ldots(13)$$

$$\delta^3\,\mathrm{Re}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v=-\frac{\partial p}{\partial y}+\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}-\frac{1}{\alpha^2}\left(\delta^4\frac{\partial^4 v}{\partial x^4}+\frac{\partial^4 v}{\partial x^4}+2\delta^2\frac{\partial^4 v}{\partial x^2\partial y^2}\right) \quad \ldots(14)$$

$$P_r\delta\,\mathrm{Re}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\theta=\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+P_rE_c\left\{2\delta^2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)^2\right.$$

$$\left.+\frac{1}{\alpha^2}\left[\delta^2\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)^2+\left(\frac{\partial^2 u}{\partial y^2}+\delta^2\frac{\partial^2 u}{\partial x^2}\right)^2\right]\right\} \quad \ldots (15)$$

Under lubrication approach (negligible inertia $\mathrm{Re}\to 0$ and long wavelength $\delta < 1$), Writing eqs. (12) – (15) in terms of the stream function $\psi(x,y)$ defined by $u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}$ we get

$$0=-\frac{\partial p}{\partial x}+\frac{\partial^3\psi}{\partial y^3}-\frac{1}{\alpha^2}\frac{\partial^5\psi}{\partial y^5} \quad \ldots (16)$$

$$0 = -\frac{\partial p}{\partial x} \qquad \dots (17)$$

$$0 = \frac{\partial^2 \theta}{\partial y^2} + P_r E_c \left\{ \left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 + \frac{1}{\alpha^2} \left(\frac{\partial^3 \psi}{\partial y^3} \right)^2 \right\} \qquad \dots (18)$$

The dimensionless boundary conditions in the wave frame are

$$\psi = \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -\beta_1 \frac{\partial^2 \psi}{\partial y^2} - \beta_2 \frac{\partial^3 \psi}{\partial y^3} - 1 \quad \text{at} \quad y = h_1 \qquad \dots (19)$$

$$\psi = -\frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = \beta_1 \frac{\partial^2 \psi}{\partial y^2} + \beta_2 \frac{\partial^3 \psi}{\partial y^3} - 1 \quad \text{at} \quad y = h_2 \qquad \dots (20)$$

where β_1, β_2 represent the first order slip parameter, second order slip parameter respectively.

Solution of the problem

The solution of Eqn. (17), (18), (19) with the boundary conditions (19), (20) is given by

$$u = \alpha \cosh \alpha y + \alpha A_1 (\sinh \alpha y) - 2 y A_5 + A_7 + P\left(\frac{y^2}{2} - 2 y A_6 - A_8 + A_0 \alpha \sinh \alpha y - 1 \right) \qquad \dots (21)$$

$$\Psi = \frac{F}{2} + \sinh \alpha y + A_1 \cosh \alpha y - A_5 y^2 - A_7 y - A_{10} + P\left(\frac{y^3}{6} - A_6 y^2 - A_8 y - A_{11} + A_0 \cosh \alpha y - A_{12} - y \right) \qquad \dots (22)$$

$$\theta = \frac{1}{4}(\cosh(2\alpha y)(-P_r\ E_c)(\alpha^2 + (1 + A_1)^2)) + \frac{A_1 \alpha^3 \sinh 2\alpha y}{2} - 4 A_1 A_5 \cos 2\alpha y - 4 A_5 \sinh \alpha y$$
$$+ 2 y^2 A_5{}^2 + \frac{A_{25} + y A_{27}}{h_1 - h_2 + 2\gamma} + P^2 \left(\frac{A_0{}^2\, \alpha^2 \cosh 2\alpha y}{4} + 2 A_0 \cosh 2\alpha y (y - 2 A_6 \alpha^2) - \frac{6 A_0 \sinh \alpha y}{\alpha} \right.$$
$$\left. - \frac{y^2}{12} - \frac{2 A_6 y^3}{3} + 4 A_6{}^2 + \frac{y^2}{2\alpha^2} + \frac{y A_{26} - A_{28}}{h_1 - h_2 + 2\gamma} \right) \qquad \dots (23)$$

where

$$I_1 = \cos[\alpha h_1] - \cosh[\alpha h_2]$$

$$I_2 = \alpha\ \sinh[\alpha h_1] + \beta_1\, \alpha^2\ \cosh[\alpha h_1] + \beta_1\, \alpha^3\ \sinh[\alpha h_1]$$

$$I_3 = \sin[\alpha h_1] - \sinh[\alpha h_2]$$

$$I_4 = \alpha \cosh[\alpha h_1] + \beta_1\, \alpha^2\ \sinh[\alpha h_1] + \beta_1\, \alpha^3\ \cosh[\alpha h_1]$$

$$I_5 = \cos[\alpha h_1] + \cosh[\alpha h_2]$$

$$I_6 = \sin\left[\alpha h_1\right] - \sinh\left[\alpha h_2\right]$$

$$I_7 = \frac{(-I_3(h_1-h_2) - 2I_1 + 2I_2 - \beta_1\alpha^2(\cosh[\alpha h_1] + \cosh[\alpha h_2]) - \beta_2(\alpha^3)(\sinh[\alpha h_1] + \sinh[\alpha h_2]))}{(2I_3 - 2I_4 + \alpha I_1(h_1-h_2) + \beta_1\alpha\left(\sinh\left[\alpha h_1\right] + \sinh\left[\alpha h_2\right]\right) + \beta_2(\alpha^3)(\cosh[\alpha h_1] + \cosh[\alpha h_2]))}$$

$$A_0 = \frac{\dfrac{\left(h_1-h_2\right)^2}{6} + \beta_1(h_1-h_2) + \dfrac{2F}{h_1-h_2} + 2}{\left(2I_3 - 2I_4 + \alpha I_1\left(h_1-h_2\right) + \beta_1\alpha\left(\sinh\left[\alpha h_1\right] + \sinh\left[\alpha h_2\right]\right) + \beta_2\alpha^3\left(\cosh\left[\alpha h_1\right] + \cosh\left[\alpha h_2\right]\right)\right)}$$

$$A_1 = I_7\left(\alpha I_3 + \beta_1\alpha^2 I_5 + \beta_2\alpha^3 I_6\right)$$

$$A_2 = \alpha I_1 + \beta_1\alpha^2 I_6 + \beta_2\alpha^3 I_5$$

$$\left(\left(\alpha I_3 + \beta_1\alpha^2 I_5 + \beta_2\alpha^3 I_6\right) + \frac{\left(h_1^2 - h_2^2\right)}{2} + \beta_1\left(h_1+h_2\right) + 2\beta_2\right)$$

$$A_4 = \left(2h_1 - 2h_2 + 4\beta_1\right),\ A_5 = \frac{A_1 + A_2}{A_4},\ A_6 = \frac{A_3}{A_4}$$

$$A_7 = \left(I_4 + I_7 I_2 - 2h_1 + \beta_1\right)A_5$$

$$A_8 = \left(\frac{h_1^2}{2} + \beta_1 h_1 + \beta_2 - 2A_6\left(h_1 + \beta_1\right) + A_0 I_2\right)$$

$$A_{10} = \left(\sinh\left[\alpha h_1\right] + I_7\cosh\left[\alpha h_1\right] - A_5 h_1^2 - A_7 h_1\right)$$

$$A_{11} = \frac{h_1^3}{6} - A_6 h_1^2 - A_8 h_1 + A_0\cos\left[\alpha h_1\right],\ A_{12} = \left(-h_1\right)$$

$$A_{13} = \frac{\left(-\Pr E_c\alpha^2\right) + \left(1+I_7\right)^2}{4}(\cosh[2\alpha h_1]) + \left(\frac{I_7(\alpha)^3}{2}\left(\sinh\left[2\alpha h_1\right]\right)\right) - \left(\left(4I_7A_5\right)\left(\cosh\left[2\alpha h_1\right]\right)\right)$$
$$-4A_5(\sinh[\alpha h_1]) + (2A_5^2 h_1^2)$$

$$A_{14} = \frac{A_0^2\alpha^2}{4}(\cosh[2\alpha h_1]) + 2A_0(h_1 - 2A_6\alpha^2)(\cosh[2\alpha h_1]) - \frac{6A_0(\sinh[\alpha h_1])}{\alpha} - \frac{h_1^2}{12}$$
$$-\frac{2A_6h_1^3}{3} + 4A_6^2 + \frac{h_1^2}{2\alpha^2}$$

$$A_{15} = -\Pr E_c\left[\frac{\alpha^3 + \left(1+I_7\right)^2}{2}\sinh[2\alpha h_1] + I_7\left(\alpha\right)^4\cosh[2\alpha h_1] - 4I_7A_5\alpha\sinh[2\alpha h_1]\right.$$
$$\left.-4A_5\alpha\cosh[\alpha h_1] + 4A_5^2 h_1\right]$$

$$A_{16}=\frac{A_0{}^2\alpha^3}{2}\cosh[2\alpha h_1]+2A_0\alpha(h_1-2A_6\alpha^2)(\sinh[2\alpha h_1])-\frac{6A_0\sinh[\alpha h_1])}{\alpha}$$
$$-\frac{h_1{}^2}{12}-\frac{2A_6h_1{}^3}{3}+4A_6{}^2+\frac{h_1{}^2}{2(\alpha)^2}$$

$$A_{17}=A_{13}+\gamma A_{15},\ \ A_{18}=A_{14}+\gamma A_{16}$$

$$A_{19}=-\Pr E_c\left[\left(\frac{\alpha^2+(1+I_7)^2}{4}\right)\left(\cosh\left[2\alpha h_2\right]\right)+\frac{I_7\alpha^3}{2}(\sinh[2\alpha h_2])-4I_7A_5(\cosh[2\alpha h_2])\right.$$
$$\left.-4A_5(\sinh[\alpha h_2])+2A_5{}^2h_2{}^2\right]$$

$$A_{20}=\frac{A_0{}^2\alpha^2}{4}(\cosh[2\alpha h_2]))+2A_0(h_2-2A_6\alpha^2(\cosh[2\alpha h_2])-\frac{6A_0(\mathrm{cinh}[\alpha h_2])}{\alpha}-\frac{h_2{}^2}{12}$$
$$-\frac{2A_6h_2{}^3}{3}+4A_6{}^2+\frac{h_2{}^2}{2\alpha^2}$$

$$A_{21}=-\Pr E_c\left[\frac{\alpha^3+(1+I_7)^2}{2}\sinh[2\alpha h_2]+I_7(\alpha)^4\cosh[2\alpha h_2]-4I_7\mathrm{A}_5\alpha\sinh[2\alpha h_2]\right.$$
$$\left.-4A_5\alpha\cosh[\alpha h_2])+4A_5{}^2h_2\right]$$

$$A_{22}=\frac{A_0{}^2\alpha^3}{2}\cosh[2\alpha h_2]+2A_0\alpha(h_2-2A_6\alpha^2)\sinh[2\alpha h_2]-\frac{6A_0\left(\sinh\left[\alpha h_2\right]\right)}{\alpha}$$
$$-\frac{h_2{}^2}{12}-\frac{2A_6h_2{}^3}{3}+4A_6{}^2\ \ +\frac{h_2{}^2}{2\alpha^2}$$

$$A_{23}=A_{19}-\gamma A_{21},\quad A_{24}=A_{20}-\gamma A_{22},\ A_{25}=\left(-A_{17}-1+A_{22}\right),\ \ A_{26}=\left(A_{24}-A_{18}\right)$$
$$A_{27}=A_{17}\left(h_1-h_2+2\gamma\right)-\left(h_1+\gamma\right)A_{25},\ \ A_{28}=A_{18}(h_1-h_2+2\gamma)+(h_1+\gamma)A_{26}$$

The non-dimensional expression for pressure rise per wavelength is defined as

$$\Delta P=\int_0^{2\pi}\frac{dp}{dx}dx$$

3.1 Rate of volume flow and boundary conditions

The dimensional rate of fluid flow in the fixed frame $\left(\bar{X},\bar{Y}\right)$ is

$$Q=\int_{\bar{h}_1(\bar{X})}^{\bar{h}_2(\bar{X})}\bar{U}(\bar{X},\bar{Y},t)\,d\bar{Y} \qquad \dots (24)$$

In the wave frame $\left(\bar{x},\bar{y}\right)$ eq. (19) reduces to

$$q = \int_{\bar{h}_1(\bar{X})}^{\bar{h}_2(\bar{X})} \bar{u}(\bar{x}, \bar{y}) d\bar{y} \quad \ldots (25)$$

By eq. (15), the above rates are related in the following expression

$$Q = q + c\bar{h}_1 - c\bar{h}_2 \quad \ldots (26)$$

Applying the averaged flow

$$Q = \frac{1}{T}\int_0^T Q dt \quad \ldots (27)$$

over a period T at a fixed position $\bar{X}$, we receive

$$\bar{Q} = q + c\bar{d}_1 + c\bar{d}_2 \quad \ldots (28)$$

with the definition of the dimensionless time averaged flows

$$\theta \equiv \frac{\bar{Q}}{c\bar{d}_1}, \quad F \equiv \frac{q}{c\bar{d}_1} \quad \ldots (29)$$

in the fixed and moving frames, respectively, we can write eq. (25) as

$$\theta = F + 1 + d, \quad \ldots (30)$$

where $$F = \int_{h_1(x)}^{h_2(x)} \frac{\partial \psi}{\partial y} dy = \psi(h_1) - \psi(h_2) \quad \ldots (31)$$

NUMERICAL RESULTS AND DISCUSSION

4.1. Trapping : Another interesting phenomenon in peristaltic motion is trapping. In the wave frame, streamlines under certain conditions split to trap bolus which moves as a whole with the speed of the wave. Figure 2, shows that there is no bolus for $b = 0$ (when there is no wave on the lower wall) and a bolus is observed with an increase in b. Figure 3, is made in order to see the effects of β_2 on trapping. No bolus is observed for second order slip $\beta_2 = 0$. A trapped bolus is observed for $\beta_2 = 1.5$.

4.2. The Pressure rise Δp : In Figure 4, the pressure rise decreases with increase in *a.* In Figure 5, the pressure rise increases with increase in ϕ in pumping region, free pumping region $(\Delta p \geq 0)$, and pressure rise decreases with increase in ϕ in co-pumping region $(\Delta p < 0)$. In Figure 6, the pressure rise increases with increase in α in pumping region, free pumping region $(\Delta p \geq 0)$, and pressure rise decreases with increase in α in co-pumping region $(\Delta p < 0)$. In Figure 7, the pressure rise increases with increase in β_1 in all three regions.

4.3. The Pressure Gradient dp/dx : In figures 8, 9 we observe the variation of $\frac{dp}{dx}$ with Q the pressure gradient increases with increase in a, β_1 in $\Delta p \geq 0$ pumping region and pressure gradient decreases with increase with increase in a, β_1 in $\Delta p < 0$ (co-pumping region).

Figures 10, 11 portrait graphs for $\frac{dp}{dx}$ vs Q for parameters β_2 and α. we observe that, pressure gradient decreases with increase in parameters β_2, α in $\Delta p \geq 0$ and the pressure gradient increases with increase in β_2, α in $\Delta p < 0$.

4.4 The Temperature θ : In Figure 12, the variation of θ vs y for different values of 'a' is observed. The temperature increases with increase in 'a' in the upper portion where as the effect is negligible in the middle portion. In Figure 13 the variation of θ vs y for different values of 'ϕ' is observed. The temperature increases with increase in 'ϕ' in the upper portion and the effect is negligible in the middle portion. In Figure 14, depicts the graph for θ vs y for different values of β_2, it is observed that the temperature increases with the increases of negativity of β_2 and it is negligible in the middle portion. In Figure 15, we see the variation of θ vs y for different values of β_1, it can be observed that, the temperature increases with the increases in β_1 for $\theta > 0$ and it reverses for $\theta < 0$.

4.5. The Velocity u (y) : To study the behaviour of the distributions of the axial velocity u, numerical calculations for several values of β_1, β_2, φ and α are carried out from Figure 16-19 represents graphs for different parameters of interest. It is observed that the velocity increases with increase in parameter values.

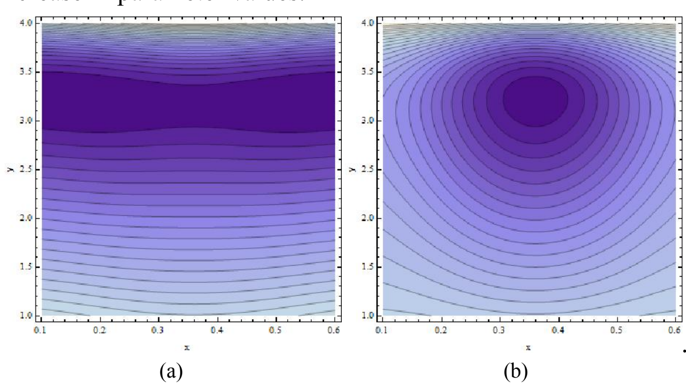

Fig. 2. Stream lines a = 0.1; $\phi = \pi/6$; β1 = 0.4; β2 = 2; α = 3; Q = 0.3; and for different b (a) b = 0 (b) b = 0.1

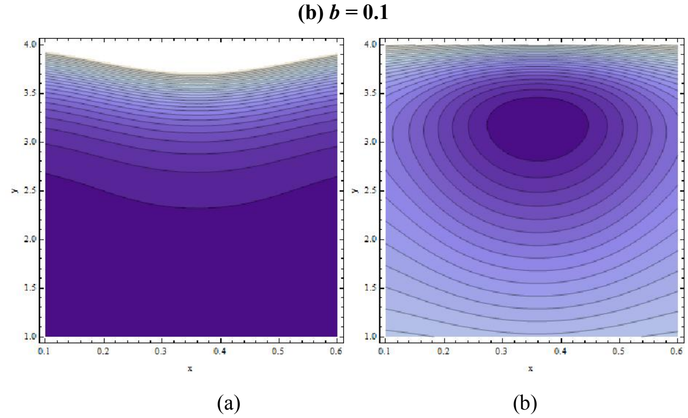

Fig. 3. Stream lines for $a = b$ = 0.1; β1 = 0.4; β2 = 2.2; α = 3; Q = 0.3; and for different β2 (a) β2 = 0 (b) β2 = 1.5

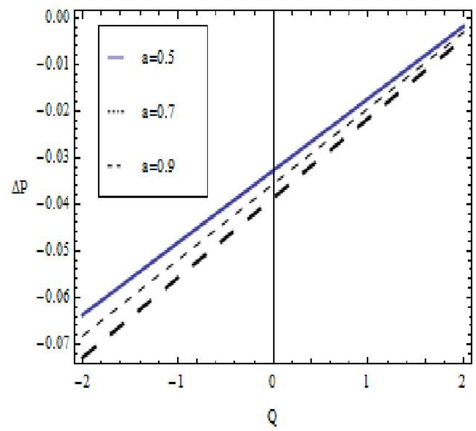

Fig. 4. The variation of Δp Vs Q for different values of a at $b = 0.5$; $\phi = \pi/2$; $\alpha = 0.3$; $d = 1$

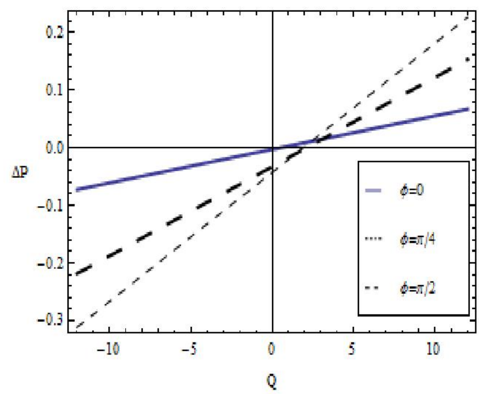

Fig. 5. The variation of Δp Vs Q for different values of ϕ at $a = 0.5$; $b = 0.5$; $\alpha = 0.3$; $d = 1$

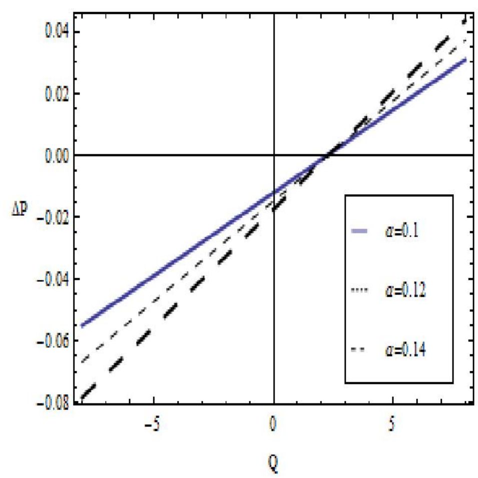

Fig. 6: The Variation of Δp Vs Q for different values of α at $a = 0.5$; $b = 0.5$; $d = 1$; $\phi = \pi/2$;

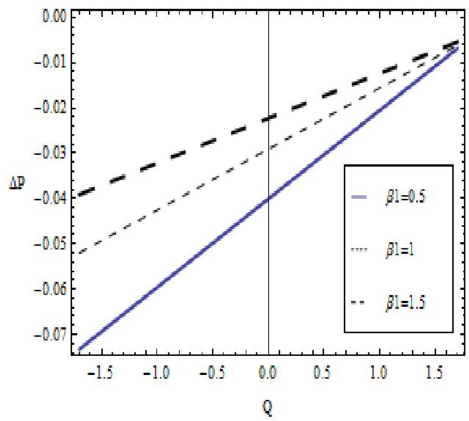

Fig. 7. The variation of Δp Vs Q for different values of β1at $a = b = 0.5$; $b = \alpha = 0.3$; $d = 1$; $\phi = \pi/2$;

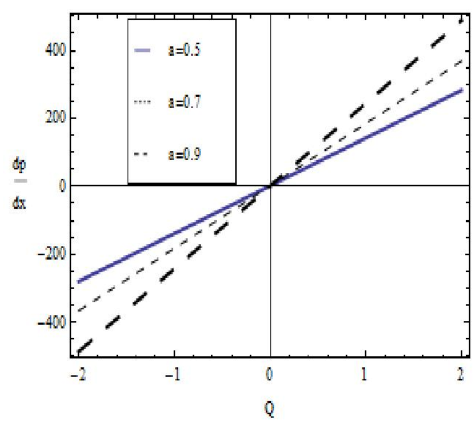

Fig. 8. The variation of $\frac{dp}{dx}$ with Q for different values of a at $b = 0.5$; $\phi = \pi/2$; $d = 1$;

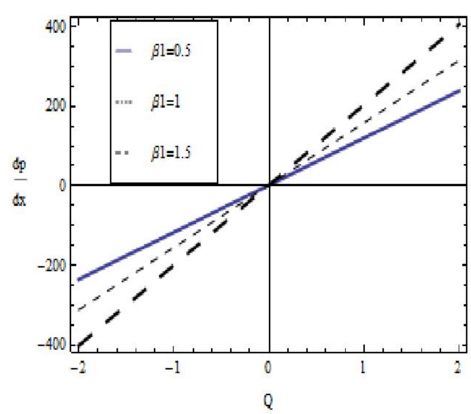

Fig. 9. The variation of $\frac{dp}{dx}$ with Q for different values of β1 at $a = b = 0.5$; $d = 1$; $\phi = 0$;

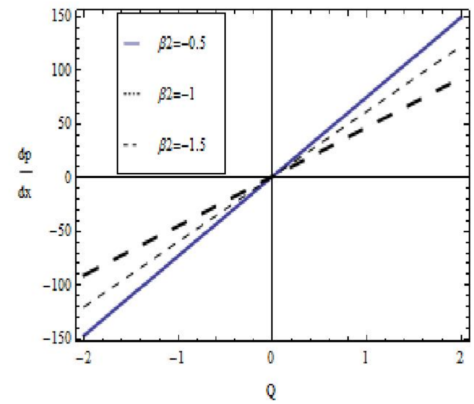

Fig. 10. The variation of $\frac{dp}{dx}$ with Q for different values of β2 at $a = b = 0.5; d = 1; \phi = 0;$

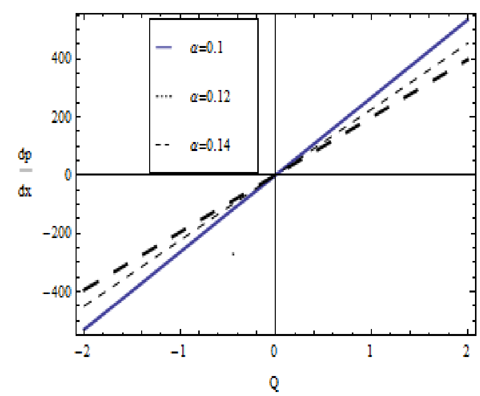

Fig. 11. The variation of $\frac{dp}{dx}$ with Q for different values of α at $b = 0.5; d = 1; \phi = 0;$

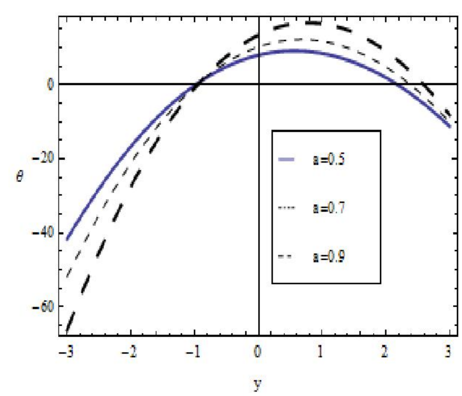

Fig. 12. The variation of θ with y for different values of a at $b = 0.5; d = 1; Q = 0.5; E_c = 0.2;$ $\mathrm{Pr} = 0.2; \alpha = 0.3; \phi = \pi/2;$

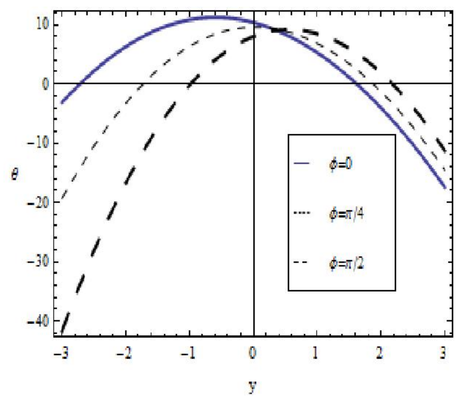

Fig. 13. The variation of θ with y for different values of ϕ at $a = b = 0.5$; $d = 1$; $Q = 0.5$; $E_c = 0.2$; $\mathrm{Pr} = 0.2$; $\alpha = 0.3$;

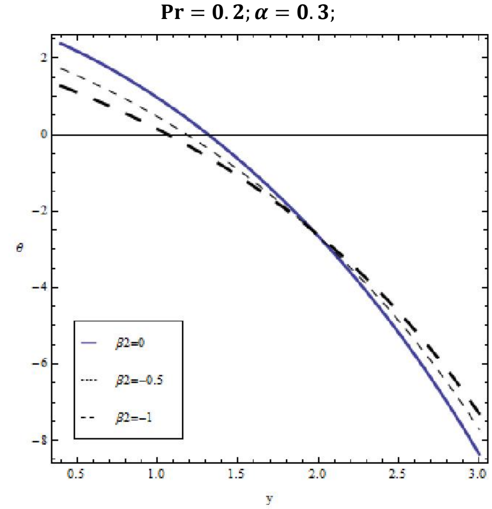

Fig. 14. The variation of θ with y for different values of β2 at $a = b = 0.5$; $d = 1$; $Q = 0.5$; $E_c = 0.2$; $\mathrm{Pr} = 0.2$; $\alpha = 0.3$; $\phi = 0$;

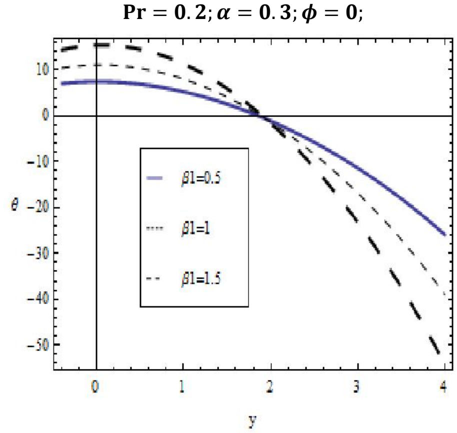

Fig. 15. The variation of θ with y for different values of β1 at $a = b = 0.5$; $d = 1$; $Q = 0.5$; $E_c = 0.2$; $\mathrm{Pr} = 0.2$; $\alpha = 0.3$; $\phi = 0$;

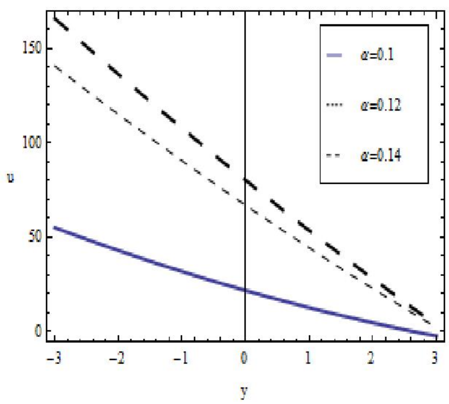

Fig. 16. **The variation of u with y for different values of a at $b = 0.5; d = 1; \Pr = 0.2; \alpha = 0.3;$ $Q = 0.5; \phi = 0;$**

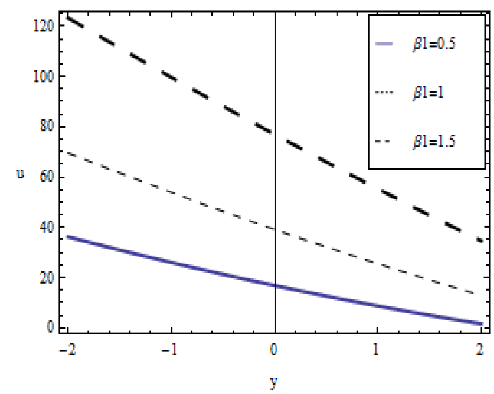

Fig. 17. **The variation of u with y for different values of β1 at $a = b = 0.5; d = 1; \Pr = 0.2;$ $\alpha = 0.3; \phi = 0; Q = 0.5;$**

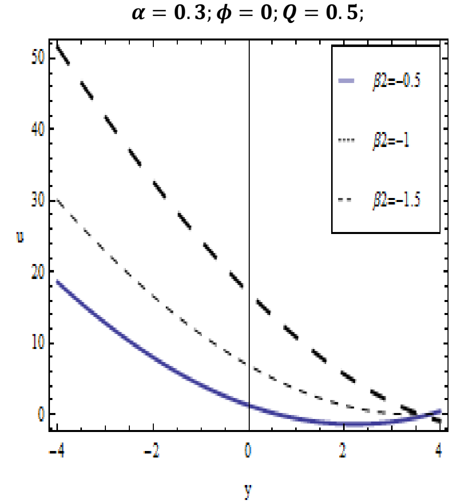

Fig. 18: The variation of u with y for different values of β2 at $a = b = 0.5; d = 1; \Pr = 0.2;$ $\alpha = 0.3; \phi = 0; Q = 0.5;$

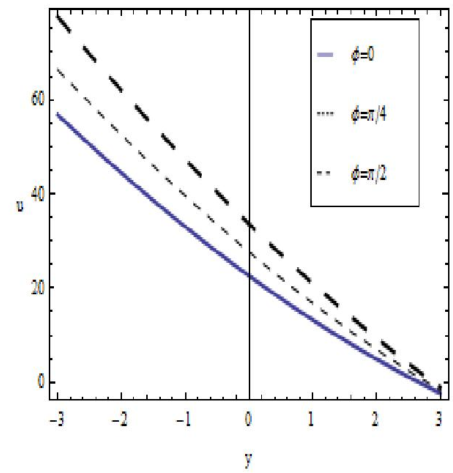

Fig. 19. **The variation of u with y for different values of ϕ at $a = b = 0.5$; $d = 1$; Pr $= 0.2$; $\alpha = 0.3$; $Q = 0.5$;**

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