AN INVESTIGATION ON SOME LRS BIANCHI TYPE-I COSMOLOGICAL MODELS WITH ZERO-MASS SCALAR FIELD

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In this paper, we have considered some LRS Bianchi-I cosmological models in the presence of zero-mass scalar fields associated with a perfect fluid distribution in it. We choose $\mu = t^{\frac{1}{2}(b-af)}$ (where *a* and *b* are any arbitrary constants) and calculate the metric, pressure and density respectively. We have also discussed various physical and geometrical properties of the models.

KEY WORDS : LRS Bianchi-I cosmological models, Zero-mass scalar field, Energy momentum tensor, Four vector velocity, Perfect fluid.

INTRODUCTION

Several investigations have been made in higher dimensional cosmology in the frame work of different scalar tensor theories and cosmological models. Recently there has been considerable interest in cosmology with LRS Bianchi type-I cosmological model in the presence of zero mass scalar field and scalar meson fields. Because of the fact that our universe is currently undergoing on accelerated expansion which has been confirmed by a host of observation such as type-I_a Supernovae (SN - I_a) [24, 31, and 41].

Many researchers in relativity have focussed their mind in the study of scalar meson field. Brahamachary [2] considered the massive, whereas Bergmann and Leipnik [1] considered the mass-less scalar field coupled to spherically symmetric gravitational fields. Janis *et. al.* [15] have further considered the problem from the point of view of singularities and Gautreau [11] and Singh [37] have extended the study to the case of non-spherical Weyl and plane symmetric fields respectively. Later on, the workers in the field , with a few exceptions (Stephenson [40]) have directed their efforts to the study of the mass less scalar fields coupled to gravitational and electromagnetic fields (Mishra and Pandey [20]); Rao, *et. al*. [28], [29], Roy, *et. al*. [33], Singh [38]. Janis *et. al*. [16] obtained the solutions of the Einstein-scalar and Brans-Dicke [3] field equations for static space-time and also gave a procedure to generate static solutions of the coupled Einstein-Maxwell scalar field equations. The solutions of axially symmetric Einstein-Maxwell scalar field equations have been given by Eris and Gurses [10]. Singh *et. al* [38] has found a method to obtain solutions to the cylindrically symmetric gravitational field coupled to mass less scalar and non-null Maxwell fields. They have further applied the technique to the solution due to Chitre *et. al*. [8] and have also obtained the dual solution by an extension of Bonnor's theorem [4].

As a matter of fact following the development of inflationary models, the importance of scalar field (mesons) in cosmology has become well known [17]. Bergmann and Leipnik [1] and Brahmachary [2] have investigated the spherically symmetric field associated with zero rest mass. The static solutions for axially symmetric field have been investigated by Buchdahl [6], Janis *et. al*. [15,16], in an attempt to present an extension of Israel's idea of a singular even horizons [14] have considered the spherically symmetric solutions of the field equation of general relativity containing zero rest-mass meson fields. Singh [37], Patel [21] and Reddy [30] have investigated plane symmetric solutions of the field equations corresponding to the zero mass scalar fields. Stephenson [41], Rao *et. al*. [29], Sharma and Yadav, Chatterjee and Roy [7], Reddy and Rao [30], Pradhan *et. al*. [25, 26], Yadav and Pradhan *et. al* [44], Purushottam and Yadav [27], Roy and Neelima [33], Yadav and Saha [43], Riess *et. al*. [32] Tagmark and Blanton [41] and Perlmutter *et. al.* [24], Ellis [9], Hawking *et. al*. [13], Mac Callum [19], Satchel [39], Benkeinstein [5] are some of the authors who have studied various aspects of interacting fields in the framework of general relativity.

In real way at the present state of evolution, the universe is spherically symmetric and the matter distribution in it is isotropic and homogeneous. But in its early stages of evolution, it could have not had a smoothed out picture. Close to the big-bang singularity, neither the assumption of spherically symmetric nor of isotropy can be strictly valid. So we consider plane symmetry, which is less restrictive than spherical symmetry and provide an avenue to study in homogeneities. For simplification and description of the large scale behaviour of the actual universe, locally rotationally symmetric [henceforth referred as LRS] Bianchi-I space time has been widely studied. Mazumdar [18] has obtained solution of LRS Bianchi-I space-time filled with a perfect fluid. Hajj-Boutros and Sfeila [12] and Sri Ram [34] have also obtained some solution for the same field equations by using their solution-generating techniques. Pradhan *et. al*. [25] have studied LRS Bianchi-I space-time with zero mass scalar field. In fact cosmological models based on scalar fields of various kinds have had enormous success in solving cosmological problems among which are the causality, entropy, initial singularity and cosmological constant problem.

In this paper, we have considered some LRS Bianchi-I cosmological models in the presence of zero-mass scalar fields associated with a perfect fluid distribution in it. We have also discussed various physical and geometrical properties of the models have been also calculated and discussed.

THE FIELD EQUATIONS

The metric for the LRS Bianchi-I space-time is of the form [19].

$$ds^2 = -dt^2 + \lambda^2 dx^2 + \mu^2 (dy^2 + dz^2) \qquad \ldots (2.1)$$

where λ and μ are function of the cosmic time t. The energy momentum tensor of a perfect fluid together with a zero mass scalar field is given by

$$T_{ij}^{(m)} T_{ij}^{(s)} \qquad \ldots (2.2)$$

where

$$T_{ij}^{(m)} = (\rho + p)\, u_i u_j + p g_{ij} \qquad \ldots (2.3)$$

Is the energy momentum tensor corresponding to perfect fluid distribution with the four vector velocity u^i satisfying $u_i u^j = -1$, p the pressure and ρ the mass-energy density. The energy momentum tensor $T_{ij}^{(s)}$ corresponding to zero mass scalar fields φ and is

$$T_{ij}^{(s)} = \varphi_{ij}\ \varphi_j - \frac{1}{2}\ g_{ij}\ g^{\alpha\beta}\varphi, \alpha^{\varphi}, \beta \qquad \ldots (2.4)$$

where φ (*t*) (a function of *t* only) is the zero-mass scalar field which satisfies the wave equation.

$$g^{ij}\,\varphi_{;ij} = 0 \qquad \dots (2.5)$$

The scalar field φ is not directly coupled to matter. It interacts with matter indirectly through gravity. The Einstein's field equations

$$R_{ij} - \frac{1}{2} R g_{ij} = kT_{ij} \qquad \dots (2.6)$$

together with energy momentum tensor defined by equation (2.2) gives the following equations

$$-Kp + \varphi^2 = \frac{2\ddot{\mu}}{\mu} + \frac{\dot{\mu}^2}{\mu^2} \qquad \dots (2.7)$$

$$-Kp + \varphi^2 = \frac{\ddot{\mu}}{\mu} + \frac{\dot{\lambda}\dot{\mu}}{\lambda\mu} + \frac{\ddot{\lambda}}{\lambda} \qquad \dots (2.8)$$

$$K\rho - \varphi^2 = \frac{2\dot{\lambda}\dot{\mu}}{\lambda\mu} + \frac{\dot{\mu}^2}{\mu^2} \qquad \dots (2.9)$$

where $k = 8\pi G$, *G* is the gravitational constant. The over dot indicates a derivative with respect to time *t*. The wave equation (2.5) yields

$$\left(\frac{\dot{\lambda}}{\lambda} + \frac{2\dot{\mu}}{\mu}\right)\dot{\varphi} + \ddot{\varphi} = 0 \qquad \dots (2.10)$$

and the energy conservation for the matter $T^{(m)}_{ij,i} = 0$ leads to

$$\dot{\rho} + \left(\frac{\dot{\lambda}}{\lambda} + \frac{2\dot{\mu}}{\mu}\right)(\rho + p) = 0 \qquad \dots (2.11)$$

SOLUTIONS OF THE FIELD EQUATIONS

From equation (2.7) and (2.8), we obtain

$$\frac{\ddot{\mu}}{\mu} + \frac{\dot{\mu}^2}{\mu^2} - \frac{\ddot{\lambda}}{\lambda} - \frac{\dot{\lambda}\dot{\mu}}{\lambda\mu} = 0 \qquad \dots (3.1)$$

which has first integral.

$$\mu^2\dot{\lambda} - \lambda\mu\dot{\mu} = A \qquad \dots (3.2)$$

where *A* is an integrating constant.

Equation (2.3) is a linear differential equation in λ (*t*) and has an exact solution,

$$\lambda = C_1\mu + A\mu \int \frac{dt}{\mu^3(t)} \qquad \dots (3.3)$$

Similarly equation (3.2) is also a linear differential equation in μ (*t*), which has an exact solution,

$$\mu_2 = C_2\lambda_2 - 2A\lambda^2 \int \frac{dt}{\lambda^3(t)} \qquad \dots (3.4)$$

On integration, equation (2.10) yields,

$$\varphi = C_2 + \int \frac{C_3 dt}{\lambda(t)\mu^2(t)} \quad \text{... (3.5)}$$

where C_1, C_2, C_3 and C_4 are integration constants.

Thus for any arbitrary μ (t), equation (3.3) gives λ (t) and then φ is known from equation (3.5). Similarly for an arbitrary λ (t) one can calculate μ (t) and φ from equation (3.4) and (3.5). Then from equation (2.7) and (2.9), p and ρ can be obtained and hence the solution of the field equations is completely known.

To illustrate our problem, we choose $\boldsymbol{\mu = t^{\frac{1}{2}(b-af)}}$ (where a and b are any arbitrary constants) From equation (3.3) and (3.5) we obtain

$$\lambda = C_1\, t^{\mathcal{K}(b-af)} + \frac{2At^{(af-b+1)}}{3(af-b)+2} \quad \text{... (3.6)}$$

$$\varphi^2 = C_3^2 \left[t^{\mathcal{K}(b-af)} + \frac{2At^{af(b-af)}}{3(af-b)+2} \right] \quad \text{... (3.7)}$$

where $\mathcal{K} = \frac{1}{2}$ and $f \neq \frac{b}{3a}$ are real constants. So in this case, the geometry of our universe is given by metric

$$ds^2 = -dt^2 + \left[C_1 t^{\mathcal{K}(b-af)} + \frac{2At^{(af-b+1)}}{3(af-b)+2} \right] dx^2 + \left[t^{2\mathcal{K}(b-af)}(dy^2 + dz^2) \right] \quad \text{... (3.8)}$$

For the metric (2.8) from the equation (2.7) – (2.9), find the expression for p and ρ

$$Kp = C_3^2 \left[C_1 t^{\mathcal{K}(b-af)^2} + \frac{2At^{af(b-af)}}{3(af-b)+2} \right]^{-2} + \left[\frac{(b-af)\{3(af+b)-2\}}{4t^2} \right] \quad \text{... (3.9)}$$

$$K\rho = C_3^2 \left[C_1 t^{\mathcal{K}(b-af)^2} + \frac{2At^{af(b-af)}}{3(af-b)+2} \right]^{-2} + \left[\frac{2A(b-af)\{3(af+b)-2\} t^{\mathcal{K}\{3(af-b)+2\}} + 3C_1 (b-af)^2 \{3(af-b)+2\}}{4t^2 [2At^{\mathcal{K}\{3(af-b)+2\}} + C_1\{3(af-b)+2\}]} \right] \quad \text{... (3.10)}$$

When $\mathcal{K} = \frac{1}{2}$, we get solution due to Pradhan *et. al*. [25] by suitable adjustment of constants. However, when $\mathcal{K} = \frac{1}{2}$ we get

$$ds^2 = -dt^2 + \left[C_1 t^{\frac{1}{2}(b-af)} + \frac{2At^{(af-b+1)}}{3(af-b)+2} \right] dx^2 + \left[t^{(b-af)}(dy^2 + dz^2) \right] \ldots \quad (3.11)$$

$$Kp = C_3^2 \left[C_1 t^{\frac{1}{2}(b-af)^2} + \frac{2At^{af(b-af)}}{3(af-b)+2} \right]^{-2} + \left[\frac{(b-af)\{3(af+b)-2\}}{4t^2} \right] \quad \text{... (3.12)}$$

$$K\rho = C_3^2\left[C_1 t^{\frac{1}{2}(b-af)^2} + \frac{2At^{af(b-af)}}{3(af-b)+2}\right]^{-2}$$

$$+\left[\frac{2A(b-af)\{3(af+b)-2\}t^{\frac{1}{2}\{3(af-b)+2\}} + 3C_1(b-af)^2\{3(af-b)+2\}}{4t^2\left[2At^{\frac{1}{2}\{3(af-b)+2\}} + C_1\{3(af-b)+2\}\right]}\right] \quad \ldots (3.13)$$

when $f \neq \dfrac{b}{a}$, $p = \rho =$ constant, whereas in the absence of scalar field we get, $p = \rho = 0$ [18].

The energy conditions [30(a)]

(i) $(\rho + p) > 0$

(ii) $(\rho + 3p) > 0$ and

(iii) $\rho > 0$

are satisfied when $C_1 > 0$, A > 0 than $\dfrac{1}{3a} < f < \dfrac{1}{a}$ and the dominant energy conditions [13].

(i) $(\rho - p) \geq 0$ and

(ii) $(\rho + p) \geq 0$

when $C_1 > 0$, A > 0 and $\dfrac{1}{3a} < f < \dfrac{2}{3a}$.

The expansion scalar θ, the shear tensor $\sigma_{\alpha\gamma}$, the rotation $\omega_{\alpha\gamma}$ and acceleration vector a_α for the velocity field u_α are defined by

$$\theta = u^{\alpha}_{;\alpha} \quad \ldots (3.14)$$

$$\sigma_{\alpha\gamma} = \frac{1}{2}\left(u_{\alpha\gamma} + u_{\gamma;\alpha}\right) - \frac{1}{2}\left(u_\alpha a_\gamma + u_\gamma a_\alpha\right) - \frac{1}{3}\theta\left(g_{\alpha\gamma} + u_\alpha u_\gamma\right) \quad \ldots (3.15)$$

$$\omega_{\alpha\gamma} = u_{\alpha;\gamma} - \sigma_{\alpha\gamma} - u_{\alpha;\beta}u^{\beta}u_\gamma - \frac{1}{3}\theta\left(g_{\alpha\gamma} + u_\alpha u_\gamma\right) \quad \ldots (3.16)$$

and

$$a_\alpha = u^{\gamma}u_{\alpha;\gamma} \quad \ldots (3.17)$$

Here the semicolon indicates covariant differentiation. The spatial volume is given by

$$V = \boldsymbol{\lambda\mu^2}$$

For the velocity field μ_α these kinematical parameters are found to have the following expressions:

$$V = \frac{t\left[C_1\{3(af-b)+2\} + 2At^{\frac{1}{2}\{3(af-b)+2\}}\right]}{\left[C_1\{3(af-b)+2\}t^{\frac{1}{2}\{3(af-b)+2\}}\right]} \quad \ldots (3.18)$$

$$\theta = \frac{t\left[3C_1(b-af)\{3(af-b)+2\} + 4At^{\frac{1}{2}\{3(af-b)+2\}}\right]}{2t\left[C_1\{3(af-b)+2\} + 2At^{\frac{1}{2}\{3(af-b)+2\}}\right]} \quad \ldots (3.19)$$

$$\sigma = \frac{1}{\sqrt{6}}\left[\frac{\left[2At\{3(af-b)+2\}t^{\frac{1}{2}\{3(af-b)+2\}}\right]}{t\left[C_1\{3(af-b)+2\} + 2At^{\frac{1}{2}\{3(af-b)+2\}}\right]}\right] \quad \ldots (3.20)$$

$$\omega = 0 \quad \ldots (3.21)$$

$$a_\alpha = [0,0,0,0]. \quad \ldots (3.22)$$

Discussion and conclusion

From above equations [3.19 – 3.22] it is clear that our model is expanding, shearing and non rotating. The acceleration vector a_α is zero and consequently the stream links of perfect fluid are geodetic. As the shear tensor is not zero, the model is clearly anisotropic.

For $f = \frac{b}{a}$, the metric (3.11) represent a non static cosmological model filled with stiff fluid, the pressure and density of which are given by

$$Kp = K\rho = \frac{C_3^2}{(C_1 + A)^2} \quad \ldots (4.1)$$

The model with equal pressure and density *i.e.* $p = \rho$ are important in relativistic cosmology for the description of very early stages of the universe.

For $f \neq \frac{b}{a}, \frac{-b}{3a}, \frac{b}{3a}$ from equation [3.12-3.13, 3.18-3.20] it is seen that at the singularity $t = 0$, $V \to 0$ and p, ρ, θ and σ are infinitely large. As $t \to \infty, V \to \infty$ and p, ρ, θ and σ vanish. Therefore, for $f \neq \frac{b}{a}, \frac{-b}{3a}, \frac{b}{3a}$ the solution [3.11] represents an anisotropic universe exploding from $t = 0$ which expands for $0 < t < \infty$ and after a large time t, would give essential an isotropic empty universe.

Choosing $\boldsymbol{\lambda = h_1 t^{k(b-af)} + h_2 t^{af}}$ and $A = 0$ in equation (3.3), we find

$$\mu^2 = g_1 t^{2k(b-af)} + g_2 t^{k+af(1-k)} + g_3 t^{2af} \quad \ldots (4.2)$$

where $g_1 = C_2 h_1^2$, $g_2 = 2C_2 h_1 h_2$, $g_3 = C_2 h_2^2$

Hence, in this case the geometry of our universe is given by metric

$$ds^2 = -dt^2 + (h_1 t^{k(b-af)} + h_2 t^{af})dx^2 + (g_1 t^{2k(b-af)} + g_2 t^{k+af(1-k)} + g_3 t^{2af})(dy^2 + dz^2) \quad \ldots (4.3)$$

From equation [3.5], we can obtain the value of φ^2, and equation [2.8-2.9] give us the values of the physical parameters p and ρ.

Putting $k = \frac{1}{2}$, in above results we get

$$\lambda = h_1 t^{\frac{1}{2}(b-af)} + h_2 t^{af} \quad \ldots (4.4)$$

$$\mu^2 = g_1 t^{(b-af)} + g_2 t^{\frac{1}{2}(1+af)} + g_3 t^{2af} \quad \ldots (4.5)$$

and $$ds^2 = -dt^2 + (h_1 t^{\frac{1}{2}(b-af)} + h_2 t^{af})dx^2 + (g_1 t^{(b-af)} + g_2 t^{\frac{1}{2}(b+af)} + g_3 t^{2af})(dy^2 + dz^2) \quad \ldots (4.6)$$

In this chapter we have generalised the solution of Refs [18], [12], [34], [25]. For $= 0, f = 0$ and $f = -\frac{b}{3a}$, from [4.6], we obtain the solutions of Sri Ram [34].

For $\varphi = 0$, from eq. [3.8] we recover the model of Mazumdar [18], and thus our solutions represent a generalization of Mazumdar [18].

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