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CERTAIN INVESTIGATIONS ON SOME BIANCHI TYPE-III STRING COSMOLOGICAL MODELS WITH BULK VISCOSITY AND MAGNETIC FIELD

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In this paper, we study Bianchi type-III string cosmological model in the presence of bulk viscosity and magnetic field. To obtain an explicit solution, we consider an equation of state $\rho = a + b\lambda$ and an assumption that the scalar of expansion is proportional to the shear scalar $\sigma \propto \theta$, which leads to the relation between metric potentials $\beta = a + b\gamma^{\mu+1}$. The physical and geometric aspects of the model in the presence and absence of magnetic field are also discussed.

KEY WORDS : Bianchi type-III string cosmological models, Zero-mass scalar field, scalar of expansion, Bulk viscosity, Magnetic field

INTRODUCTION

In recent years, there has been considerable interest is string cosmology, as string are believed to have played an important role during early stages of the universe [30] and can generate density fluctuations which lead to galaxy formation [40].The strings have stress-energy and they can couple to the gravitational field, hence it may be interesting to study the gravitational effects that arise from strings. In fact, the general relativistic treatment of strings was initiated by Letelier [14] and Satchel [26]. This model was used as a source for Bianchi type-1 and Kantowski-Sachs cosmologies by Letelier. More recently, Krori *et. al* [12, 13] and Wang [36, 31, 38] have discussed the solutions of Bianchi type I, VI, VIII and IX for a cloud string, and Tikekar and Patel [28] and Chackraborty and Chackraborty [9] have presented the exact solutions of Bianchi type-III and spherically symmetric cosmology respectively for a cloud string. It is well known that in an early stage of universe when neutrino decoupling occurs, the matter behaves like a viscous fluid. The cosmological models of a fluid with viscosity play a significant role in study the evolution of the universe. Recently, string cosmological models of Bianchi types I, II, III with bulk viscosity have been discussed by several authors [4, 32-37].

On the other hand, the magnetic field has an important role at the cosmological scale and is present in galactic and intergalactic spaces. The importance of the magnetic field for various astrophysical phenomena has been studied in many papers. Melvin [16] has pointed out that during the evolution of the universe, the matter was in a highly ionized state and is smoothly coupled with the field; subsequently forming neutral matter as a result of universe expansion. Therefore the possibility of the presence of magnetic field in the cloud string universe is not unrealistic and has been investigated by many authors [10, 21, 35]. Bahera [3], Bali and Dave [5], Bali [7], Kibble [11], Takabayski [27], Yadav [39], Zimadahl [41] are the some workers in this line.

In this paper, we study Bianchi type-III string cosmological model in the presence of bulk viscosity and magnetic field. To obtain an explicit solution, an equation of state $\rho = a + b\lambda$ and an assumption that the scalar of expansion is proportional to the shear scalar $\sigma \propto \theta$, which leads to the relation between metric potentials $\beta = a + b\gamma^{\mu+1}$. The physical and geometric aspects of the model in the presence and absence of magnetic field are also discussed.

THE FIELD EQUATIONS AND ITS SOLUTIONS

The Bianchi type-III space-time metric we considered here is [37]

$$ds^2 = -dt^2 + \alpha^2 dx^2 + \beta^2 e^{2x} dy^2 + \gamma^2 dz^2 \qquad \dots (2.1)$$

where α, β, and γ are only the functions of time *t*.

The energy-momentum tensor for a cloud of string with bulk viscosity and magnetic field [35].

$$\mathcal{T}_{ij} = \rho u_i u_j - \lambda \mathcal{X}_i \mathcal{X}_j - \xi\theta(u_i u_j + g_{ij}) + E_{ij} \qquad \dots (2.2)$$

where $\rho = \rho_p + \lambda$, is the rest energy density of the cloud of strings with particles attached to them , ρ_p is the rest energy density of particles, λ is the tension density of the cloud of strings, $\theta = u^i_{;i}$, is the scalar of expansion, and ξ is the coefficient of bulk viscosity. According to Letelier [14] the energy density for the coupled system ρ and ρ_p is restricted to be positive, while the tension density λ may be positive or negative. The vector u^i describes the cloud four-velocity and $\mathcal{X}^i$ represents a direction of anisotropy, *i.e.* the direction of string. They satisfy the standard relations [14].

$$u^i u_j = -\mathcal{X}^i \mathcal{X}_j = -1, \quad u^i \mathcal{X}_i = 0 \qquad \dots (2.3)$$

E_{ij} is the energy-momentum tensor for the magnetic field

$$E_{ij} = \frac{1}{4\pi} g^{hk} F_{ih} F_{jk} - \frac{1}{4} g_{ij} F_{hk} F^{hk} \qquad \dots (2.4)$$

where F_{ij} is the electromagnetic field tensor, which satisfies the Maxwell equations

$$F_{[ij;h]} = 0, \qquad \left(F^{ij}\sqrt{-g}\right)_{;j} = 0 \qquad \dots (2.5)$$

Einstein's equation we consider here is

$$R_{ij} - \frac{1}{2} R g_{ij} = \mathcal{T}_{ij} \qquad \dots (2.6)$$

where we have choose the units such that $c = 1$ and $8\pi G = 1$. In the co-moving coordinates $u^i = \delta^i_0$ and $u^i = -\delta^0_i$, and the incident magnetic field taken along the *z*-axis, with the help of Maxwell equations (2.5), the only non-vanishing component of F_{ij} is [35].

$$F_{ij} = \text{constant} = M \qquad \dots (2.7)$$

The Einstein equation (2.6) for the metric (2.1) can be written as following system of equations [35, 37]:

$$\frac{\ddot{\beta}}{\beta}+\frac{\ddot{\gamma}}{\gamma}+\frac{\dot{\beta}\dot{\gamma}}{\beta\gamma}=\xi\theta-\frac{M^2}{8\pi\alpha^2\beta^2e^{2x}} \quad \text{... (2.8)}$$

$$\frac{\ddot{\alpha}}{\alpha}+\frac{\ddot{\gamma}}{\gamma}+\frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma}=\xi\theta-\frac{M^2}{8\pi\alpha^2\beta^2e^{2x}} \quad \text{... (2.9)}$$

$$\frac{\ddot{\alpha}}{\alpha}+\frac{\ddot{\beta}}{\beta}+\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta}-\frac{1}{\alpha^2}=\lambda+\xi\theta+\frac{M^2}{8\pi\alpha^2\beta^2e^{2x}} \quad \text{... (2.10)}$$

$$\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta}+\frac{\dot{\beta}\dot{\gamma}}{\beta\gamma}+\frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma}-\frac{1}{\alpha^2}=\rho+\frac{M^2}{8\pi\alpha^2\beta^2e^{2x}} \quad \text{... (2.11)}$$

$$\frac{\dot{\alpha}}{\alpha}-\frac{\dot{\beta}}{\beta}=0 \quad \text{... (2.12)}$$

where the dot denotes the differentiation with respect to time *t*. From Eq. (2.12), we have

$$\boldsymbol{\alpha=H\beta} \quad \text{... (2.13)}$$

where H is the constant of integration. In order to obtain a more general solution, we assume Takabayasi's equation of state [27]

$$\boldsymbol{\rho=a+k\lambda} \quad \text{... (2.14)}$$

where a and k are the positive constants.

The expression for scalar of expansion and shear scalar are

$$\theta=u^i_{;i}=\frac{\dot{\alpha}}{\alpha}+\frac{\dot{\beta}}{\beta}+\frac{\dot{\gamma}}{\gamma} \quad \text{... (2.15)}$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij}$$
$$=\frac{1}{3}\left(\frac{\dot{\alpha}^2}{\alpha^2}+\frac{\dot{\beta}^2}{\beta^2}+\frac{\dot{\gamma}^2}{\gamma^2}-\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta}-\frac{\dot{\beta}\dot{\gamma}}{\beta\gamma}-\frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma}\right) \quad \text{... (2.16)}$$

We note that the five independent equations (2.9)-(2.12) and (2.14) connecting six unknown variables $(\alpha,\beta,\gamma,\lambda,\rho,\xi)$. Thus, one more relation connecting these variables is needed to solve these equations. In order to obtain explicit solutions, one additional relation is needed and we adopt an assumption that the shear scalar of expansion is proportional to the shear scalar of expansion $\sigma\propto\theta$, which leads to

$$\boldsymbol{\beta=a+b\gamma^{\mu+1}} \quad \text{... (2.17)}$$

where a, b and μ is a constant.

Now we consider $\boldsymbol{a=0}$ and $\boldsymbol{b=1}$, then Equation (2.14) and (2.17) reduces to-

$$\rho=k\lambda \quad \text{... (2.18)}$$

$$\beta=\gamma^{\mu+1} \quad \text{... (2.19)}$$

From equation (2.9), (2.10), (2.11), with the help of Eq. (2.18) eliminating ρ, λ and $\xi\theta$, we obtain

$$k\frac{\ddot{\alpha}}{\alpha}-k\frac{\ddot{\gamma}}{\gamma}+(k-1)\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta}-(k+1)\frac{\dot{\beta}\dot{\gamma}}{\beta\gamma}-\frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma}=(k-1)\frac{1}{\alpha^2}+\frac{M^2}{8\pi}\frac{(2k-1)}{\alpha^2\beta^2e^{2x}} \quad \text{... (2.20)}$$

Substituting Equation (2.13) and (2.19) into eq. (2.20), we have

$$\ddot{\gamma} + \frac{(\mu+1)[2k\mu-(\mu+3)]}{k\mu}\frac{\dot{\gamma}^2}{\gamma^2} = \frac{(k-1)}{k\mu H^2}\gamma^{-(2\mu+1)} + \frac{M^2}{8\pi}\frac{(2k-1)}{k\mu H^2 e^{2x}}\gamma^{-(4\mu+3)} \quad \text{... (2.21)}$$

To solve Eq. (2.21), we denote $\dot{\gamma} = \vartheta$, then $\ddot{\gamma} = \vartheta \frac{d\vartheta}{d\gamma}$, and the Eq. (2.21) can be reduced to the first-order differential equation in the following form:

$$\vartheta \frac{d\vartheta}{d\gamma} + l\frac{\vartheta^2}{\gamma} = \frac{(k-1)}{k\mu H^2}\gamma^{-(2\mu+1)} + \frac{(2k-1)\mathcal{K}}{k\mu H^2 e^{2x}}\gamma^{-(4\mu+3)} \quad \text{... (2.22)}$$

where
$$l = \frac{(\mu+1)[2k\mu-(\mu+3)]}{k\mu} \quad \text{... (2.23)}$$

$$\mathcal{K} = \frac{M^2}{8\pi} \quad \text{... (2.24)}$$

Equation (2.22) can be written as

$$\frac{d}{d\gamma}(\vartheta^2\gamma^{2l}) = \frac{2(k-1)}{k\mu H^2}\gamma^{2l-(2\mu+1)} + \frac{2(2k-1)\mathcal{K}}{k\mu H^2 e^{2x}}\gamma^{2l-(4\mu+3)} \quad \text{... (2.25)}$$

Thus we obtained:

$$dt = \left[\frac{(k-1)}{\{k(\mu^2+2\mu)-(\mu^2+4\mu+3)\}H^2}\gamma^{-2\mu} + \frac{(2k-1)\mathcal{K}}{\{k\mu-(\mu^2+4\mu+3)\}H^2 e^{2x}}\gamma^{-(4\mu+2)} + N\gamma^{-2l}\right]^{-\frac{1}{2}} d\gamma \quad \text{... (2.26)}$$

where N is the constant of integration. For this solution, the geometry of the universe is described by the metric

$$ds^2 = -\left[\frac{(k-1)}{\{k(\mu^2+2\mu)-(\mu^2+4\mu+3)\}H^2}\gamma^{-2\mu} + \frac{(2k-1)\mathcal{K}}{\{k\mu-(\mu^2+4\mu+3)\}H^2 e^{2x}}\gamma^{-(4\mu+2)} + N\gamma^{-2l}\right]^{-1} d\gamma^2 + H^2\gamma^{2\mu+2}dx^2 + \gamma^{2\mu+2}e^{2x}dy^2 + \gamma^2 dz^2 \quad \text{... (2.27)}$$

Under suitable transformation of coordinates, Eq. (2.27) reduces to

$$ds^2 = -\left[\frac{(k-1)}{\{k(\mu^2+2\mu)-(\mu^2+4\mu+3)\}H^2}\mathcal{T}^{-2\mu} + \frac{(2k-1)\mathcal{K}}{\{k\mu-(\mu^2+4\mu+3)\}H^2 e^{2x}}\mathcal{T}^{-(4\mu+2)} + N\mathcal{T}^{-2l}\right]^{-1} d\mathcal{T}^2 + H^2\mathcal{T}^{2\mu+2}dx^2 + \mathcal{T}^{2\mu+2}e^{2x}dy^2 + \mathcal{T}^2 dz^2 \quad \text{... (2.28)}$$

For the model of Eq. (2.28), the order physical and geometrical parameters can easily be obtained. The expressions for the energy density ρ, the string tension density λ, the particle density ρ_p the coefficient of bulk viscosity ξ, the scalar of expression θ and the shear scalar σ^2 are, respectively, given by

$$\rho = \frac{k(2\mu+3)}{\{k(\mu^2+2\mu)-(\mu^2+4\mu+3)\}H^2}\mathcal{T}^{-2(\mu+1)} + \frac{\{2k(\mu+1)^2+3k\mu+3k+3\}\mathcal{K}}{\{k\mu-(\mu^2+4\mu+3)\}H^2e^{2x}}\mathcal{T}^{-(4\mu+4)} + (\mu+1)(\mu+3)N\mathcal{T}^{-2(l+1)} \quad \dots(2.29)$$

$$\lambda = \frac{\rho}{k} \quad \dots(2.30)$$

$$\rho_p = \rho - \lambda = \left(1-\frac{1}{k}\right)\rho \quad \dots(2.31)$$

$$\xi\,\theta = \frac{(k-1)}{\{k(\mu^2+2\mu)-(\mu^2+4\mu+3)\}H^2}\mathcal{T}^{-2(\mu+1)} + \frac{(\mu^2+3\mu+1)(1-2k)}{\{k\mu-(\mu^2+4\mu+3)\}H^2e^{2x}}\mathcal{K}\mathcal{T}^{-(4\mu+4)} + \frac{\{(\mu+1)(\mu+2)-k\mu(\mu+1)\}(\mu+3)}{k\mu}N\mathcal{T}^{-(2l+2)} \quad \dots(2.32)$$

$$\theta = (2\mu+3).\left[\frac{(k-1)}{\{k(\mu^2+2\mu)-(\mu^2+4\mu+3)\}H^2}\mathcal{T}^{-2(\mu+1)} + \frac{(2k-1)\mathcal{K}}{\{k\mu-(\mu^2+4\mu+3)\}H^2e^{2x}}\mathcal{T}^{-(4\mu+4)} + N\mathcal{T}^{-(2l+2)}\right]^{\frac{1}{2}} \quad \dots(2.33)$$

$$\sigma^2 = \frac{\mu^2}{3}.\left[\frac{(k-1)}{\{k(\mu^2+2\mu)-(\mu^2+4\mu+3)\}H^2}\mathcal{T}^{-2(\mu+1)} + \frac{(2k-1)\mathcal{K}}{\{k\mu-(\mu^2+4\mu+3)\}H^2e^{2x}}\mathcal{T}^{-(4\mu+4)} + N\mathcal{T}^{-(2l+2)}\right] \quad \dots(2.34)$$

RESULTS AND DISCUSSION

From Equations (2.29) and (2.31) it is observed that the energy condition $\rho \geq 0$ and ρ_p are fulfilled, provided

$$N \geq 0,\ \mu > 0 \text{ and } k > (\mu+1)(\mu+3)/\mu$$

or $$N \geq 0,\ \mu > 0 \text{ and } k < -1/(\mu+1)(2\mu+5)$$

when $N \geq 0,\ \mu > 0\ and\ k > (\mu+1)(\mu+3)/\mu$, the string tension density $\lambda > 0$; however, $\lambda < 0$ when $N \geq 0,\ \mu > 0\ and\ k < -1/(\mu+1)(2\mu+5)$.

The above expressions (2.29 - 2.32) indicate that the magnetic field is relates with ρ, λ, ρ_p and ξ. Here a term of $\mathcal{K}$ is involved in the expression for ρ, λ, ρ_p, ξ, θ and σ^2 respectively, and it represents the effect of magnetic field on the model.

It is seen that in the case $\mu > 0$, whether $k > (\mu+1)(\mu+3)/\mu$ or $k < -\frac{1}{(\mu+1)}(2\mu+5)$ we have $l+1 > 0$. Hence equation (2.33) shows that the scalar of expansion θ tends to infinitely large when $\mathcal{T} \to 0$, but $\theta \to 0$ or tends to finite when $\mathcal{T} \to \infty$. The energy density ρ tends to finite when $\mathcal{T} \to \infty$ and $\rho \to \infty$ when $\mathcal{T} \to 0$, therefore the model describes a shearing non rotating expanding universe with the big-bang start. We can see from the above discussion that the bulk viscosity plays a significant role in the evolution of universe [1, 15]. Furthermore, since $\lim_{\mathcal{T} \to \infty} \frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large values of $\mathcal{T}$.

In the special case $k = 1$, the model represents a geometric string model [38]. In the absence of magnetic field, $\mathcal{K} = 0$, the metric (2.28) reduces to the string model with bulk viscosity i.e.,

$$ds^2 = -\left[\frac{(k-1)}{\{k(\mu^2 + 2\mu) - (\mu^2 + 4\mu + 3)\}H^2}\mathcal{T}^{-2\mu} + N\mathcal{T}^{-2l}\right]^{-1} d\mathcal{T}^2$$

$$+ H^2\mathcal{T}^{2\mu+2}dx^2 + \mathcal{T}^{2\mu+2}e^{2x}dy^2 + \mathcal{T}^2 dz^2 \qquad \ldots (3.1)$$

Conclusion

In this paper, we study Bianchi type-III string cosmological model in the presence of bulk viscosity and magnetic field. To obtain an explicit solution, an equation of state $\rho = a + b\lambda$ and an assumption that the scalar of expansion is proportional to the shear scalar $\sigma \propto \theta$, which leads to the relation between metric potentials $\beta = a + b\gamma^{\mu+1}$. Then the cosmological model for a cosmic string with bulk viscosity and magnetic field is obtained. The physical and geometric aspects of the model in the presence and absence of magnetic field are also discussed. Our model describes a shearing non-rotating continuously expanding universe with a big-bang start. In the absence of magnetic field it reduces to the string model with bulk viscosity.

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