Acta Ciencia Indica, Vol. XLI M, No. 2, 85 (2015)

THERMO DIFFUSION EFFECT ON MHD FLOW OF HEAT AND MASS TRANSFER OVER A VERTICAL ISOTHERMAL POROUS SURFACE

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RECEIVED : 4 February, 2015

Thermo diffusion effect on MHD flow of heat and mass transfer on continuously moving isothermal vertical porous surface is considered. The boundary layer equations are transformed into ordinary differential equations of second order and solutions obtained for velocity, temperature and concentration. The velocity, temperature and concentration profiles are studied for different non dimensional parameters like Grashof number, Schmidt number, Prandtl number, porosity parameter, Hartmann number and Soret number. The results are discussed with the help of graphs.

KEYWORDS : MHD, Heat transfer, Mass transfer, Isothermal surface, Soret effect

Introduction

The study of boundary layer behaviour over a moving continuous solid surface is an important type of flow occurring in several engineering process. To be more specific, heat-treated materials travelling between a feed roll and a wind-up roll or materials manufactured by extrusion, glass-fibre and paper production, the boundary layer along a liquid film in condensation process, the material handling conveyors, cooling of metallic sheets or electronic chips etc are few examples.

There are many natural phenomena and engineering problems susceptible to magneto hydrodynamics. MHD finds practical uses in many areas such as pumping orientation and confinement of extremely hot ionized gases or plasmas in thermonuclear fusion experiments and space propulsion resulting from the electromagnetic acceleration of ionized gases. Magneto hydrodynamics finds applications in Ion Propulsion, electromagnetic pumps, MHD power generators, controlled fusion research, MHD couples and bearings, plasma jets and chemical synthesis, etc.

Chenna Kesavaiah D., Sudhakaraiah A. [1] studied the Effects of heat and mass flux to MHD flow in vertical surface with radiation and absorption . Hassan A.M.El [2]-Arabawy obtained exact solutions of Mass transfer over a stretching surface with chemical reaction and suction/injection. Soret and Dufour effects on natural convection heat and mass transfer flow past a horizontal surface in a porous medium with variable viscosity has studied by M.B.K. Moorthy *et. al* [3].

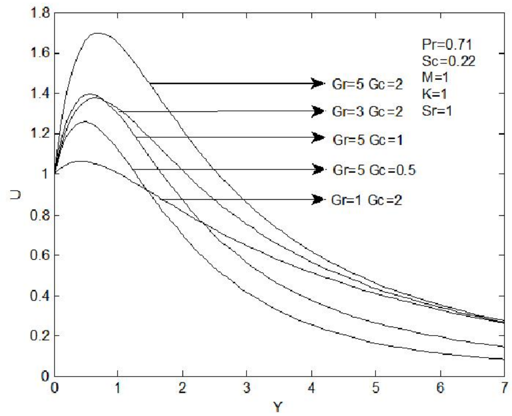

Fig. 1. Velocity Profile for different *Gr* and *Gc*

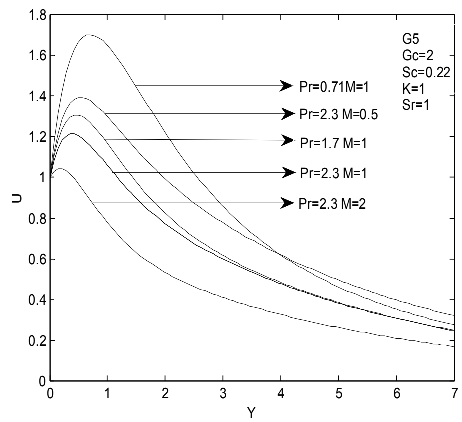

Fig. 2. Velocity Profile for different *Pr* and *M*

Thermal boundary layer on a continuously moving semi-infinite horizontal plate in the presence of transverse magnetic field with heat flux was studied by Murthy [4]. Muthucumaraswamy R. [5] analyzed heat and Mass Transfer Effects on Moving Isothermal Vertical Surface in the Presence of Magnetic Field. The similarity solutions and the resulting equations were integrated numerically. Omokhuale E., *et. al*. [6] Effect of Jeffery fluid on heat and mass transfer past a Vertical porous plate with soret and variable thermal conductivity .

Magnetic field effect on transient free convection flow through porous medium past an impulsively started vertical plate with fluctuating temperature and mass diffusion is studied by Ravikumar V., *et. al.* [7]. Seethamahalakshmi B. D., Prasad. C.N, and Ramana Reddy. G.V. [8] analyzed MHD Free Convective Mass Transfer Flow Past an Infinite Vertical Porous Plate with Variable Suction and Soret Effect. Vajravelu [9] studied the exact solution for hydrodynamic boundary layer flow and heat transfer over a continuous, moving, horizontal flat surface with uniform suction and internal heat generation/absorption. In all these studies, the authors have taken the continuous moving surface to be oriented in the horizontal direction. Again, Vajravelu [10] extended the problem of [9] to vertical surface.

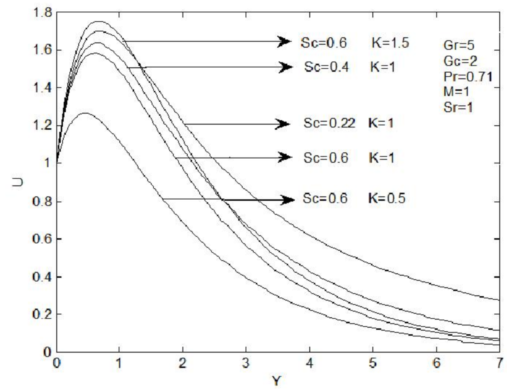

Fig. 3. Velocity Profile for different *Sc* and *K*

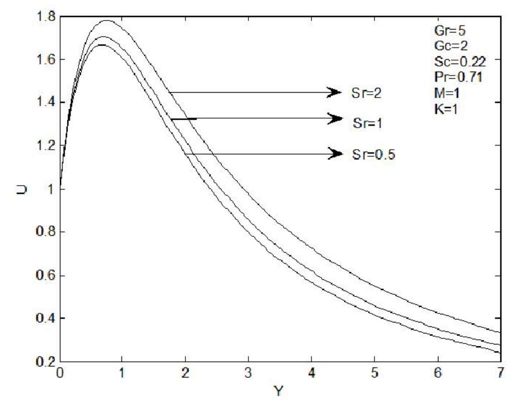

Fig. 4. Velocity Profile for different *Sr*

However, the theoretical solution for hydromagnetic convection on continuously moving isothermal vertical surface with uniform suction and mass diffusion is not studied in the literature. The present study deals with heat and mass transfer effects on flow past an impulsively started vertical porous surface in the presence of magnetic field and Thermo diffusion effect. The effect of velocity for different non dimensional parameters like Grashof number, Prandtl number, Schmidt number, Hartmann number, porosity parameter and soret number are analyzed graphically. In future, this work may be extended with hall effect, chemical reaction etc.

Analysis

Consider a two-dimensional steady incompressible flow of a viscous fluid on a continuous vertical porous surface, issuing from a slot and moving with a uniform velocity u_w, in a fluid at rest, in the presence of a transverse magnetic field of strength B_o. Let the *x*-axis be taken along the direction of motion of the sheet in the upward direction and the *y*-axis is taken normal to it. The surface temperature and concentration level near the surface are raised uniformly. If σ is the electrical conductivity of the fluid, then the governing equations are as follows:

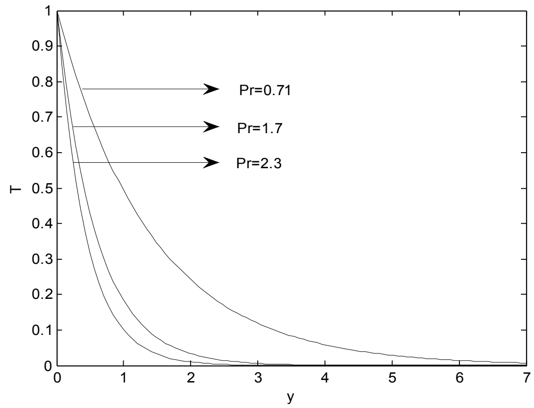

Fig. 5. Temperature Profile for different Pr

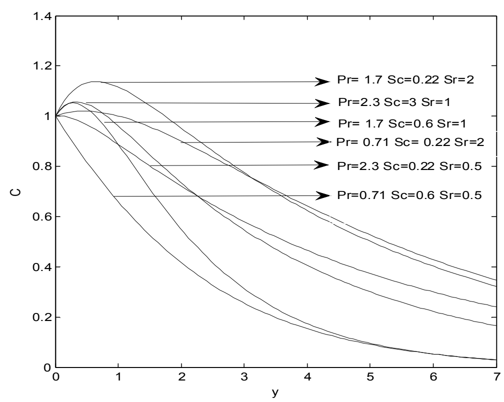

Fig. 6. Concentration Profile for different *Sc, Pr* and *Sr*

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots (1)$$

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2 u}{\rho} - \frac{\upsilon}{k}u \qquad \ldots (2)$$

$$\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T'}{\partial y} \right) = k\frac{\partial^2 T'}{\partial y^2} \qquad \ldots (3)$$

$$u\frac{\partial C'}{\partial x} + v\frac{\partial C'}{\partial y} = D\frac{\partial^2 C'}{\partial y^2} + \frac{D_m K_T}{T_m}\frac{d^2 T'}{dy^2} \qquad \ldots (4)$$

The initial and boundary conditions are

$$u = u_w, v = v_o = \text{const.} < 0,\ T' = T'_w, C' = C'_w \quad \text{at} \quad y = 0$$

$$u \to 0, \qquad T' \to T'_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty \qquad \ldots (5)$$

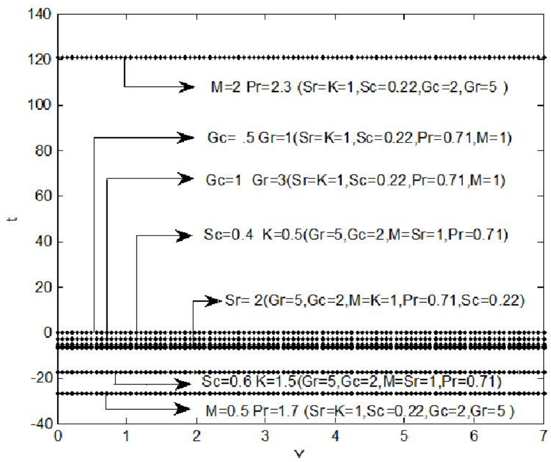

Fig. 7. Skin-friction for different *Pr, Gc, Gc, M, K, Sc* and *Sr*

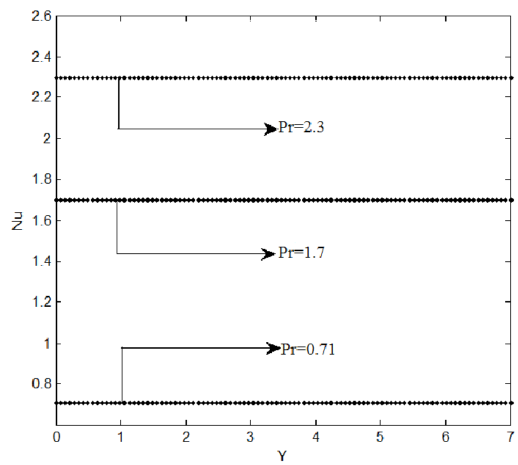

Fig. 8. Nusselt number for different *Pr*

Making use of the assumptions that the velocity, temperature and concentration fields are independent of the distance parallel to the surface and the Boussinesq's approximation taken into account, equations (1) to (4) and the boundary conditions (5) can be written as

$$-v_o \frac{du}{dy} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \upsilon \frac{d^2u}{dy^2} - \frac{\sigma B_o^2}{\rho} u - \frac{\upsilon}{k} u \quad \ldots (6)$$

$$-\sigma C_p v_o \frac{dT'}{dy} = k \frac{d^2 T'}{dy^2} \quad \ldots (7)$$

$$-v_o \frac{dC'}{dy} = D \frac{d^2 C'}{dy^2} + \frac{D_m K_T}{T_m} \frac{d^2 T'}{dy^2} \quad \ldots (8)$$

and the corresponding initial and boundary conditions are

$$u = u_w, \qquad T' = T'_w, C' = C'_w \quad \text{at} \quad y = 0$$

$$u \to 0, \qquad T' \to T'_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty \qquad \ldots (9)$$

On introducing the following non-dimensional quantities:

$$Y = \frac{yv_o}{\upsilon}, U = \frac{u}{u_w}, Gr = \frac{\upsilon g\beta(T'_w - T'_\infty)}{u_w v_o^2}, Gc = \frac{\upsilon g\beta^*(C'_w - C'_\infty)}{u_w v_o^2},$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \Pr = \frac{\mu C_p}{k}, M = \frac{\sigma B_o^2 \upsilon}{\rho v_o^2}, Sc = \frac{\upsilon}{D},$$

$$Sr = \frac{D_m K_T}{T_m \upsilon}\left[\frac{T'_w - T'_\infty}{C'_w - C'_\infty}\right], K = \frac{v_o^2 k}{\upsilon^2} \qquad \ldots (10)$$

Equations (6) to (8) are reduced to the following non-dimensional form

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} + GrT + GcC - \left(M + \frac{1}{K}\right)U = 0 \qquad \ldots (11)$$

$$\frac{d^2T}{dY^2} + \Pr\frac{dT}{dY} = 0 \qquad \ldots (12)$$

$$\frac{d^2C}{dY^2} + Sc\frac{dC}{dY} + SrSc\frac{d^2T}{dY^2} = 0 \qquad \ldots (13)$$

and the corresponding initial and boundary conditions in non-dimensional form are

$$U = 1, \quad T = 1, \quad C = 1 \qquad \text{at} \quad Y = 0$$

$$U \to 0, T \to 0, C \to 0 \qquad \text{as} \quad Y \to \infty \qquad \ldots (14)$$

Solving equations (11) to (13) with boundary conditions (14), we get

$$U(Y) = \left[1 + \frac{Gr}{\Pr^2 - \Pr - M - \frac{1}{K}} + \left\{\frac{Gc}{Sc^2 - Sc - M - \frac{1}{K}}\left(1 + \frac{ScSr\Pr}{\Pr - Sc}\right)\right\}\right]\exp\left[-\frac{1}{2}\left(1 + \sqrt{\left(1 + 4\left(M + \frac{1}{K}\right)\right)}\right)Y\right]$$

$$-\left[\frac{GcSrSc\Pr}{\left(\Pr^2 - \Pr - M - \frac{1}{K}\right)(\Pr - Sc)}\right]\exp\left[-\frac{1}{2}\left(1 + \sqrt{\left(1 + 4\left(M + \frac{1}{K}\right)\right)}\right)Y\right] - \left[\frac{Gr}{\Pr^2 - \Pr - M - \frac{1}{K}}\right]\exp(-\Pr Y)$$

$$-\left[\frac{Gc}{Sc^2 - Sc - M - \frac{1}{K}}\right]\exp(-ScY) + \left[\frac{GcSrSc\Pr}{\left(\Pr^2 - \Pr - M - \frac{1}{K}\right)(\Pr - Sc)}\right]\exp(-\Pr Y) \qquad \ldots (15)$$

$$T(Y) = \exp(-\Pr Y) \qquad \ldots (16)$$

$$C(Y) = \left[1 + \frac{ScSr\Pr}{\Pr - Sc}\right]\exp(-ScY) - \left[\frac{ScSr\Pr}{\Pr - Sc}\right]\exp(-\Pr Y) \qquad \ldots (17)$$

Skin- friction coefficient

The dimensionless skin-friction at the surface are given by

$$\tau = \left(\frac{dU}{dY}\right)_{Y=0} = \frac{1}{2}\left[1+\sqrt{\left(1+4\left(M+\frac{1}{K}\right)\right)}\right]\left[1+\frac{Gr}{\Pr^2-\Pr-M-\frac{1}{K}}\right.$$

$$\left.+\left[\frac{Gc}{Sc^2-Sc-M-\frac{1}{K}}\left(1+\frac{ScSr\Pr}{\Pr-Sc}\right)\right]\right]-\left[\frac{GcSrSc\Pr}{\left(\Pr^2-\Pr-M-\frac{1}{K}\right)(\Pr-Sc)}\right]$$

$$\left[\frac{1}{2}\left[1+\sqrt{\left(1+4\left(M+\frac{1}{K}\right)\right)}\right]\right]-\left[\frac{Gr\Pr}{\Pr^2-\Pr-M-\frac{1}{K}}\right]-\left[\frac{GcSc}{Sc^2-Sc-M-\frac{1}{K}}\right]$$

$$+\left[\frac{GcSrSc\Pr^2}{\left(\Pr^2-\Pr-M-\frac{1}{K}\right)(\Pr-Sc)}\right] \qquad \ldots (18)$$

Nusselt number

The rate of heat transfer coefficient at the vertical plate in non-dimensional form is given by

$$Nu = -\left(\frac{\partial T}{\partial Y}\right)_{Y=0} = \Pr \qquad \ldots (19)$$

Sherwood number

The rate of mass transfer coefficient at the vertical plate in non-dimensional form is given by

$$Sh = -\left(\frac{\partial C}{\partial Y}\right)_{Y=0} = Sc\left[1+\frac{Sc\Pr Sr}{\Pr-Sc}-\frac{Sr\Pr^2}{\Pr-Sc}\right] \qquad \ldots (20)$$

Results and Discussion

The computed solutions for the velocity, temperature, concentration and skin-friction are valid at some distance from the slot, even though suction is applied from the slot onward. Making use of the assumption that the velocity is independent of the distance parallel to the surface.

Fig. 1 shows the velocity profile for various thermal and mass Grashof number. From the profile, it is clear that the velocity profile increases with the increase of *Gr* and *Gc*. Fig. 2 gives the velocity profile for different Prandtl number and Hartmann number. The velocity profile decreases with the increase of Prandtl number and Hartmann number.

Fig. 3 exhibits the velocity profile for various Schmidt number and Porosity parameter. It is clear from the graph that with the increase in Schmidt number the profile decreases and the increase of porosity parameter increases the velocity profile. Fig. 4 gives the velocity profile for different soret number. The velocity profile increases with the increase of Soret number.

Fig. 5 shows that with the increase in Prandtl number the temperature profile decreases. Fig. 6 exhibits the concentration profile for various Schmidt number, Prandtl number and

Soret number. It is clear from the graph that with increase in Soret number the concentration profile increase and the increase in Schmidt number decrease the profile of concentration.

Fig. 7 shows the skin-friction profile for various *Pr*, *Gc*, *Gr*, *K*, *Sc*, *Sr* and *M*. It is clear from the graph that with the increase of Pr and M the profile increases and with the increase of *Gc, Gr, K, Sc*, and *Sr* the profile decreases. Fig. 8 shows the Nusselt number for different *Pr*. It is clear from the graph that with increase in Prandtl number the profile increases. Fig. 9 shows the Sherwood number for different *Pr* and *Sr*. It is clear from the graph that with increase in Soret number and Prandtl number the profile decrease.

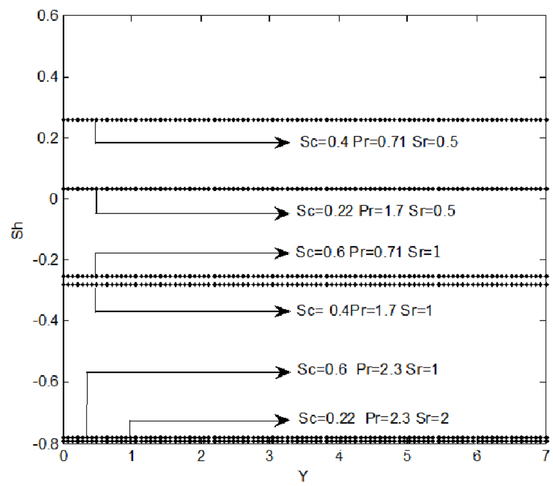

Fig. 9. Sherwood number for different *Pr* and *Sr*

Conclusions

Theoretical solution for Thermo diffusion effects on Heat and mass transfer on flow past an impulsively started isothermal vertical porous surface in the presence of magnetic field with uniform suction is obtained. The effects of different Grashof number, Prandtl number, Hartmann number, Schmidt number, Porosity parameter and Soret number are studied in detail. The solutions are in terms of exponential functions. The study concludes the following results:

(i) Velocity profile increases with increasing thermal Grashof number, mass Grashof number, Porosity parameter and Soret number. The trend is just reversed with respect to Schmidt number, Prandtl number and Hartmann number.

(ii) Temperature profile decreases with the increase of Prandtl number.

(iii) Concentration profile decreases with the increase of Schmidt number. Concentration profile increases with the increase of Soret number.

(iv) There is a fall in the skin-friction profile due to the increase of Schmidt number, Porosity parameter, thermal Grashof number, mass Grashof number and soret number. The skin-friction profile increases with increase of Prandtl number and Hartmann number.

(v) Nusselt number profile increases with the increase of Prandtl number.

(vi) Sherwood number profile decreases with the increase of Soret number and Prandtl number.

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