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PRE CONTINUOUS MAPPINGS IN A TOPOLOGICAL SPACES

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The present paper "Pre Continuous Mapping in a Topological Spaces" is a study of pre continuous mapping. The concepts of pre open subsets gives, the concepts of pre continuous mapping. In this paper we defined some definitions and then we discuss the characteristic properties of the pre continuous mapping. At last we give a number of references to help the other new researchers.

KEYWORDS : (i) Strongly continuous (ii) Pre continuous mapping

Introduction

The concept of pre-open subsets gives rise to the concept of pre-continuous mappings. We define some definition and its characteristic its properties. First of all we discuss continuity and strong continuity in to topological space.

Preliminaries

Let $f:(X, T) \rightarrow (Y, S)$ be a mapping in a topological space (X, T) to are other topological space (Y, S).

Definition (1.1) : A mapping $f:(X, T) \rightarrow (Y, S)$ of a topological space (X, T) into an other topological space (Y, S) is said to be

(i) Continuous on X iff $f^{-1}(v)$ is open in X, whenever is open in y.

(ii) Strogly continuous on X iff $f^{-1}(\text{int } A) \text{ int } f^{-1}(A)$, ... for every subset A of Y.

Definition (1.2) : Let $f: x \rightarrow y$ be a mapping then f is said to be pre-continuous at $x \in X$ if for every open set v containing $f(x)$ there exists a pre-open set L containing x s.t. $f(L) \subset v$.

Remarks (1.1) : If f is pre-continuous at every point $x \in X$ then f is said to be pre-continuous on X.

It is easy to see that.

Note (1.1) : Every strongly continous mapping is continuous but the converse is not true.

CHARACTERISTIC PROPERTIES

Theorem (1.1) : Let $f:(X, T) \to (Y, S)$ be a mapping then -

(i) f is continuous on X iff $f^{-1}(X)$ is closed in X for every closed set F in Y.

(ii) f is continuous on X iff.

$$f(\overline{A}) \subseteq \overline{f(A)}$$

for every subset A of X.

(iii) f is strongly continuous on X iff $f^{-1}(clA) = clf^{-1}(A)$, for every subset A of Y.

Now we prove the following results :

Theorem (1.2) : If $f:(X, T) \to (Y, S)$ strongly continuous on X then -

(i) $f^{-1}(\text{int } cl\ A) = \text{int } cl\ f^{-1}(A)$

(ii) $f^{-1}(cl \text{ int } A) = cl \text{ int } f^{-1}(A)$

for every subset A of Y.

Proof : Let cl $A = B$. then

$$\begin{aligned} f^{-1}(\text{int } cl\ (A)) &= f^{-1}(\text{int } B) \\ &= \text{int } f^{-1}(B) \\ &= \text{int } f^{-1}(cl\ A) \\ &= \text{int } cl\ f^{-1}(A) \end{aligned}$$

Similarly we can prove (ii)

Theorem (1.3) : A mapping $f: x \to y$ defined from a topological space X into a topological space Y is pre-continuous on X iff $f^{-1}(v)$ is pre open in X for every open set v in y.

Proof : Suppose that f is pre-continuous on X. Let v be an open set in y. We consider $f^{-1}(v)$. If $x \in f^{-1}(v)$ then there exists a pre open set L_x such that $f(L_3) \subseteq v$. This means that $L_x \subseteq f^{-1}(v)$.

If we do this for all $x \in f^{-1}(v)$ then we see that -

$$U\{L_x\{x \in f^{-1}(v)\} \text{ is pre open and } U\{L_x : x \in f^{-1}(v)$$

Since $f^{-1}(v) \subseteq U\{L_x : x \in f^{-1}(v)\}$, we see that $f^{-1}(v) = U\{L_x : x \in f^{-1}(v)\}$ is pre open in X.

Conversely, suppose that the given condition is satisfied, then for every $x \in X$ and every open set v containing $f(x)$, $f^{-1}(v)$ is a pre open set containing x s.t. $f(f^{-1}(v)) = v$.

So f is pre-continuous at every point $x \in X$. This proves that f is pre-continuous on X.

Theorem (1.4) : A mapping $f : x \to y$ defined from a topological space X into a topological space Y is pre-continuous on X iff $f^{-1}(v)$ is pre open in X for every open set V in Y.

Proof : Suppose that f is pre-continuous on X. Let v be an open set in Y. We consider $f^{-1}(v)$. If $x \in f^{-1}(v)$ then there exists a pre open set L_x such that $f(L_x) \subseteq v$. This means that $L_x \subseteq f^{-1}(v)$.

If we do this for all $x \in f^{-1}(v)$ then we see that -

$$U\{L_x : x \in f^{-1}(v)\} \text{ is pre open and } U\{L_x : x \in f^{-1}(v)\} \subset f^{-1}(v)$$

Since $f^{-1}(v) \subseteq \cup\{L_x : x \in f^{-1}(v)\}$, we see that $f^{-1}(v) = \cup\{L_x : x \in f^{-1}(v_e)\}$ is pre open in X.

Conversely, suppose that the given condition is satisfied. Then every $x \in X$ and every open set v containing $f(x)$, $f^{-1}(v)$ is a pre open set containing x s.t.

So f is pre-continuous at every point $x \in X$. This proves that f is pre-continuous on X.

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