

DIFFERENT TYPES OF CONVERGENCES, CORRESPONDING CONTINUITIES AND IRRESOLUTES IN TOPOLOGICAL SPACES

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The required paper "Different types of Convergences, Corresponding Continuities and Irresolutes" is discussed of different types of convergence corresponding continuities due to different types of open sets in a topological space. We have tried to find relation between them. We also try to find a relation with corresponding continuities and irresolutes. The paper contains definitions, example, preliminaries, a number of characteristic properties and their proof. At last there is a number of references.

KEYWORDS : L -Open Set, P -Open Set, L -Continuous Function, L -Irresolute.

INTRODUCTION

Here we discuss different types of convergences due to different type of open sets in a topological space and we try to find their relations with corresponding continuities and irresolute.

PRELIMINARIES

Let (X, T) be a topological space.

Definition (1.1). A sequence $\{x_n\}$ in a topological space X is said to converge to a point x w.r.t. L -open sets; written as $x_n \xrightarrow{L} x$ if for every L -open set K containing x there exists a positive integer N s.t. $x_n \in K$ for all $n \geq N$.

Characteristics Properties and Important Results : (Here L -Stands for pre, semi, α , β , generalized etc.)

Theorem (1.1). If an open subset is L -open then $x_n \xrightarrow{L} x \Rightarrow x_n \rightarrow x$.

Proof : Let K be an open set. Then K is L -open. So $x_n \xrightarrow{L} x$ implies that there exists a positive integer N such that $x_n \in K$ for all $n \geq N$. This is true for every open set K . Hence $x_n \rightarrow x$.

Remarks (1.1). The converse may not be true for this we consider the following example.

Example (1.1). For this we recall a topological space. If we consider the sequence $\{x_n\}$ with $x_n = c$ for all n then $x_n \rightarrow d$.

Because open subsets containing c and d ,

But $x_n \xrightarrow{P} D$ because p -open set containing d is $\{d\}$ which does into contain c , we can also prove that :

Theorem (1.2). If L_1 -open $\Rightarrow L_2$ -open then $x_n \xrightarrow{L_2} x \Rightarrow x_n \xrightarrow{L_1} x$.

Proof : This can be proof similar o that of theorem (1.1).

Now we prove the following theorem :

Theorem (1.3). Let $f : (X, T) \rightarrow (Y, S)$ be a mapping from X into Y . If f is L -continuous at $x \in X$ then $x_n \xrightarrow{L} x \Rightarrow f(x_n) \rightarrow f(x)$.

Proof : Suppose that f is L -continuous at $x \in X$. Also suppose that $x_n \xrightarrow{L} x$. Let v be an open set in Y containing $f(x)$. Then the L -continuity of at x implies that there is an L -open set U in X containing x such that $f(U) \subseteq v$.

Since $x_n \xrightarrow{L} x$, there exists a natural number N

Such that $n \geq N \Rightarrow x_n \in U \Rightarrow f(x_n) \in v$.

This proves that $f(x_n) \rightarrow f(x)$.

Theorem (1.4). Let $f : (X, T) \rightarrow (y, s)$ be a mapping from X into Y . If f is L -irrsolute at $x \in X$ then $x_n \xrightarrow{L} x \Rightarrow f(x_n) \xrightarrow{L} f(x)$.

Proof : Suppose that f is L -irresolute at $x \in X$. Suppose that $x_n \xrightarrow{L} x$. Let V be an L -Open set in y containing $f(x)$. Then there exist and L -openset U in x , containing x which that $f(U) \subseteq V$. Since $x_n \xrightarrow{L} x$. There exist a natural number N such that $n \geq N \Rightarrow x_n \in U \Rightarrow f(x_n) \in V \Rightarrow f(x_n) \xrightarrow{L} f(x)$.

Hence the result.

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