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HOST–MORTAL COMMENSAL SPECIES PAIR WITH HOST IMMIGRATED AT CONSTANT RATE-A SPECIAL CASE WITH NUMERICAL STUDY

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The present paper aims to analyze an ecological model which comprises the mortal commensal species and host species with a constant immigration of the host species by the numerical solutions. Further both the species are restricted to have limited resources. The concerned trajectories of this model are illustrated in the wide range of the parameters. The dominance reversal time of the host species over the commensal species is derived and vice versa is also done. The sustainability of this ecological commensal model in terms of various interactions between the species is discussed.

KEYWORDS : Non-linear system, Host species, Commensal Species, Mortal coefficient, Dominance Reversal time, Stability.

2010 Mathematics Subject Classification : 92D25, 92D40

Introduction

Many biological phenomena can be internally involved in the system of non-linear ordinary differential equations [6]. Multifarious situations with multiple conditions may influence directly or indirectly on the ecological models. The construction of Mathematical models [9] and their simulations are to be understood qualitatively and quantitatively. The study of biological phenomena such as harvesting of species and availability of biological resources is relevant in the ecological life of all human activities. Therefore, it is important to investigate the real life models which emphasize the possible interactions between species. There are different kinds of interactions [7-8] between species like Mutualism, Neutralism, Ammensalism [1-5], Commensalism [11-19], prey-predators [10], competition etc. in mathematical ecology the work related to continuous and discrete models have been devoted to investigate these models regarding periodicity, global stability, boundedness and others features.

The aim of this paper is to examine a mathematical model of ecological commensalism between two species with the help of classical method of 4^{th} order Runge-Kutta method. This model comprises a Host-Commensal species pair with limited resources in which the host species is being immigrated at constant rate. The model is characterized by a couple of first order non linear ordinary differential equations. The effect of changing is observed in the growth, balanced and mortal coefficients of the commensal species over the host species by fixing other parameters as constants. The interaction between the species under these considered conditions are discussed. The dominance reversal time is derived in all possible cases. The trajectories of this model are drawn by using DE Discover software and the conclusions are given.

Nomenclature

$N_1(t),\ N_2(t)$: The population rates of the commensal (S_1) and host (S_2) at time *t*.

d_1 : The mortal rate of the commensal (S_1).

a_2 : The rate of natural growth of the host (S_2).

a_{11} : The rate of decrease of the commensal (S_1) due to the limitations of its natural resources.

a_{22} : The rate of decrease of the host (S_2) due to the limitations of its natural resources.

a_{12} : The rate of increase of the commensal (S_1) due to the support given by the host.

$k_2 (= a_2 / a_{22})$: The carrying capacity of S_2.

$c (= a_{12} / a_{11})$: The coefficient of the commensal.

$e_1 (= d_1 / a_{11})$: The mortality coefficient of S_1.

$i_2 (= a_{22} I_2)$: The coefficient of renewal /immigration of the host.

I_2 : The renewal/replenishment/immigration of S_2 per unit time.

t^* : The dominance reversal time.

$t^*_{g_1}$: The dominance reversal time of the host over the commensal when birth rate is greater than the death rate.

t^*_0 : The dominance reversal time of the host over the commensal when death rate is equal to the birth rate.

$t^*_{e_1}$: The dominance reversal time of the host over the commensal when death rate is greater than the birth rate.

The state variables $N_1(t)$ and $N_2(t)$ as well as all the model parameters $d_1,\ a_2,\ a_{11},\ a_{22},\ a_{12},\ e_1,\ k_2,\ c,\ i_2$ are assumed to be non-negative constants.

Basic model equations

The model equations for the two species commensal-host ecosystem in which the host is being immigrated at constant rate by employing the above notations are given by the following system of non-linear coupled ordinary differential equations.

(i) Growth rate equation for the Mortal-Commensal species (S_1)

$$\frac{dN_1}{dt} = a_{11}N_1[-e_1 - N_1 + cN_2] \quad \ldots(1)$$

(ii) Growth rate equation for the Host species (S_2)

$$\frac{dN_2}{dt} = a_{22}[k_2 N_2 - N_2{}^2 + I_2] \quad \ldots(2)$$

with the initial conditions $N_i(0) = N_{i0} \geq 0, \ (i = 1, 2)$ ...(3)

The growth rate equations for **Balanced** (*i.e.* birth rate of the commensal is equal to its death rate) and **Growing** species (*i.e.* birth rate of the commensal is greater than its death rate) can be obtained by taking $e_1 = 0$ and $e_1 = -g_1$ in equation (1).

Numerical solutions of the growth rate equations for the ecological model

Numerical solution of the coupled non-linear basic differential equations (1) and (2) have been computed in the time interval [0, 10] in steps of one each employing Runge - Kutta system for a wide range of the model characterizing parameter for the mortal coefficient of the commensal e_1, keeping the commensal coefficient $c = 2.2$, the self inhibition coefficients $a_{11} = 0.03$, $a_{22} = 0.05$, the host carrying capacity $k_2 = 8$ and the immigrating rate for the host $I_2 = 5$ constants. The graphical illustrations of the obtained results are illustrated from Figs. 1 to 6.

Case : 1

In this case the host species out-numbers the commensal species irrespective of the nature of the commensal (*i.e.*, growing, balanced and mortal) till the time instants $t^*_{g_1} = 7.903$, $t^*_0 = 8.466$ and $t^*_{e_1} = 9.292$ respectively after which the out-numbering is reversed. Further the commensal species grows unbounded at a lower rate initially and the host species rises also at a lower rate to approach it's asymptotic value. (Vide Fig. 1)

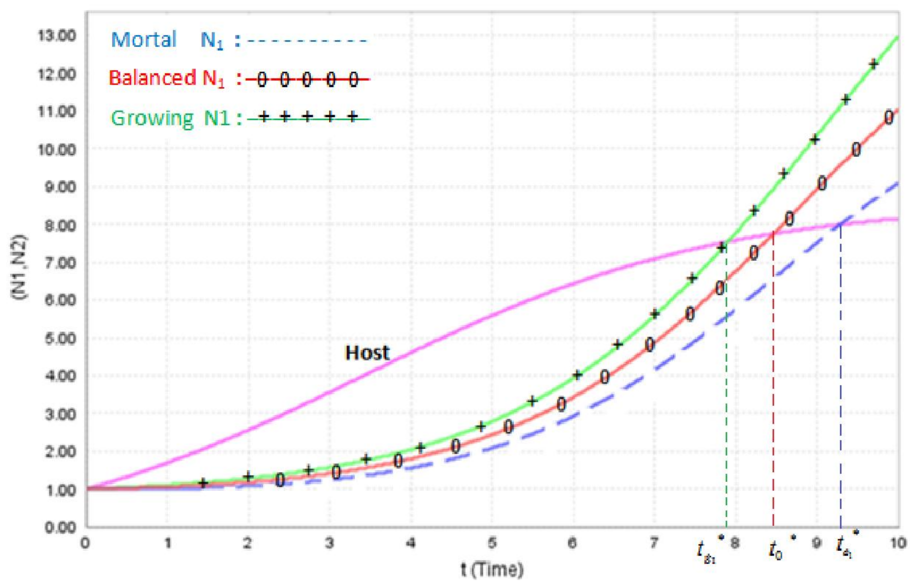

Fig. 1

Variation of N_1, N_2 vs. t for $a_{11} = 0.03$, $e_1 = 1.5$, $g_1 = 1.5$, $c = 2.2$, $a_{22} = 0.05$, $k_2 = 8$, $I_2 = 5$, $N_{10} = 1$, $N_{20} = 1$

Case : 2

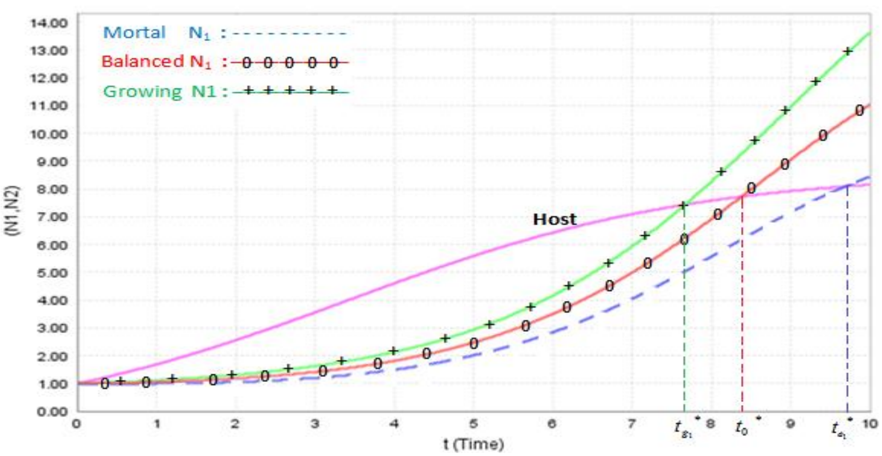

Fig. 2

Variation of N_1, N_2 vs. t for $a_{11} = 0.03$, $e_1 = 1.9$, $g_1 = 1.9$, $c = 2.2$, $a_{22} = 0.05$, $k_2 = 8$, $I_2 = 5$, $N_{10} = 1$, $N_{20} = 1$

Here the host species eclipses over the commensal species irrespective of the nature of the commensal up to the time instants $t_{g_1}^* = 7.668$, $t_0^* = 8.383$ and $t_{e_1}^* = 9.684$ respectively there after the dominance is reversed. The commensal species initially decreases slightly and then rises steeply where as the host species slowly increases to approach its asymptotic value. (as shown in Fig. 2).

Case : 3

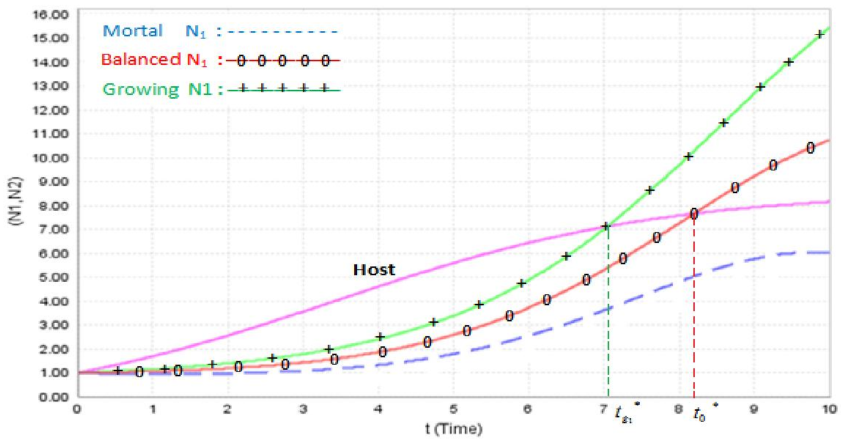

Fig. 3

Variation of N_1, N_2 vs. t for $a_{11} = 0.03$, $e_1 = 3$, $g_1 = 3$, $c = 2.2$,

$a_{22} = 0.05$, $k_2 = 8$, $I_2 = 5$, $N_{10} = 1$, $N_{20} = 1$

Initially the host species prevails over both the growing and balanced commensal up to the time instants $t_{g_1}^* = 7.05$ and $t_0^* = 8.2$ respectively after which the dominance is reversed. Also it is observed that the growing, balanced commensal slowly increase up to the time instants $t_{g_1}^*$, t_0^* there after rise steeply where as the mortal commensal gradually increases and then reaches its asymptotic value. (Vide Fig. 3)

Case: 4

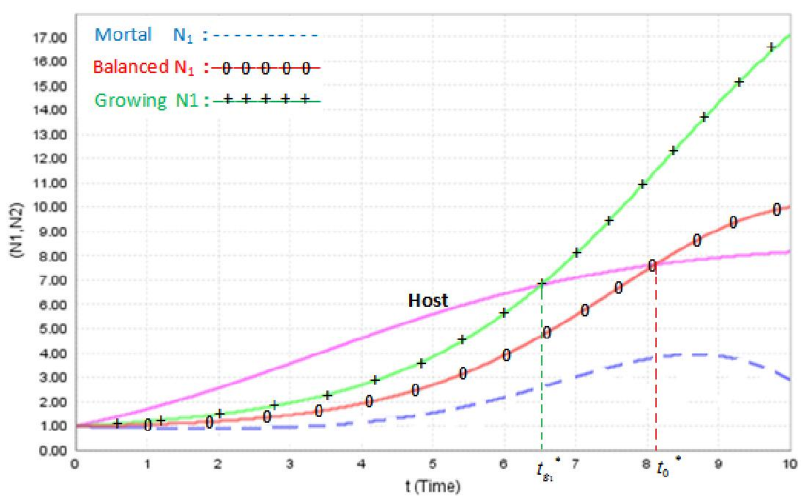

Fig. 4

Variation of N_1, N_2 vs. t for $a_{11} = 0.03$, $e_1 = 4$, $g_1 = 4$, $c = 2.2$,

$a_{22} = 0.05$, $k_2 = 8$, $I_2 = 5$, $N_{10} = 1$, $N_{20} = 1$

In this case, the growing and balanced commensal species dominate over that of the host species after the time instants $t_{g_1}^* = 6.517$ and $t_0^* = 8.108$ respectively. Also it is identified that the mortal commensal species has very low growth rate up to the time $t = 9$ and then declining further as shown in Fig. 4.

Case : 5

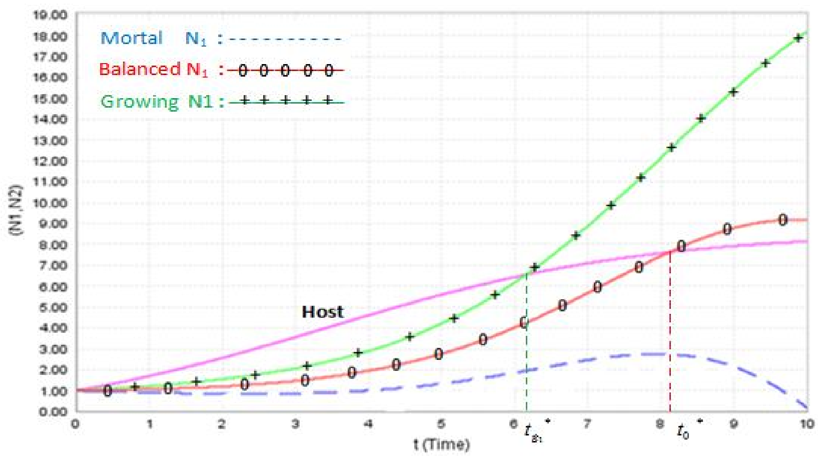

Fig. 5

Variation of N_1, N_2 vs. t for $a_{11} = 0.03$, $e_1 = 4.7$, $g_1 = 4.7$, $c = 2.2$,

$a_{22} = 0.05$, $k_2 = 8$, $I_2 = 5$, $N_{10} = 1$, $N_{20} = 1$

In this case, the host species out-numbers the growing and balanced commensal till the time instants $t_{g_1}^* = 6.157$ and $t_0^* = 8.114$ respectively after which the out-numbering is reversed. It is also noticed that the mortal commensal species is almost extinct where as the growing commensal species rises steeply and there is no appreciable growth rate in the host species. (Fig. 5).

Case : 6

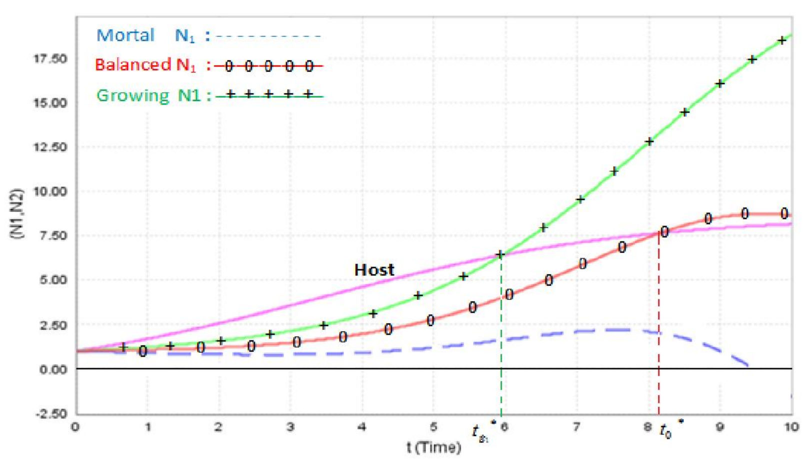

Fig. 6

Variation of N_1, N_2 vs. t for $a_{11} = 0.03,\ e_1 = 5.09,\ g_1 = 5.09,\ c = 2.2,$

$a_{22} = 0.05,\ k_2 = 8,\ I_2 = 5,\ N_{10} = 1,\ N_{20} = 1$

The host dominates over both the growing and balanced commensal till the time instants $t^*_{g_1} = 5.961$ and $t^*_0 = 8.164$ respectively there after the dominance is reversed. It can be noticed that the mortal commensal species extinct (at a time *t* = 9.472) earlier than the other species as shown in Fig. 6.

Conclusions

In this paper, some observations are identified in the mathematical model of commensal ecological Model which comprises the mortal commensal species and host species with a constant immigration of the host species. The analysis on various interactions is classified with obtained numerical solutions. Moreover, the impact of dominance reversal time in various situations is established. In addition to this, the nature of the stability in this model is also discussed.

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