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CHARACTERIZATIONS OF LOW SEPARATION AXIOMS VIA α^{s*}OPEN AND α^{s*} CLOSURE OPERATOR

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In this paper we introduce the concepts of using $\alpha^{s*}T_i$ and $\alpha^{s*}D_i$ spaces using – α^{s*} open sets and investigate some of their basic properties and give characterizations for these spaces. We also study the relationships among themselves T_i, Pre* T_i and $\alpha - T_i$ spaces.

KEYWORDS AND PHRASES : α^{s*}-open, α^{s*-} closed, α^{s*}-continuous, α^{s*}-T_i, α^{s*}-D_i (i = 0, 1, 2)

Introduction

The notion of α-open sets was introduced by O. Njastad [12] in 1965. M. Cladas *et al* introduced the notion α-T_i, (i = 0, 1, 2) and α-D_i, (i = 0, 1, 2) spaces using α-open sets. Maheswari and Prasad [9] introduced the notion of semi-T_i, (i = 0, 1, 2) spaces using semi open sets in 1975. Askishkar and Bhattacha introduced the concepts of Pre- T_i, (i = 0, 1, 2) spaces. T. Selvi and A. Punitha Tharani introduced the concepts of Pre*-T_i, (i = 0, 1, 2) spaces. Quite recently , the authors introduced a new class of nearly open set namely α-open sets and studied some functions using these sets.

In this paper, we introduce α^{s}*-T_i, and α^{s}*-D_i (i = 0, 1, 2) spaces using α^{s}*-open sets and investigate some of their basic properties. We also study the relationships among themselves and with known separation axioms T_i.α-T_i, pre*-T_i, D_i and α-D_i (i = 0, 1, 2).

Preliminaries

Throughout this paper cl$A$ and int$A$ respectively closure and the interior of the set A where A is a subset of a Topological spaces (X, τ) on which no separation axioms are assumed unless explicitly stated. The following definitions and results are listed because of their use in the sequel.

Definition 2.1. A subset A of a topological space (X, τ) is called

(i) generalized closed (briefly g-closed) if cl (A) $\subseteq U$ whenever $A \subseteq U$ and U is open in X.

(ii) generalized open (briefly g-open) if $X \backslash A$ is g-closed in X.

Definition 2.2. Let A be a subset of X. The generalized closure of A is defined as the intersection of all g-closed sets containing A and is denoted by cl*(A).

Definition 2.3. A subset A of a topological space (X, τ) is called

(i) semi-open [7] (resp. pre-open [10], α-open [12] and α^{s}* open [5]) if $A \subseteq$ cl (int (A)) (resp. $A \subseteq$ int (cl (A)), $A \subseteq$ int (cl (int (A))) and $A \subseteq$ int*(cl (int (A))).

(ii) semi-closed [7](resp. pre-closed [10], α-closed [12] and α^{s}* closed [5]) if $X \backslash A$ is semi-open (resp. pre-open, α-open and α^{s}* open) or equivalently if int (cl (A)) $\subseteq A$ (resp. cl (int (A)) $\subseteq A$, cl (int (cl (A))) $\subseteq A$ and cl*(int (cl (A))) $\subseteq A$),

Definition 2.4. Let A be a subset of X. Then the α^{s}*-closure of A is defined as the intersection of all – α^{s}*closed sets containing A and it is denoted by α^{s}*cl (A).

Definition 2.5. A space X is said to be T_0 [14] (resp, semi T_0 [9], pre-T_0 [1], α-T_0 [11]) if for every pair of distinct points x and y in X, there is an open (resp, semi open, pre-open, α-open) set in X containing one of x and y but not the other.

Definition 2.6. A space X is said to be T_1 [14] (resp, semi T_1 [10], pre-T_1 [1], α-T_1 [12]) if for every pair of distinct points x and y in X, there are an open (resp, semi open, pre-open, α-open) sets U and V such that U contains x but not y and Vcontains y but not x.

Definition 2.7. A space X is said to be T_2 [14] (resp, semi T_2 [19], pre-T_2 [1], α-T_2 [12]) if for every pair of distinct points x and y in X, there are disjoint open (resp, semi-open, pre-open, α-open) sets U and V in X containing x and y respectively.

Definition 2.8 [3]. A subset A of a topological space X is said to be αD-set if there are two $U, V \in \alpha O(X, \tau)$ such that $U \neq X$ and $A = U - V$.

Definition 2.9 [3]. A space X is said to be α-D_0 if for every pair of distinct points x and y in X, there exists an αD-set of X containing x but not y or an αD–set of X containing y but not x.

Definition 2.10 [3]. A space X is said to be α-D_1 if for every pair of distinct points x and y in X, there exists an αD-set of X containing x but not y and an αD-set of X containing y but not x.

Definition 2.11 [3]. A space X is said to be α-D_2 if for every pair of distinct points x and y in X, there exists disjoint αD-sets G and E of X containing x and y respectively.

Theorem 2.12 [5].

(i) Every α-open set is α^{s*} open and every α-closed set is α^{s*}closed.

(ii) Every α^{s*}-open is pre*open and every α^{s*}-closed set is pre*-closed.

(iii) Every open set is α^{s*} open and every closed set is α^{s*}closed

Theorem 2.13 [5]. Let $A \subseteq X$ and let $x \in X$. Then $x \in \alpha^{s*}$cl (A) if and ony if every α^{s*}-open set in X containing x intersects A.

Theorem 2.14 [5]. If $\{A_\alpha\}$ is a collection of α^{s*}-open sets in X, then $\cup A\alpha$ is also α^{s*}–open in X.

Lemma 2.15. A topological space (X, τ) is T_1 if and only if $\{x\}$ is closed for every $x \in X$. [15]

Definition 2.16 [6]. A function $f : X \rightarrow Y$ is said to be

(i) α^{s*}-continuous if $f^{-1}(V)$ is α^{s*}open in X. for every open set V in Y.

(ii) $M\alpha^{s*}$-continuous if $f^{-1}(V)$ is α^{s*}open in X. for every α^{s*}-open set V in Y.

(iii) α^{s*}-open if $f(V)$ is α^{s*}open in Y.for every open set V in X.

(iv) $M\ \alpha^{s*}$-open if $f(V)$ is α^{s*}open in Y.for every α^{s*}-open set V in X.

(v) α^{s*}-closed if $f(V)$ is α^{s*}closed in Y.for every closed set V in X.

(vi) $M\ \alpha^{s*}$-closed if $f(V)$ is α^{s*}closed in Y.for every α^{s*}-closed set V in X.

Theorem 2.17 [14]. A space X is α-T_0 iff $\alpha\text{cl}\{x\} \neq \alpha\text{cl}\{y\}$ for every pair of distinct points of x, y of X.

α^{s*}-T_0 SPACE

Definition 3.1. A topological space X is said to be an α^{s*}-T_0 if for any two distinct points x and y of X, there exists an α^{s*}-open set G in X such that $x \in G$ and $y \notin G$ or ($y \in G$ and $x \notin G$)

Theorem 3.2. Every α–T_0 space is α^{s*}–T_0 space

Proof : Let X be a α-T_0 space. Let x and y be two distinct points in X. since X is α-T_0, there exists an α-open set U such that ($x \in U$ and $y \notin U$) or ($y \in U$ and $x \notin U$). By Theorem 2.12, U is α^{s*}-open set such that ($x \in U$ and $y \notin U$) or ($y \in U$ and $x \notin U$). Thus X is α^{s*}–T_0.

Conversely, suppose for any $x, y \in X$ with $x \neq y$, $\alpha^{s*}\text{cl}\{x\} \neq \alpha^{s*}\text{cl}\{y\}$. Without any loss of generality, Let $z \in X$ such that $z \in \alpha^{s*}\text{cl}\{x\}$ but $z \notin \alpha^{s*}\text{cl}\ \{y\}$. Now we claim that $x \notin \alpha^{s*}\text{cl}\{y\}$. For if $x \in \alpha^{s*}\text{cl}\{y\}$ then $\{x\} \subseteq \alpha^{s*}\text{cl}\ \{y\}$ which implies that $\alpha^{s*}\text{cl}\ \{x\} \subseteq \alpha^{s*}\text{cl}\ \{y\}$. This contradicts the fact that $z \notin \alpha^{s*}\text{cl}\{y\}$. Consequently x belongs to the α^{s*}-open set $[\alpha^{s*}\text{cl}\{y\}]^c$ to which y does not belong . Hence the space is an α^{s*}–T_0-space

Theorem 3.3. A space X is α^{s*}-T_0 iff the α^{s*}-closures of distinct points are distinct.

Proof : Let x and y be two distinct points of a space X. Since X is α^{s*}-T_0, Now by definition, there exists a α^{s*}-open set U such that $x \in U$ but $y \notin U$ or $y \in U$ but $x \notin U$. If $x \in U$ and $y \notin U$, then U is a α^{s*}-open set containing x that does not intersect $\{y\}$. By using Theorem 2.13, it follows that $x \notin \alpha^{s*}\text{cl}\ (\{y\})$. But $x \in \alpha^{s*}\text{cl}\ (\{x\})$, so we get $\alpha^{s*}\text{cl}\ (\{x\}) \neq \alpha^{s*}\text{cl}\ (\{y\})$. The proof of the other case is similar.

α^{s*}–T_1 SPACE

In this section we introduce α^{s*}–T_1 spaces and investigate some of their basic properties.

Definition 4.1. A topological space X is said to be an α^{s*}-T_1 if for every pair of distinct points x and y of X, there exists an α^{s*}-open sets U and V such that $x \in U$ and $y \notin U$ and $y \in V$ but $x \notin V$.

Theorem 4. 2

(i) Every α–T_1 space is α^{s*}–T_1 space

(ii) Every T_1 space is α^{s*}–T_1 space

(iii) Every α^{s*}–T_1 space is pre*–T_1

Proof : (i) Suppose X is α–T_1 space. Let x and y be two distinct points in X. since X is α–T_1 space, there exists α-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. By Theorem 2.12 U and V are α^{s*}-open sets such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. Hence X is α^{s*}–T_1 .

(ii) Suppose X is T_1 space. Let x and y be two distinct points in X. since X is T_1 space, there exists open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. By Theorem 2.12 U and V are α^{s*}-open sets such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. Hence X is α^{s*}–T_1.

(iii) Suppose X is α^{s*}–T_1 space. Let x and y be two distinct points in X. since X is α^{s*}–T_1 space, there exists α^{s*}-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. By Theorem 2.12 U and V are pre*-open sets such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. Hence X is pre*-T_1.

Remark 4.3. The converse of the above theorem is not true as shown in the following Example.

Example 4.4. Consider the space (X, τ) where $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ then (X, τ) is α^{s*}–T_1 but not α–T_1. Hence α^{s*}–T_1 does not implies α–T_1.

Example 4.5. Consider the space (X, τ) where $X = \{a, b, c, d, e, f, g\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, X\}$ then (X, τ) is α^{s*}–T_1 but not T_1 . Hence α^{s*}–T_1 does not implies T_1.

Example 4.6. Consider the space (X, τ) where $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a, b, c\}, X\}$ then (X, τ) is pre*–T_1 but not α^{s*}–T_1. Hence pre*–T_1 does not implies α^{s*}–T_1.

Theorem 4.7. For a topological space X, the following are equivalent.

(i) X is α^{s*}–T_1 space.

(ii) Each singleton set is α^{s*}-closed in X.

(iii) The intersection of all α^{s*}-open sets containing the set A is A.

(iv) The intersection of all α^{s*}-open sets containing the point $x \in X$ is $\{x\}$.

Proof : (i) ⇒ (ii)

Let X be a α^{s*}–T_1 space and $x \in X$. Then for every $y \neq x$ there exists an α^{s*}-open set U_y in X such that containing y but not x. That is $y \in U_y \subseteq X \backslash \{x\}$. Therefore, $X \backslash \{x\} = \cup \{U_y : y \in X \backslash \{x\}\}$ is α^{s*}-open in X. By Theorem (2.14), it follows that $\{x\}$ is α^{s*}-closed.

(ii) ⇒ (iii)

Let $A \subseteq X$ then for each $x \in X \backslash A$, $\{x\}$ is α^{s*}-closed in X and hence $X \backslash \{x\}$ is α^{s*}-open. Clearly $A \subseteq X \backslash \{x\}$ for each $x \in X \backslash A$. Therefore $A \subseteq \cap \{X \backslash \{x\} : x \in X \backslash A\}$. On the other hand, if $y \notin A$ then $y \in X \backslash A$ and $y \notin X \backslash \{y\}$. Therefore $y \notin \cap \{X \backslash \{x\} : x \in X \backslash A\}$ and hence $\cap \{X \backslash \{x\} : x \in X \backslash A\} \subseteq A$. This proves (iii).

(iii) ⇒ (iv)

Take $A = \{x\}$. Then $A = \{x\} = \cap \{U : U$ is α^{s*}-open and $x \in U\}$. This proves (iv).

(iv) ⇒ (i)

Let $x, y \in X$ and $y \neq x$. Then $y \notin \{x\} = \cap \{U : U$ is α^{s*}-open and $x \in U\}$. Hence there exists α^{s*}-open set U containing x but not y. Similarly, there exists a α^{s*}-open set V containing y but not x. Thus X is α^{s*}–T_1 space.

Theorem 4.8. Let $f : X \to Y$ be a function

(i) If f is a α^{s*}-closed surjection and X is T_1, then Y is α^{s*}–T_1.

(ii) If f is a M- α^{s*}closed surjection and X is α^{s*}–T_1, then Y is α^{s*}–T_1.

(iii) If f is a α^{s*}-continuous bijection and Y is T_1 then X is α^{s*}–T_1.

(iv) If f is a M-α^{s*} continuous bijection and Y is α^{s*}–T_1, then X is α^{s*}–T_1.

Proof: (i) Suppose $f : X \to Y$ is α^{s*}-closed and X is T_1. Let $y \in Y$. Since f is onto, there exists $x \in X$, such that $f(x) = y$. Since X is T_1, By Lemma (2.15), $\{x\}$ is closed in X. Since f

is α^{s*}-closed map, $f(\{x\}) = \{y\}$ is α^{s*}-closed. Since every singleton set in Y is α^{s*}-closed, by Theorem Y is α^{s*}–T_1.

(ii) Suppose f is M-α^{s*}closed and X is α^{s*}–T_1. Let $y \in Y$. since f is onto, there exists $x \in X$, such that $f(x) = y$. since X is α^{s*}–T_1, by Theorem (4.7), $\{x\}$ is α^{s*}-closed in X. Since f is M-α^{s*}cosed, $f(\{x\}) = \{y\}$ is α^{s*}-closed in Y. Since every singleton set in Y is –α^{s*}closed, again by Theorem 4.7, Y is α^{s*}–T_1.

(iii) Suppose $f : X \to Y$ is α^{s*}-continuous bijection and Y is T_1. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is one-one, $y_1 \neq y_2$. Since Y is T_1, there exists open sets U and V such that $y_1 \in U$ but $y_2 \notin U$ and $y_2 \in V$ but $y_1 \notin V$. Again since f is a bijection, $x_1 \in f^{-1}(U)$ but $x_2 \notin f^{-1}(U)$ and $x_2 \in f^{-1}(V)$ but $x_1 \notin f^{-1}(V)$. since f is α^{s*}-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are α^{s*}-open sets in X. This shows that X is α^{s*}–T_1.

(iv) Suppose $f : X \to Y$is a M-α^{s*}continuous bijection and Y is α^{s*}–T_1. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is one-one, $y_1 \neq y_2$. Since Y is α^{s*}–T_1. There exists α^{s*}-open sets U and V such that $y_1 \in U$ but $y_2 \notin U$ and $y_2 \in V$ but $y_1 \notin V$. Again since f is a bijection, $x_1 \in f^{-1}(U)$ but $x_2 \notin f^{-1}(U)$ and $x_2 \in f^{-1}(V)$ but $x_1 \notin f^{-1}(V)$. Since f is M-α^{s*}continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are α^{s*}-open sets in X. This shows that X is α^{s*}–T_1.

α^{s*}–T_2 SPACE

In this section we introduce α^{s*}–T_2 spaces and investigate some of their basic properties.

Definition 5.1. A topological space X is said to be α^{s*}-T_2 if for every pair of distinct points x and y in X, there are disjoint α^{s*}-open sets U and V in X containing x and y respectively.

Theorem 5.2.

(i) Every α–T_2 space is α^{s*}–T_2 space.

(ii) Every T_2 space is α^{s*}–T_2 space.

(iii) Every α^{s*}–T_2 space is pre*–T_2.

Proof : (i) Let X be a α–T_2 space. Let x and y be two distinct points in X. since X is α–T_2 space, there exists disjoint α-open sets U and V such that $x \in U$ and $y \in V$. By Theorem (2.12), U and V are disjoint α^{s*}-open sets such that $x \in U$ and $y \in V$. Hence X is α^{s*}–T_2.

(ii) Suppose X is T_2 space. Let x and y be two distinct points in X. since X is T_2 space, there exists disjoint open sets U and V such that $x \in U$ and $y \in V$. By theorem (2.12) U and V are disjoint α^{s*}-open sets such that $x \in U$ and $y \in V$. Hence X is α^{s*}–T_2.

(iii) Suppose X is α^{s*}–T_2 space. Let x and y be two distinct points in X. since X is α^{s*}–T_2 space, there exists disjoint α^{s*}–open sets U and V such that $x \in U$ and $y \in V$. By Theorem (2.12) U and V are pre*-open sets such that $x \in U$ and $y \in V$. Hence X is pre*-T_2.

Remark 5.3. The converse of the statements (i), (ii) and (iii) of the above theorem is not true as shown in the following example.

Example 5.4. Consider the space (X, τ) where $X = \{a, b, c, d, e, f, g\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, X\}$ then (X, τ) is α^{s*}–T_2 but not α-T_2 and not T_2. Hence α^{s*}–T_2 does not implies α-T_2 and T_2.

Example 5.5. Consider the space (X, τ) where $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a, b\}, \{c, d\}, X\}$ then (X, τ) is pre*-T_2 but not α^{s*}–T_2 . Hence pre*-T_2 does not implies α^{s*}–T_2.

Theorem 5.6. Every α^{s*}–T_2 space is α^{s*}–T_1.

Proof : Let X be a α^{s*}–T_2. Let x and y be two distinct points in X. Since X is α^{s*}–T_2. There exists disjoint α^{s*}-open sets U and V such that $x \in U$ and $y \in V$. Since U and V are disjoint, $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. Hence X is α^{s*}–T_1.

However the converse is not true as shown in the following

Example 5.7. Consider $X = \{a, b, c, d\}$ where $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then X is α^{s*}–T_1 but not α^{s*}–T_2.

Theorem 5.8. For a topological space X, the following are equivalent.

(i) X is α^{s*}–T_2 space

(ii) Let $x \in X$, then for each $y \neq x$ there exists a α^{s*}-open sets U such that $x \in U$ and $y \notin \alpha^{s*}\mathrm{cl}\,(U)$.

(iii) for each $x \in X$, $\cap\{\alpha^{s*}\mathrm{cl}\,(U) : U \in \alpha^{s*}\,O\,(X)$ and $x \in U\} = \{x\}$

Proof : (i) ⇒ (ii)

Let X be a α^{s*}–T_2 space. then for every $y \neq x$ there exists disjoint α^{s*}-open set U and V such that $x \in U$ and $y \in V$. Since V is α^{s*}-open, $X \backslash V$ is α^{s*}-closed and $U \subseteq X \backslash V$. This implies that $\alpha^{s*}\mathrm{cl}\,(U) \subseteq X \backslash V$. Since $y \notin X \backslash V$, $y \notin \alpha^{s*}\mathrm{cl}\,(U)$.

(ii) ⇒ (iii)

If $y \neq x$, then there exists an α^{s*}-open set U such that $x \in U$ and $y \notin \alpha^{s*}\mathrm{cl}\,(U)$. Therefore $y \notin \cap\,\{\alpha^{s*}\mathrm{cl}\,(U) : U \in \alpha^{s*}\,O\,(X)$ and $x \in U\}$. This proves (iii).

(iii) ⇒ (i)

Let $y \neq x$ in X . then $y \notin \{x\} = \cap\,\{\alpha^{s*}\mathrm{cl}\,(U) : U \in \alpha^{s*}O\,(X)$ and $x \in U\}$. This implies that there exists an α^{s*}-open set U such that $x \in U$ and $y \notin \alpha^{s*}\mathrm{cl}\,(U)$. Let $V = X \backslash \alpha^{s*}\,\mathrm{cl}\,(U)$. Then V is $-\alpha^{s*}$open and $y \in V$. Now $U \cap V = U \cap (X \backslash \alpha^{s*}\,\mathrm{cl}\,(U)) \subseteq U \cap (X \backslash U) = \emptyset$. Therefore X is a α^{s*}-T_2 space.

Theorem 5.9. Let $f : X \to Y$ be a bijection

(i) If f is a α^{s*}-open and X is T_2, then Y is α^{s*}–T_2.

(ii) If f is a M- α^{s*}open and X is α^{s*}–T_2, then Y is α^{s*}–T_2.

(iii) If f is α^{s*}-continuous and Y is T_2 then X is α^{s*}–T_2.

(iv) If f is M-α^{s*} continuous and Y is α^{s*}–T_2, then X is α^{s*}–T_2.

Proof : Let $f : X \to Y$ be a bijectioin.

(i) Suppose f is α^{s*}-open and X is T_2. Let $y_1 \neq y_2 \in Y$. Since f is a bijection function, there exists $x_1, x_2 \in X$, such that $f(x_1) = y_1$ and $f(x_2) = y_2$ with $x_1 \neq x_2$. since X is T_2, There exists disjoint open sets U and V in X such that $x_1 \in U$ and $x_2 \in V$. since f is α^{s*}-open map, $f(U)$ and $f(V)$ are α^{s*}-open in Y such that $y_1 = f(x_1) \in f(U)$ and $y_2 = f(x_2) \in f(V)$. Again since f is a bijection $f(U)$ and $f(V)$ are disjoint in Y. Thus Y is α^{s*}–T_2.

(ii) Suppose f is M-α^{s*}open and X is $\alpha^{s*} - \mathrm{T}_2$. Let $y_1 \neq y_2 \in Y$. Since f is a bijection function, there exists $x_1, x_2 \in X$, such that $f(x_1) = y_1$ and $f(x_2) = y_2$ with $x_1 \neq x_2$. Since X is $\alpha^{s*} - T_2$. There exists disjoint α^{s*}open sets U and V in X such that $x_1 \in U$ and $x_2 \in V$. Since f is M-α^{s*}open map, $f(U)$ and $f(V)$ are disjoint α^{s*}-open in Y containing y_1 and y_2. This shows that X is α^{s*}–T_2.

(iii) Suppose $f : X \to Y$ is a α^{s*}continuous bijection and Y is T_2. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is one-one, $y_1 \neq y_2$. Since Y is T_2. There exists disjoint open sets U and V containing y_1 and y_2 respectively. Again since f is α^{s*}continuous bijection, f

$^{-1}$ (U) and f^{-1} (V) are disjoint α^{s*}open sets in X containing x_1 and x_2 respectively. Thus X is α^{s*}–T_2

(iv) Suppose $f : X \rightarrow Y$ is a M-α^{s*}continuous bijection and Y is $\alpha^{s*} - T_2$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is one-one, $y_1 \neq y_2$. Since Y is $\alpha^{s*} - T_2$. There exists disjoint α^{s*}-open sets U and V containing y_1 and y_2 respectively. Again since f is M-α^{s*}continuous bijection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint α^{s*}open sets in X containing x_1 and x_2 respectively. Thus X is α^{s*}–T_2.

α^{s*}–D

Definition 6.1. A subset A of a Topological space X is called an α^{s*}–D set if there are two U, $V \in \alpha^{s*}O(X, \tau)$ such that $U \neq X$ and $A = U - V$.

Observe that every α^{s*}-open set U different from X is an α^{s*} –D set if $A = U$ and $V = \emptyset$.

Theorem 6.2. Every α–D set is α^{s*}–D set.

Proof : Let $\underline{A} \subseteq X$ and A is an α–D set . then there are two U, $V \in \alpha\ O(X, \tau)$ such that $U \neq X$ and $A = U - V$. By Theorem (2.12), U, $V \in \alpha^{s*}\ O(X, \tau)$. Hence A is α^{s*}–D set.

But the converse is not true as shown in the following example.

Example 6.3. Let $X = \{a, b, c, d\}$ $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Here $\{c, d\}$ is α^{s*}–D set but not α–D set.

Definition 6.4. A topological space X is called α^{s*}–D_0 if for any distinct pair of points x and y in X there exists an α^{s*}–D set of X containing x but not y or an α^{s*}–D set of X containing y but not x.

Definition 6.5. A topological space (X, τ) is called α^{s*}–D_1 if for any distinct pair of points x and y of X there exists an α^{s*}–D set of X containing x but not y and an α^{s*}–D set of X containing y but not x.

Definition 6.6. A topological space (X, τ) is called α^{s*}–D_2 if for any pair of distinct points x and y of X there exists disjoint α^{s*}–D sets G and E of X containing x and y respectively.

Theorem 6.7. Every α-D_i is α^{s*}-D_i, $(i = 0, 1, 2)$.

Proof : Form the Theorem (2.12) and Definition of α-D_i and α^{s*}-D_i, $(i = 0, 1, 2)$.

Remark 6.8.

(i) If (X, τ) is α^{s*}-T_i then (X, τ) is α^{s*}-D_i,$(i = 0, 1, 2)$

(ii) If (X, τ) is α^{s*}-D_i then (X, τ) is α^{s*}-D_{i-1}, $(i = 0, 1, 2)$

Theorem 6.9.

(i) (X, τ) is α^{s*}-D_0 iff it is α^{s*}-T_0

(ii) (X, τ) is α^{s*}-D_1 iff it is α^{s*}-D_2

Proof : (i) The sufficiency is stated in Remark (6.10). The prove necessity, let (X, τ) be α^{s*}-D_0. then for each distinct pair of $x, y \in X$, at least one of x, y say x belongs to an α^{s*}-D-set G where $y \notin G$. Let $G = U_1 \backslash U_2$ such that $U_1 \neq X$ and $U_1, U_2 \in \alpha^{s*}O(X, \tau)$ then $x \in U_1$. For $y \notin G$ we have two cases (a) $y \notin U_1$ (b) $y \in U_1$ and $y \in U_2$.

In case (a), $x \in U_1$ but $y \notin U_1$

In case (b), $y \in U_2$ but $x \notin U_2$. Hence X is α^{s*}-T_0.

(ii) Sufficiency: Remark (6.10)

Necessity. Suppose that X is α^{s*}-D_1. then for each distinct pair $x, y \in X$. We have α^{s*}–D sets G_1, G_2 such that $x \in G_1$, $y \notin G_1$, $y \in G_2$, $x \notin G_2$. Let $G = U_1 \backslash U_2$, $G = U_3 \backslash U_4$. By $x \notin G_2$, it follows that either $x \in U_3$ or $x \in U_3$ and $x \in U_4$.Now we consider two cases.

(i) $x \notin U_3$. By $y \notin G_1$, we have two sub cases

(a) $y \notin U_1$. But $x \in U_1 \backslash U_2$, it follows that $x \in U_1 \backslash (U_2 \cup U_3)$ and by $y \in U_3 \backslash U_4$ we have $y \in U_3 \backslash (U_4 \cup U_1)$. Hence $(U_1 \backslash (U_2 \cup U_3) \cap U_3 \backslash (U_1 \cup U_4) = \emptyset$.

(b) $y \in U_1$ and $y \in U_2$, we have $x \in U_1 \backslash U_2$, $y \in U_2$, $(U_1 \backslash U_2) \cap U_2 = \emptyset$.

(ii) $x \in U_3$ and $x \in U_4$. we have $y \in U_3 \backslash U_4$, $x \in U_4$, $(U_3 \backslash U_4) \cap U_4 = \emptyset$.

Therefore X is α^{s*}-D_2.

Theorem 6.10. If $(X,)$ is α^{s*}-D_1 then it is α^{s*}-T_0

Proof : Follows Remark 6.10 and Theorem 6.9.

Example 6.11. Consider the space (X, τ), where $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a, b\}, \{c, d\}, \{b\}, \{b, c, d\}, X\}$. Then X is $\alpha^{s*}T_0$ but not $\alpha^{s*}D_1$.

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