RINGS ON PRE *A**-ALGEBRAS

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This paper is a study on algebraic structure of Pre *A**-algebra and proves basic theorems on Pre *A**-algebra. This paper defines *p*-ring, Boolean ring and 3-ring. Establish Pre *A**-algebra as a Boolean ring and Boolean ring as a Pre *A**-algebra. It includes to prove Pre *A**-algebra as a 3-ring and 3-ring as a Pre *A**-algebra.

KEYWORDS : Pre A^*-algebra, *p*-ring, ring on Pre *A**-algebra, Boolean ring, 3-ring.

Introduction

In a draft paper [4], *The Equational theory of Disjoint Alternatives*, around 1989, E.G. Manes introduced the concept of Ada (Algebra of disjoint alternatives) $(A, , V, (–)^I, (–), 0, 1, 2)$ (Where, V are binary operations on A, $(–)^I$, $(–)$ are unary operations and 0, 1, 2 are distinguished elements on A) which is however differ from the definition of the Ada of his later paper [5] *Adas and the equational theory of if-then-else* in 1993. While the Ada of the earlier draft seems to be based on extending the If-Then-Else concept more on the basis of Boolean algebras and the later concept is based on *C*-algebras $(A, , V, (–)^{\sim})$) (where , V are binary operations on A, $(–)^{\sim}$ is a unary operation) introduced by Fernando Guzman and Craig C. Squir [2]. In 1994, P. Koteswara Rao [3] first introduced the concept of *A**-algebra $(A, , V, *, (–)^{\sim}, (–), 0, 1, 2)$) (where, V, $*$ are binary operations on A, $(–)^{\sim}$, $(–)$ are unary operations and 0, 1, 2 are distinguished elements on A) not only studied the equivalence with Ada, *C*-algebra, Ada's connection with 3-Ring, Stone type representation but also introduced the concept of *A**-clone, the If-Then-Else structure over *A**-algebra and Ideal of *A**-algebra. In 2000, J. Venkateswara Rao [6] introduced the concept Pre *A**-algebra $(A, , , (–)^{\sim})$ (where, V are binary operations on A, $(–)^{\sim}$ is a unary operation on A analogous to *C*-algebra as a reduct of *A**- algebra, studied their subdirect representations, obtained the results that 2 = {0, 1} and 3 = {0, 1, 2} are the subdirectly irreducible Pre-*A**-algebras and every Pre-*A**-algebra can be imbedded in 3^X (where 3^X is the set of all mappings from a nonempty set X into 3 = {0, 1, 2}**).** Praroopa, Y. [13] introduced the specific concepts on Pre *A**-algebra and of the papers [8], [9], studied Pre *A**-algebra as a semilattice, lattice in Pre *A**-algebra.

Preliminaries

1.1 Definition : An algebra $(A, , , (–)^{\sim})$ satisfying

- $(x^{\sim})^{\sim} = x, \ x \ A$
- $x \ x = x, \ x \ A$

- $x \;\; y = y \;\; x, \;\; x, y \;\; A$
- $(x \;\; y)^{\sim} = \; x^{\sim} \;\; y^{\sim}, \;\; x, y \;\; A$
- $x \;\; (y \;\; z) = (x \;\; y) \;\; z, \;\; x, y, z \;\; A$
- $x \;\; (y \;\; z) = (x \;\; y) \; (x \;\; z), \;\; x, y, z \;\; A$
- $x \;\; y = x \;\; (x^{\sim} \;\; y), \;\; x, y \;\; A$

is called a Pre A* - algebra.

1.2 Example :

3 = {0, 1, 2} with operations $\wedge, \vee \; (-)^{\sim}$ defined below is a Pre A*-algebra.

∧	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

∨	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

x	x˜
0	1
1	0
2	2

1.3 Note :

The elements 0, 1, 2 in the above example satisfy the following laws:

(a) $2^{\sim} = 2$ (b) $1 \wedge x = x$ for all $x \in 3$ (c) $0 \vee x = x, \; \forall \; x \in 3$

(d) $2 \wedge x = 2 \vee x = 2, \;\; \forall \; x \in 3.$

1.4 Example : 2 = {0,1} with operations $\wedge, \vee, (-)^{\sim}$ defined below is a Pre A*-algebra.

∧	0	1
0	0	0
1	0	1

∨	0	1
0	0	1
1	1	1

x	x˜
0	1
1	0

1.5 Note : $(2, \vee, \wedge, (-)^{\sim})$ is a Boolean algebra. So every Boolean algebra is a Pre A*-algebra

1.6 Note : If (mn) is an axiom in Pre A*-algebra, then $(mn)^{\sim}$ is its dual.

PRE *A**-ALGEBRAS AND RINGS

2.1 Basic Theorems in Pre *A**-algebra :

Theorem 1 : De-Morgan laws :

Let $(A, \wedge, (-)^{\sim}, 1,)$ be a Pre A*-algebra. Then,

(i) $(a \wedge b)^{\sim} = a^{\sim} \vee b^{\sim}$

(ii) $(a \vee b)^{\sim} = a^{\sim} \wedge b^{\sim}$

2.2 Lemma 1 : Uniqueness of identity in a Pre A*-algebra :

Let $(A, \wedge, (-)^{\sim}, 1)$ be a Pre A^*-algebra and $a \in B(A)$ be an identity for $\wedge$, then $a^{\sim}$ is an identity for $\vee$, a is unique if it exists, denoted by 1 and a $^{\sim}$ by 0 where $B(A) = \{x/x \vee x^{\sim} = 1\}$ *i.e.*, (a) $1 \wedge x = x, \forall\, x \in A$ (b) $0 \vee x = x, \forall\, x \in A$.

2.3 Lemma 2 : Let A be a Pre A^*-algebra with 1 and 0 and let $x, y \in A$.

(i) If $x \vee y = 0$, then $x = y = 0$

(ii) If $x \vee y = 1$, then $x \vee x^{\sim} = 1$

2.4 Theorem 2 : Let A, be a Pre A^*-algebra with 1 and $x, y \in A$.

If $x \wedge y = 0$, $x \vee y = 1$, then $y = x^{\sim}$

2.5 Theorem 3 : Let $(A, \wedge, (-)^{\sim}, 1)$ be a Pre A^*-algebra.

Then we have the following (i) Involution law : $(a^{\sim})^{\sim} = a, \ \forall\, a \in A$

(ii) $0^{\sim} = 1, \ 1^{\sim} = 0$

2.6 Pre A*-algebra as a ring : Theorem 4: If $(A, \wedge, (-)^{\sim}, 1)$ is a Pre A* - algebra, then $(A, +, ., 1)$ is a ring where +, . are defined as follows.

(i) $a + b = (a \wedge b^{\sim}) \vee (b \wedge a^{\sim})$, where

$a \vee b = (a^{\sim} \wedge b^{\sim})^{\sim}$

(ii) $a \,.\, b = a \wedge b$

2.7 *p*-ring : *p* is an integer. A ring $(R, +, ., 0)$ is called a p-ring if

(i) $x^p = x, \forall\, x \in R$,

(ii) $px = 0, \forall\, x \in R$

2.8 Example : If $p = 3$ then $(R, +, ., 0)$ is called 3-ring.

PRE *A**-ALGEBRAS AND BOOLEAN RINGS, 3-RINGS

3.1 Boolean ring : A ring $(R, +, .)$ is said to be a Boolean ring if it satisfies the idempotent law *i.e.*, $x^2 = x, \ \forall\, x \in R$

3.2 Pre *A-algebra as a Boolean ring : Theorem 5 :** If $(A, \wedge, (-)^{\sim}, 1)$ is a Pre A^*-algebra, then $(A, +, ., 1)$ is a Boolean ring where +, . are defined as follows:

(i) $a + b = (a \wedge b^{\sim}) \vee (b \wedge a^{\sim})^{\sim}$, where

$a \vee b = (a^{\sim} \wedge b^{\sim})^{\sim}$

(ii) $a \,.\, b = (a \wedge b)$

3.3 Theorem 6 : If $(A, +, ., 1)$ is a Boolean ring, then $(A, \wedge, (-)^{\sim}, 1)$ is a Pre A^*-algebra, where

(i) $a^{\sim} = 1 - a$

(ii) $a \wedge b = [1 - (1 - a)]\,[1 - (1 - b)]$

3.4 Pre *A-algebra as 3-ring :Theorem 7 :** If $(A, \wedge, (-)^{\sim}, 1)$ is a Pre A^*-algebra then $(A, +, ., 1)$ is a 3-ring where +, . are defined as follows.

(i) $a + b = (a \wedge b^{\sim}) \vee (b \wedge a^{\sim})$, where

$a \vee b = (a^{\sim} \wedge b^{\sim})^{\sim}$

(ii) $a \, . \, b = a \wedge b$

3.5 Theorem 8 : If $(A, +, ., 1)$ is a 3-ring, then $(A, \wedge, (-)^{\sim}, 1)$ is a Pre A*-algebra, where

(i) $a^{\sim} = 1 - a$

(ii) $a \wedge b = -[1 - (1 - a)] [1 - (1 - b)]$

Conclusion

This paper is a study on algebraic structure of Pre A*-algebra and proves basic theorems on Pre A*-algebra. This paper defines p-ring, Boolean ring and 3-ring. Establish Pre A*-algebra as a Boolean ring and Boolean ring as a Pre A*-algebra. It includes to prove Pre A*-algebra as a 3-ring and 3-ring as a Pre A*-algebra.

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