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MOMENTUM AND HEAT TRANSMISSION IN UNSTEADY BOUNDARY LAYER FLOW OF A ROTATING NON-NEWTONIAN FLUID WITH TIME DEPENDENT SUCTION

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This paper deals with momentum and heat transmission in unsteady boundary layer flow of rotating non-Newtonian fluid with time dependent suction, when the plate velocity is constant. Constitutive equations of momentum and energy are formulated. These equations are non-dimensionalised and solved with the permissible boundary conditions. Velocity and temperature profiles are drawn varying the fluid parameters. The numerical values of shearing stresses are entered in the tables for different values of fluid parameters. It is noticed that the non-Newtonian parameter and the suction parameter influence the flow field to a great extent and the prandtl number affects the temperature field.

KEYWORDS : Unsteady flow, non-Newtonian fluid, suction, heat transfer.

Introduction

Investigations on rotating flows of non-Newtonian fluids attract the attention of researchers in view of their applications in cosmical and geophysical fluid dynamics and in mechanical and nuclear engineering. The literature is replete with copious such studies. Miles [1] discussed the Cauchyission problem for a rotating liquid. Greenspan and Howard [2] have considered unsteady rotating flow. Batchelor [3] has discussed the of rotating fluid on a plate and it has been observed there exists a layer of fluid near the plate, called the layer, where the viscous and the coriolis forces are of same order of magnitude. Chawla [4] as well as Singh and Santhi [5] have studied unsteady rotating flow in an impermeable plate under the assumption of rigid body. Thornley [6] discussed the Stokes and Ekman layers case of the plate performing non-torsional oscillations down plane. Incompressible fluid. Pop [7] has considered unsteady boundary layers in a rotating fluid. Dordjevic [8] extended the problem of Thornley [9] to the case of non-homogeneous fluid. Debnath and Mukherjee [10] have discussed unsteady multiple boundary layers on a porous plate in a rotating system. Unsteady boundary layers in a rotating flow has also been considered by Pop and Soundalgekar [11]. Soundalgekar, Martin, Gupta and Pop [12] have studied unsteady boundary-layers in a rotating fluid with time-dependent suction. The method of Fourier series suggested by Kelley [13] has been used to obtain the solution. Biswal [14] has analysed the problem of unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started porous wall. Biswal, Roy and Mishra [15] have investigated the problem of Heat transfer in unsteady axisymmetric rotational flow of Oldroyd liquid. The same authors [16] have also analysed the hydrodynamic free convection flow of a rotating visco elastic fluid past an isothermal vertical porous plate with mass transfer. Dash and Paikaray [17] have studied the heat and mass transfer in the unsteady Couette flow of Oldroyd liquid between two horizontal parallel porous plates with heat sources, chemical reaction and Soret effect when the lower plate moves with time varying velocity.

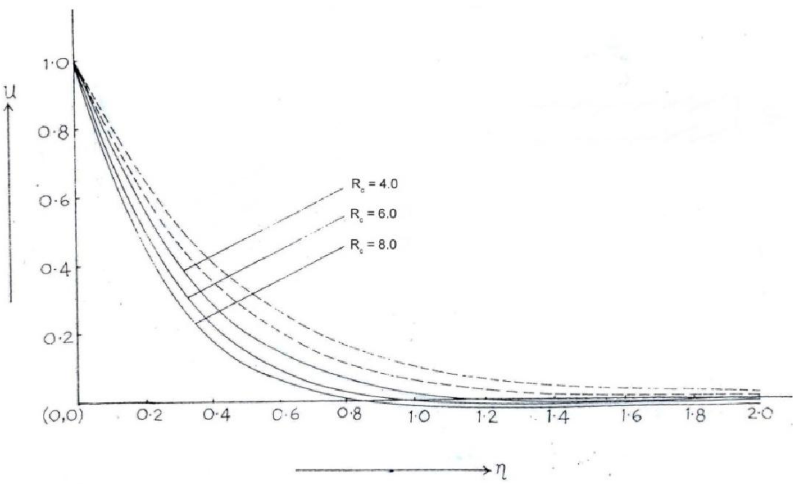

Fig 1 : Velocity component u for different values of R_c when $\Omega = \pi/2_c$ and $\omega = \pi/4_c$

In this paper, our aim is to study the momentum and heat transfer in unsteady boundary layer flow of a rotating non-Newtonian fluid with time dependent suction. Here the problem is studied, when the plate velocity is constant. Analysis of the temperature field has also been presented.

Basic educations

In this paper, the Cartesian co-ordinate system is supposed to be rotating uniformly with the fluid, with angular velocity Ω, about z-axis taken vertically upwards, *i.e.*, normal to the plate. The x-axis in taken along the plate, in the direction of motion and the y-axis is taken on the plate, perpendicular to the x-axis. Moreover, the plate being infinite in length all variables in the problem are functions of a z and to only.

Hence, the equation of continuity takes the form

$$\frac{\partial w}{\partial z} = 0\,, \qquad \ldots (2.1)$$

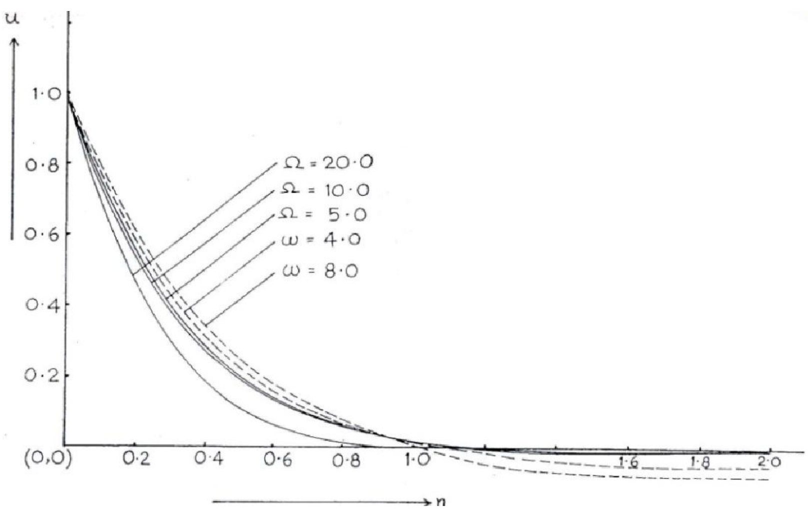

Fig. 2 : Velocity component '*u*' for different values of Ω and ω when R_c = 4.0

Which on integration gives

$$w = \text{constant} = w\,(t) \qquad \dots (2.2)$$

$$\vec{V} = (u, v, w\,(t)), \qquad \dots (2.3)$$

Let us assume $w\,(t)$ to be of the form

$$w\,(t) = -\,w_0\,[1 + \varepsilon\,(e^{iwt} + e^{-\,iwt})] \qquad \dots (2.4)$$

where w_0 is the constant suction velocity, w is the frequency of oscillation and ε (<< 1) is assumed to be a constant amplitude of suction velocity.

The equations of motion are reduced to

$$u\ - w_0\,[1 + \varepsilon\,(e^{iwt}) + e^{-\,iwt})]\ \frac{\partial u}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2} + 2\Omega v + \frac{K_0}{\rho}\frac{\partial^3 u}{\partial z^2 \partial t} \qquad \dots (2.5)$$

$$v\ - w_0\,[1 + \varepsilon\,(e^{iwt} + e^{-\,iwt})]\ \frac{\partial v}{\partial z} = \nu \frac{\partial^2 v}{\partial z^2} - 2\Omega u - \frac{K_0}{\rho}\frac{\partial^3 v}{\partial z^2 \partial t} \qquad \dots (2.6)$$

THE CASE OF CONSTANT PLATE VELOCITY

Let us formulate the equations of the problem when the plate moves with constant velocity *A*. Initially, at time $t \leq 0$, the fluid and the plate have no motion along *x* and *y* axes. At time $t > 0$, the plate is accelerated with velocity *A* along *x*-axis. Hence, equns. (2.5) and (2.6) are to be solved subject to the boundary conditions:

$$\left.\begin{array}{l} u = 0 = v, \text{ for all } z, \text{ at } t \leq 0, \\ \left.\begin{array}{l} u = A,\ v = 0, \text{ at } z = 0 \\ u = 0 = v, \text{ as } z \to \infty \end{array}\right\}\ t > 0 \end{array}\right\} \qquad \dots (3.1)$$

From equns. (4.2.5) and (4.2.6) we get

$$\frac{\partial F}{\partial t} - w_0 [1 + \varepsilon (e^{iwt} + e^{-iwt})] \frac{\partial F}{\partial z} = v \frac{\partial^2 F}{\partial z^2} - 21\Omega F - R_c \frac{\partial^3 F}{\partial z^2 \partial t} \quad \ldots (3.2)$$

where $F = u + iv$.

The boundary conditions (3.1) are now modified to

$$\left.\begin{array}{l} F = 0, \text{ at } t \le 0, \text{ for all } z \\ \left.\begin{array}{l} F = A, \text{ at } z = 0 \\ F = 0, \text{ as } z \to \infty \end{array}\right\} t > 0 \end{array}\right\} \quad \ldots (3.3)$$

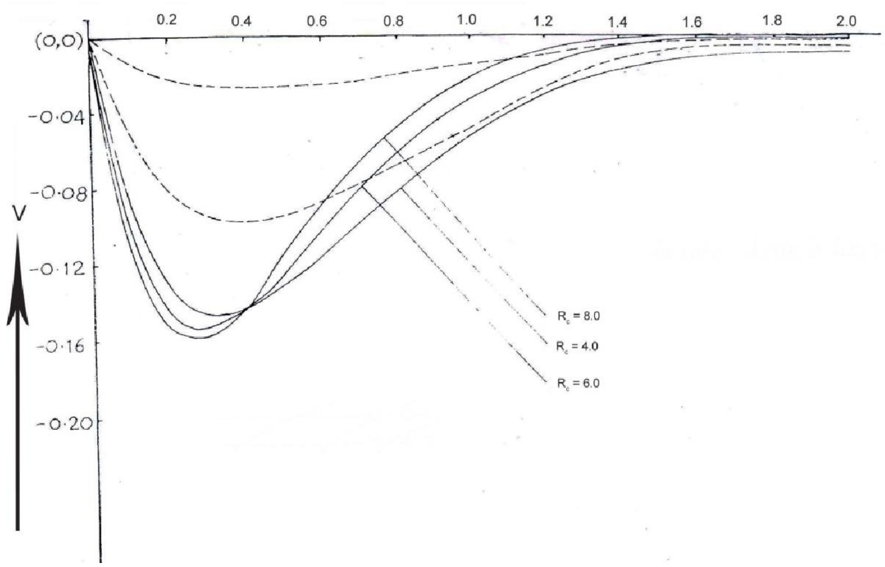

Fig. 3 : Velocity component *v* for different values of R_c when $\Omega = \pi/2_c$ and $\omega = \pi/4_c$

Since the flow field is under the influence of time dependent suction, given by eqn. (2.2), we assume the solution $F(z, t)$ of eqn. (3.2) in the form of the following Fourier series :

$$F(z, t) = F_0(z) + \sum_{n=1}^{\infty} F_n(z)\, e^{in\omega t} + \sum_{n=1}^{\infty} \overline{F}_n(\overline{z})\, e^{-in\omega t} \quad \ldots (3.4)$$

Substituting $F(z, t)$ from eqn. (3.4) in eq. (3.2) and equating the harmonic and non-harmonic terms from both sides, we get the following equations:

$$vF_0'' + w_0F_0' + w_0\varepsilon\,(F_1' + F_1' - [R_c + 2i\Omega)]F_0 = 0 \quad (3.5)$$

$$vF_1'' + w_0F_0' + w_0\varepsilon\,(F_0' + F_2') - [R_c + 2i\Omega)]F_1 - i\omega F_1 = 0 \quad (3.6)$$

$$vF_n'' + w_0F_0' + w_0\varepsilon\,(F_{n-1}' + F_{n+1}' - [R_c + i(2\Omega + n\omega)]\, F_n = 0,\ n \ge 2\,, \quad (3.7)$$

$$\left.\begin{array}{l} vF_1'' + w_0F_1' + w_0\varepsilon\,(F_0' + F_2') - [R_c + 2i\Omega - i\omega]F_1 = 0 \\ vF_n'' + w_0F_0' + w_0\varepsilon(F_{n-1}' + F_{n+1}' - [R_c + i(2\Omega + n\omega)]F_n = 0, n \ge 2 \end{array}\right\} \quad \ldots (3.8)$$

The boundary conditions (4.3.3.) yield

$$\left.\begin{array}{l} F_0\ (0) = A,\ F_n\ (0) = 0,\ \text{ for } n \geq 1 \\ F_0\ (\infty) = 0,\ F_n\ (\infty) = 0,\ \text{for } n \geq 1 \end{array}\right\} \qquad \ldots (3.9)$$

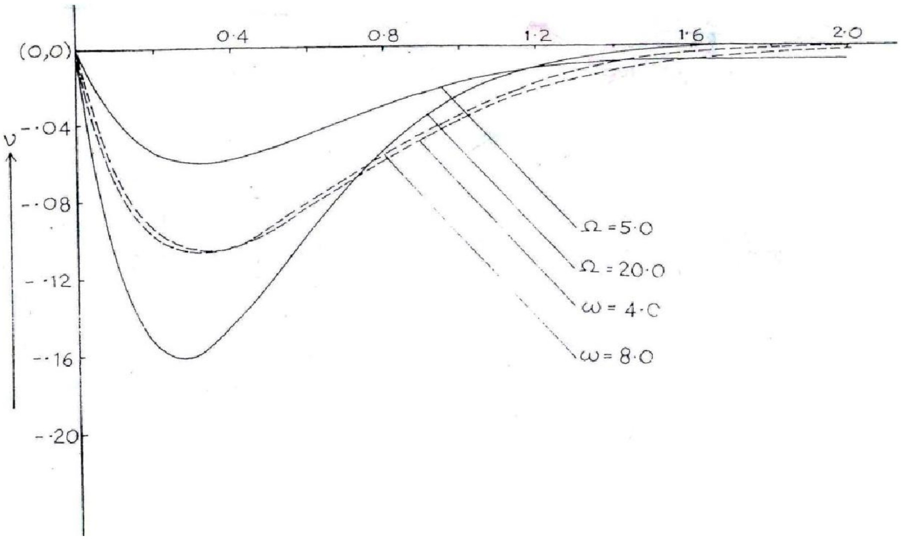

Fig. 4 : Velocity component '*v*' for different values of Ω and ω when R_c = 4.

In order to obtain the solutions F_n ($n \geq 0$), we expand these functions in powers of ε as follows:

$$F\ (z) = \sum_{r=0}^{\infty} F_{nr}(z)\ \varepsilon^r \qquad (3.10)$$

Substituting F_n from eqn. (3.10) in eqns. (3.5) – (3.7) and equating coefficients of like powers of ε (up to ε^2), we obtain the following equations:

$$vF_{00}'' + w_0 F_{00}' - [R_c + 2i\Omega]\ F_{00} = 0 \qquad (3.11)$$

$$vF_{01}'' + w_0 F_{01}' - [R_c + 2i\Omega]\ F_{01} = -w_0 (F_{10}' + \bar{F}_{10}') \qquad (3.12)$$

$$vF_{02}'' + w_0 F_{02}' - [R_c + 2i\Omega] F_{02} = -w_0 (F_{11}' + F_{11}') \qquad (3.13)$$

$$vF_{10}'' + w_0 F_{10}' - [R_c + 2i\Omega + i\omega]\ F_{10} = 0 \qquad (3.14)$$

$$vF_{11}'' + w_0 F_{11}' - [R_c + 2i\Omega + i\omega] F_{11} = -w_0 (F_{00}' + F_{20}') \qquad (3.15)$$

$$vF_{12}'' + w_0 F_{12}' - [R_c + 2i\Omega + i\omega]\ F_{12} = -w_0 (F_{01}' + F_{21}') \qquad (3.16)$$

$$vF_{20}'' + w_0 F_{20}' - [R_c + 2i\Omega + 2i\omega]\ F_{20} = 0 \qquad (3.17)$$

$$vF_{21}'' + w_0 F_{21}' - [R_c + 2i\Omega + 2i\omega]\ F_{21} = -w_0 (F_{10}' + F_{30}') \qquad (3.18)$$

$$vF_{22}'' + w_0 F_{22}' - [R_c + 2i\Omega + 2i\omega]\ F_{22} = -w_0 (F_{11}' + F_{31}') \qquad (3.19)$$

The boundary conditions (3.9) are now modified to

$$\left.\begin{array}{l} F_{00}(O) = A,\ \text{for } (O),\ r \geq 1,\ F_{nr}\ \ (O)\ = \ 0,\ \text{for all } f,\ n \geq 1 \\ F_{0r}(\infty) = 0,\ \text{for all } r,\ F_{nr}\ \ (\infty)\ = 0,\ \text{for all } r,\ n \geq 1 \end{array}\right\} \qquad \ldots (3.20)$$

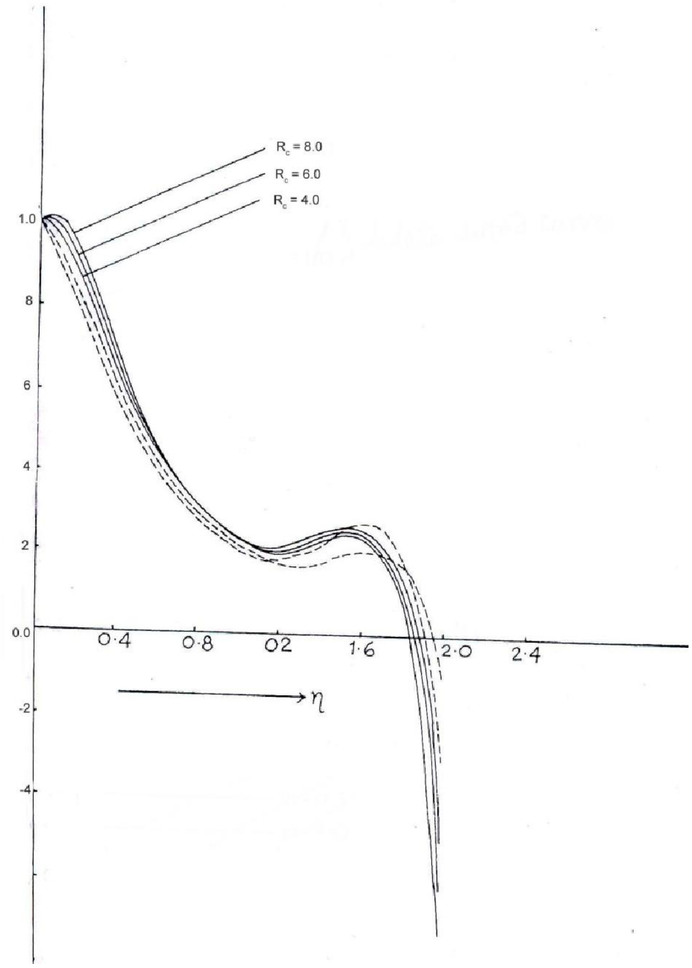

Fig. 5 : Temperature field for different values of R_c

The solution of eqn. (3.11) in given by

$$F_{00}(z) = A_1\, e^{d_2 z} \qquad \ldots (3.21)$$

where

$$\left.\begin{aligned} \alpha_1 &= \frac{1}{2}\left[-\frac{w_0}{v} + \sqrt{\frac{w_0^2}{v^2} + \frac{4}{v}\{R_c + 2i\Omega\}}\right] \\ \alpha_2 &= \frac{1}{2}\left[-\frac{w_0}{v} - \sqrt{\frac{w_0^2}{v^2} + \frac{4}{v}\{R_c + 2i\Omega\}}\right] \end{aligned}\right\}, \qquad \ldots (3.22)$$

and A_1 and A_2 are to be evaluated with the help of boundary conditions (3.20).

Hence,

$$A_1 = 0 \text{ (since } F_{00}(\infty) = 0),$$

and $A_2 = A$ ((since $F_{00}(\infty) = A$).

Thus we have,

$$F_{00}(z) = A\,e^{\alpha_2 z} \qquad \dots (3.23)$$

The solution of eqn. (3.14) is given by

$$F_{10}(a) = B_1\,e^{\beta_1 z} + B_2\,e^{\beta_2 z}$$

where

$$\left.\begin{aligned} \beta_1 &= \frac{1}{2}\left[-\frac{w_0}{v} + \sqrt{\frac{w_0^2}{v^2} + \frac{4}{v}\{R_c + 2i\Omega + i\omega\}}\right] \\ \beta_2 &= \frac{1}{2}\left[-\frac{w_0}{v} - \sqrt{\frac{w_0^2}{v^2} + \frac{4}{v}\{R_c + 2i\Omega + i\omega\}}\right] \end{aligned}\right\} \qquad \dots (3.24)$$

And B_1 and B_2 are to be determined subject to the boundary conditions (3.20). We thus obtain.

$$B_1 = B_2 = 0,$$

So that

$$F_{10}(z) = 0, \qquad \dots (3.25)$$

Similar, the solution of eqn. (3.17) is given by

$$F_{20}(z) = c_1\,e^{\gamma_1 z} + c_2\,e^{\gamma_2 z}$$

where

$$\left.\begin{aligned} \gamma_1 &= \frac{1}{2}\left[-\frac{w_0}{v} + \sqrt{\frac{w_0^2}{v^2} + \frac{4}{v}\{R_c + 2i\Omega + i\omega\}}\right] \\ \gamma_2 &= \frac{1}{2}\left[-\frac{w_0}{v} - \sqrt{\frac{w_0^2}{v^2} + \frac{4}{v}\{R_c + 2i\Omega + i\omega\}}\right] \end{aligned}\right\} \qquad \dots (3.26)$$

And from eqn. (3.20), c_1 and c_2 are determined as

$$c_1 = c_2 = 0.$$

Hence, $$F_{20}(z) = 0 \qquad \dots (3.27)$$

Substituting $F_{10}(z)$ from eqn. (3.25) in eqn. (3.12) and solving for $F_{01}(z)$ subject to the boundary conditions (3.20), obtain

$$F_{01}(z) = 0 \qquad \dots (3.28)$$

From equations (3.15), (3.23) and (3.27), we have

$$F_{11}'' + w_0 F_{11}' - [R_c + 2i\Omega + i\omega]F_{11} = -w_0 A\alpha_2 e^{\alpha_2 z} \qquad \dots (3.29)$$

Solution of eqn. (3.29) is given by

$$F_{11}''(z) = D_1 e^{\beta_1 z} + D_2 e^{\beta_2 z} + F_{11p},$$

where β_1, β_2 are given by (3.24),

$$F_{11p} = -\frac{w_0}{v} A\alpha_2 \frac{e^{\alpha_2 z}}{-\frac{i\omega}{v}} = \frac{w_0 A\alpha_2}{i\omega} e^{\alpha_2 z},$$

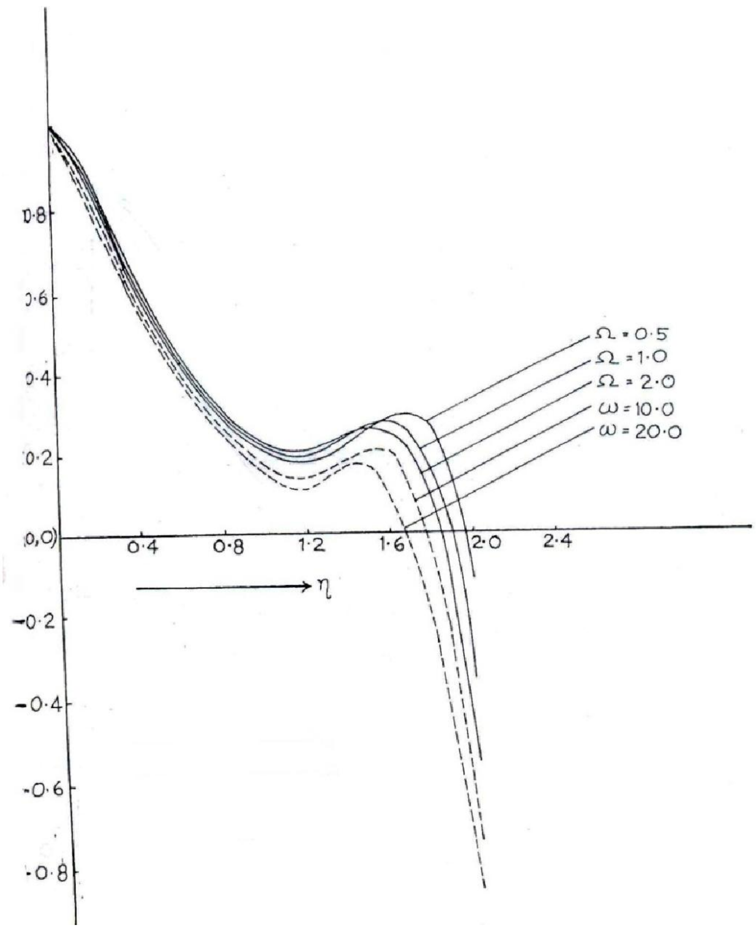

Fig. 6 : Temperature field for different values of ωand Ω, R_c = 4.0, P = 2.0

D_1, D_2 are to be determined from the boundary conditions (3.20). Hence,

$$D_1 = 0, \text{ (since } F_{11}(\infty) = 0),$$

$$D_2 = -\frac{w_0 A\alpha_2}{i\omega} \text{ (since } F_{11}(0) = 0)$$

Therefore,

$$F_{11}(z) = -\frac{w_0 A\alpha_2}{i\omega} [e^{\beta_2 z} - e^{\alpha_2 z}] \quad \dots (3.30)$$

Using equn. (3.30) in equn. (3.13) we get

$$F_{02}'' + \frac{w_0}{v} F_{02}' - \frac{1}{v}[R_c + 2i\Omega]F_{02}$$

$$= \frac{w_0^2 A}{\omega v}\left[-i\alpha_2\left\{\beta_2 e^{\beta_2 z} - \alpha_2 e^{\alpha_2 z}\right\} + i\bar{\alpha}_2\left\{\bar{\beta}_2 e^{\bar{\beta}_2 z} - \bar{\alpha}_2 e^{\bar{\alpha}_2 z}\right\}\right] \quad \ldots (3.31)$$

Hence the solution of eqn. (3.31) is given by

$$F_{02}(z) = A_3\ e^{\alpha_1 z} + A_4\ e^{\alpha_2 z} + F_{02p},$$

Where α_1, α_2 are given in eqn. (4.3.22),

$$F_{02\mathrm{p}} = \frac{w_0^2 A}{\omega}\left[-\frac{\alpha_2\beta_2}{\omega} e^{\beta_2 z} + \frac{i\alpha_2^2 z e^{\alpha_2 z}}{2\alpha_2 v + w_0} - \frac{\bar{\alpha}_2\bar{\beta}_2 e^{\bar{\beta}_2 z}}{2\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}}\right] + \frac{\bar{\alpha}_2^2 e^{\bar{\beta}_2 z}}{2\{R_c + 2\Omega\}}$$

And A_3 and A_4 are to be determined subject to boundary conditions (3.20). Thus, we get,

$A_3 = 0$, (since $F_{02}(\infty) = 0$)

$$A_4 = -\frac{w_0^2 A}{\omega}\left[-\frac{\alpha_2\beta_2}{\omega} - \frac{\bar{\alpha}_2\bar{\beta}_2}{2\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}}\right] + \frac{\bar{\alpha}_2^2}{2\{R_c + 2\Omega\}} \text{(since } F_{02}(\infty) = 0)$$

Hence

$$F_{02}(z) = \frac{w_0^2 A}{\omega}\left[-\frac{\alpha_2\beta_2}{\omega}(e^{\alpha_2 z} - e^{\beta_2 z}) + \frac{i\alpha_2^2 z e \alpha_2 z}{2\alpha_2 v + w_0}\right.$$

$$\left. + \frac{\alpha_2\beta_2(e^{\alpha_2 z} - e^{\beta_2 z})}{2\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}} + \frac{\alpha_2^{-2}(e^{\bar{\alpha}_2 z} - e^{\alpha_2 z})}{2\{R_c + 2\Omega\}}\right. \quad \ldots (3.32)$$

With the help of eqns. (3.7), (3.10) and (3.20) for $n = 3$, we can easily get

$$F_{30}(z) = 0, \quad \ldots (3.33)$$

Substituting $F_{10}(z)$ and $F_{30}(z)$ from eqns. (3.25) and (3.30) eqn. (3.18), we have

$$vF_{21}'' + w_0 F_{21}' - [R_c + 2i\Omega + 2i\omega]\ F_{21} = 0 \quad (3.34)$$

Proceeding as before the solution of eqn. (3.34) subject to boundary conditions (3.20) in given by

$$F_{21}(z) = 0, \quad \ldots (3.35)$$

Substituting $F_{01}(z)$ and $F_{21}(z)$ from eqns. (3.28) and (3.35) in eqn. (3.16), we obtain

$$vF_{12}'' + w_0 F_{12}' - [R_c + 2i\Omega + i\omega]\ F_{12} = 0 \quad \ldots (3.36)$$

The solution of eqn. (3.36) subject to boundary conditions (3.20) is

$$F_{12}(z) = 0, \quad \ldots (3.37)$$

Further eqns. (3.7), (3.10) and (3.20), for $n = 3$ and $n = 4$ yield

$$F_{40}(z) = 0, \quad \ldots (3.38)$$

And so,

$$F_{31}(z) = 0, \text{ (since } F_{20}(z) = 0 \text{ from eqn. (3.27)} \qquad \ldots (3.39)$$

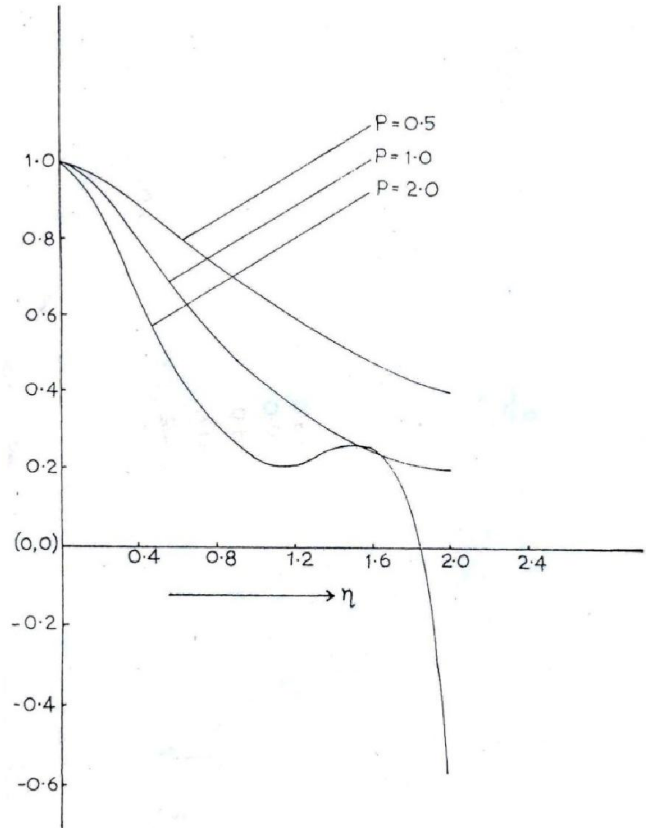

Fig. 7 : Temperature field for different value s of *P*, R_c = 4.0, Ω= 2.0, ω = 5.0

Substituting $F_{11}(z)$ and $F_{31}(z)$ from eqns. (3.30) and (3.39) in eqn. (3.19), we get

$$vF''_{22} + w_0 F'_{22} - [R_c + 2i\Omega + 2i\omega] F_{22} = \frac{w_0^2 A\alpha_2}{i\omega}\left[\beta_2 e^{\beta_2 z} - \alpha_2 e^{\alpha_2 z}\right]. \quad \ldots (3.40)$$

The solution of eqn. (3.40) is given by,

$$F_{22}(z) = K_1 e^{\gamma_1 z} + K_2 e^{\gamma_2 z} + F_{22p},$$

where $\gamma_1 + \gamma_2$ are given in eqn. (3.26),

$$F_{22p} = \frac{w_0^2 A\alpha_2}{\omega^2}\left[\beta_2 e^{\beta_2 z} - \alpha_2 e^{\alpha_2 z}\right]$$

And K_1 and K_2 are to be determined on application of boundary conditions (3.20), we thus have,

$$K_1 = 0, \qquad \text{(since } F_{22}(\infty) = 0)$$

$$K_2 = \frac{w_0^2 A\alpha_2}{\omega^2}\left[\frac{\alpha_2}{2} - \beta_2\right], \quad \text{(since } F_{22}(0) = 0\text{)}.$$

Hence,

$$F_{22}(z) = \frac{w_0^2 A\alpha_2}{\omega^2}\left[\beta_2\left(e^{\beta_2 z} - e^{\gamma_2 z}\right) + \frac{\alpha_2}{2}\left(e^{\gamma_2 z} - e^{\alpha_2 z}\right)\right] \quad \ldots (3.41)$$

Table 1. Values of θ for different values of ω*t*, when R_c= 4.0, ω = 5.0, Ω = 2.0, *P* = 2.0, *E* = 0.5

$\eta/\omega t$	0.0	$\pi/4$	$\pi/2$
0.0	1.0	1.0	1.0
0.1	0.96866	0.96869	0.96940
0.2	0.87648	0.87681	0.87856
0.3	0.76346	0.76453	0.76758
0.4	0.64978	0.65223	0.65675
0.5	0.54500	0.54960	0.55570
0.6	0.45322	0.76080	0.46830
0.7	0.37598	0.38719	0.39536
0.8	0.31377	0.32874	0.33595
0.9	0.26681	0.28467	0.28796
1.0	0.23536	0.25347	0.24821
1.1	0.21948	0.23265	0.21226
1.2	0.21860	0.21813	0.17415
1.3	0.23054	0.20344	0.12625
1.4	0.25011	0.17884	0.05941
1.5	0.26737	0.13064	– 0.03624
1.6	0.26560	0.04107	– 0.16953
1.7	0.21938	– 0.11078	– 0.34495
1.8	0.09350	– 0.34609	– 0.55747
1.9	– 0.15631	– 0.68055	– 0.78527
2.0	– 0.57918	– 1.11473	– 0.98021

From eqns. (3.10), (3.23), (3.28) and (3.32), we obtain the mean velocity $F_0(z)$ as

$$F_0(z) = A\, e^{\alpha_2 z} + \varepsilon^2 \frac{w_0^2 A}{\omega}\left[\frac{\alpha_2\beta_2}{\omega}\left(c^{\alpha_2 z} - c^{\beta_2 z}\right) + \frac{i\alpha_2^2 z e \alpha_2 z}{2\alpha_2 v + \omega_2}\right.$$

$$\left. + \frac{\bar{\alpha}_2\bar{\beta}_2\left(e^{\alpha_2 z} - e^{\bar{\beta}_2 z}\right)}{2\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}} + \frac{\alpha_2^{-2}\left(e^{\bar{\alpha}_2 z} - e^{\alpha_2 z}\right)}{2\left\{R_c + 2\Omega\right\}}\right] \quad \ldots (3.42)$$

Similarly eqns. (3.10), (3.25), (3.30) and (3.37) yield

$$F_1(z) = -\frac{w_0 A\alpha_2\varepsilon}{i\omega}\left[e^{\beta_2 z} - e^{\alpha_2 z}\right] \qquad \ldots (3.43)$$

In a similar way eqns. (3.10), (3.27), (3.35) and (3.41) give

$$F_2(z) = \frac{w_0^2 A\alpha_2\varepsilon^2}{\omega^2}\left[\beta_2(e^{\beta_2 z} - e^{\gamma_2 z}) + \frac{\alpha_2}{2}(e^{\gamma_2 z} - e^{\alpha_2 z})\right] \qquad \ldots (3.44)$$

Denoting the real and imaginary parata of $F_r(z)$ by $u_r(z)$ and $v_r(z)$ respectively, for $r = 0, 1, 2$ we get from eqns. (3.42) – (3.44)

$$u_0(z) = A\,e^{\alpha_1 z}\cos b_1 z + \frac{\varepsilon^2 w_0^2 A}{\omega}\left[\frac{a_1a_2 - b_1b_2}{\omega}(e^{\alpha_1 z}\cos b_1 z - e^{\alpha_2 z}\cos b_2 z)\right.$$

$$-\frac{a_1b_2 - a_2b_1}{\omega}(e^{\alpha_1 z}\sin b_1 z - e^{\alpha_2 z}\sin b_2 z)$$

$$+ z\,e^{\alpha_1 z}\,\{(a_1^2 - b_1^2)(2vb_1\cos b_1 z - [2va_1 + w_0]\sin b_1 z)$$

$$-2a_1b_1[(2va_1 + w_0)\cos b_1 z + 2vb_1\sin b_1 z]\}/\{(2va_1 + w_0)^2 + 4v^2b_1^2$$

$$+\frac{1}{2\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}}\{(a_1a_2 - b_1b_2)(e^{\alpha_1 z}\cos b_1 z - e^{\alpha_2 z}\cos b_2 z)$$

$$\left. + (a_1b_2 + a_2b_1)(e^{\alpha_1 z}\sin b_1 z + e^{\alpha_2 z}\sin b_2 z)\} - \frac{2a_1b_1e^{a_1 z}\sin b_1 z}{\{R_c + 2\Omega\}}\right] \qquad \ldots (3.45)$$

$$v_0(z) = Ae^{\alpha_1 z}\sin b_1 z + \frac{\varepsilon^2 w_0^2 A}{\omega}\left[\frac{a_1a_2 - b_1b_2}{\omega}(e^{a_1 z}\sin b_1 z - e^{a_2 z}\sin b_2 z)\right.$$

$$+\frac{a_1b_2 + a_2b_1}{\omega}(e^{a_1 z}\cos b_1 z - e^{a_2 z}\cos b_2 z)\;\frac{ze^{a_1 z}}{(2va_1 + w_0)^2 + (2vb_1)^2}$$

$$\{(a_1^2 - b_1^2)[(2va_1 + w_0)\cos b_1 z + 2vb_1\sin b_1 z] - 2a_1b_1[(2va_1 + w_0)\sin b_1 z$$

$$-2vb_1\cos b_1 z\} + \frac{1}{2\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}}\{(a_1a_2 - b_1b_2)$$

$$(e^{\alpha_1 z}\sin b_1 z + e^{\alpha_2 z}\sin b_2 z) - (a_1b_2 + a_2b_1)(e^{\alpha_1 z}\cos b_1 z - e^{\alpha_2 z}\cos b_2 z)\}$$

$$\left. -\frac{(a_1^2 - b_1^2)e^{a_1 z}\sin b_1 z}{\{R_c + 2\Omega\}}\right] \qquad \ldots (3.46)$$

$$u_1(z) = \frac{w_0 Ae}{\omega}[b_1(e^{\alpha_1 z}\cos b_1 z - e^{\alpha_2 z}\cos b_2 z) + a_1(e^{\alpha_1 z}\sin b_1 z - e^{\alpha_2 z}\sin b_2 z)], \qquad \ldots (3.47)$$

$$v_1(z) = \frac{w_0 Ae}{\omega}[b_1(e^{\alpha_1 z}\sin b_1 z - e^{\alpha_2 z}\sin b_2 z) + a_1(e^{\alpha_1 z}\cos b_1 z - e^{\alpha_2 z}\cos b_2 z)], \qquad \ldots (3.48)$$

$$u_2(z) = \frac{w_0^2 A\varepsilon^2}{\omega^2}[(a_1a_2 - b_1b_2)\ (e^{\alpha_2 z}\cos b_2 z - e^{\alpha_3 z}\cos b_3 z)$$

$$-(a_1b_2 + a_2b_1)\ (e^{\alpha_2 z}\sin b_2 z - e^{\alpha_3 z}\sin b_3 z)],\ + \frac{a_1^2 - b_1^2}{2}\ (e^{\alpha_3 z}\cos b_3 z - e^{\alpha_1 z}\cos b_1 z)$$

$$-(a_1b_1)\ (e^{\alpha_3 z}\sin b_3 z - e^{\alpha_1 z}\sin b_1 z)], \qquad \dots (3.49)$$

$$v_2(z) = \frac{w_0^2 A\varepsilon^2}{\omega^2}[(a_1a_2 - b_1b_2)\ (e^{\alpha_2 z}\sin b_2 z - e^{\alpha_3 z}\sin b_3 z)$$

$$-(a_1b_2 + a_2b_1)\ ((e^{\alpha_2 z}\cos b_2 z - e^{\alpha_3 z}\cos b_3 z)],$$

$$+\frac{a_1^2 - b_1^2}{2}\ (e^{\alpha_3 z}\sin b_3 z - e^{\alpha_1 z}\sin b_1 z) + (a_1b_1)\ (e^{\alpha_3 z}\cos b_3 z - e^{\alpha_1 z}\cos b_1 z)], \dots (3.50)$$

where

$$a_1 = -\frac{w_0}{2v} - \frac{1}{2}\left[\frac{1}{2}\left\{\sqrt{\left(\frac{w_0^2}{v^2} + R_c\right)^2 + \left(R_c + 2\Omega\right)^2} + \frac{w_0^2}{v^2} + R_c\right\}\right]^{1/2},$$

$$b_1 = -\frac{w_0}{2v}\left[\frac{1}{2}\left\{\left(\frac{w_0^2}{v^2} + R_c\right)^2 + \left(2\Omega\right)^2 - \left(\frac{w_0^2}{v^2} + R_c\right)^2 + \left(R_c\right)\right\}\right]^{1/2},$$

$$a_2 = -\frac{w_0}{2v} - \frac{1}{2}\left[\frac{1}{2}\left\{\left[\left(\frac{w_0^2}{v^2} + R_c\right)^2 \left(R_c + 2\Omega + \omega\right)^2\right]^{1/2} + \frac{w_0^2}{v^2} + R_c\right\}\right]^{1/2},$$

$$b_2 = -\frac{1}{2}\left[\frac{1}{2}\left\{\left[\left(\frac{w_0^2}{v^2} + R_c\right)^2 \left(R_c + 2\Omega + \omega\right)^2\right]^{1/2} - \left(\frac{w_0^2}{v^2} + R_c\right)\right\}\right]^{1/2},$$

$$a_3 = -\frac{w_0}{2v} - \frac{1}{2}\left[\frac{1}{2}\left\{\left[\left(\frac{w_0^2}{v^2} + R_c\right)^2 \left(R_c + 2\Omega + \omega\right)^2\right]^{1/2} + \frac{w_0^2}{v^2} + R_c\right\}\right]^{1/2}$$

$$b_3 = -\frac{1}{2}\left[\frac{1}{2}\left\{\left[\left(\frac{w_0^2}{v^2} + R_c\right)^2 \left(R_c + 2\Omega + \omega\right)^2\right]^{1/2} - \left(\frac{w_0^2}{v^2} + R_c\right)\right\}\right]^{1/2} \qquad \dots (3.51)$$

From equation (3.4), we obtain

$$F(z, t) = u_0(z) + iv_0(z) + (u_1(z) + iv_1(z))\, e^{i\omega t}$$

$$+ (u_1(z) - iv_1(z))\, e^{-i\omega t} + (u_2(z) + iv_2(z))\, e^{12\omega t} + (u_2(z) - iv_2(z))\, e^{-12\omega t} + \dots..$$

Hence separating real and imaginary parts

$$u(z,t) = u_0(z) + 2\,(u_1(z)\cos\omega t - v_1(z)\sin\omega t) + 2\,(u_2(z)\cos 2\omega t - v_2(z)\sin 2\omega t) + \ldots\ldots\ldots, \qquad \ldots (3.52)$$

and $$v(z,t) = v_0(z). \qquad \ldots (3.53)$$

Table 2. Values of τ_1 and τ_2 for different values of R_c (high) and $\Omega = 5.0$

	τ_1			τ_2		
ω / R_c	4.0	6.0	8.0	4.0	6.0	8.0
1.5	– 2.846	– 3.321	– 3.728	– 1.171	– 1.447	– 1.677
2.0	– 2.863	– 3.336	– 3.742	– 1.139	– 1.421	– 1.655
2.5	– 2.881	– 3.350	– 3.754	– 1.103	– 1.395	– 1.633
3.0	– 2.898	– 3.364	– 3.767	– 1.065	– 1.367	– 1.610
3.5	– 2.917	– 3.379	– 3.779	– 1.024	– 1.338	– 1.587
4.0	– 2.935	– 3.393	– 3.791	– 0.974	– 1.307	– 1.562
4.5	– 2.954	– 3.408	– 3.804	– 0.931	– 1.274	– 1.537
5.0	– 2.973	– 3.423	– 3.816	– 0.879	– 1.240	– 1.510
5.5	– 2.993	– 3.439	– 3.829	– 0.822	– 1.203	– 1.482
6.0	– 3.013	– 3.454	– 3.842	– 0.759	– 1.165	– 1.453
6.5	– 3.033	– 3.470	– 3.855	– 0.689	– 1.124	– 1.423
7.0	– 3.053	– 3.486	– 3.868	– 0.610	– 1.081	– 1.391
7.5	– 3.073	– 3.502	– 3.882	– 0.517	– 1.036	– 1.359
8.0	– 3.093	– 3.518	– 3.895	– 0.402	– 0.988	– 1.324
8.5	– 3.114	– 3.535	– 3.909	– 0.233	– 0.936	– 1.289
9.0	– 3.135	– 3.551	– 3.923	– 0.233	– 0.881	– 1.252
9.5	– 3.155	– 3.568	– 3.937	– 0.405	– 0.821	– 1.213
10.0	– 3.176	– 3.585	– 3.951	– 0.524	– 0.756	– 1.172
10.5	– 3.197	– 3.601	– 3.965	– 0.621	– 0.683	– 1.130
11.0	– 3.218	– 3.618	– 3.979	– 0.705	– 0.602	– 1.085
11.5	– 3.239	– 3.635	– 3.994	– 0.781	– 0.506	– 1.038

Introducing the non-dimensional parameters

$$\left.\begin{aligned}
&\eta = \frac{w_0}{v} z, \Omega^* = \frac{v\Omega}{w_0^2}, \omega^* = \frac{v\omega}{w_0^2}, R_c = \frac{K_0\omega_0^2}{v^2}, a_1^* = \frac{a_1 v}{w_0},\\
&b_1^* = \frac{b_1 v}{w_0}, a_2^* = \frac{a_2 v}{w_0}, b_2^* = \frac{b_2 v}{w_0}, a_3^* = \frac{a_3 v}{w_0}, b_3^* = \frac{b_3 v}{w_0},\\
&u_0^* = \frac{u_0}{A}, v_0^* = \frac{v_2}{A}, u_1^* = \frac{u_1}{A}, v_1^* = \frac{v_1}{A},\\
&u_2^* = \frac{u_2}{A}, v_2^* = \frac{v_2}{A}
\end{aligned}\right\} \qquad \ldots (4.3.54)$$

In eqns. (3.45) – (3.50) and dropping stars we obtain

$$u_0(\eta) = e^{a_1\eta}\cos b_1\eta + \frac{\varepsilon^2}{\omega}\left[\frac{a_1a_2 - b_1b_2}{\omega}\right](e^{a_1\eta}\cos b_1\eta - e^{a_2\eta}\cos b_2\eta)$$

$$-\frac{a_1b_2 - a_2b_1}{\omega}(e^{a_1\eta}\sin b_1\eta - e^{a_2\eta}\sin b_2\eta)$$

$$+\eta e^{a_1\eta}\{(a_1^2 - b_1^2)(2b_1\cos b_1\eta - [2a_1+1]\sin b_1\eta)$$

$$-2a_1b_1[(2a_1+1)\cos b_1\eta + 2b_1\sin b_1\eta]\}/\{(2a_1+1)^2 + 4b_1^{\,2}\}$$

$$+\frac{1}{2\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}}\{(a_1a_2 - b_1b_2)\,(e^{a_1\eta}\cos b_1\eta - e^{a_2\eta}\cos b_2\eta)$$

$$\left.+(a_1b_2 + a_2b_1)\,(e^{a_1\eta}\sin b_1\eta - e^{a_2\eta}\sin b_2\eta)\} - \frac{2a_1b_1e^{a_1\eta}\sin b_1\eta}{R_c + 2\Omega}\right] \quad \ldots (3.55)$$

$$v_0(\eta) = (e^{a_1\eta}\sin b_1\eta + \left[\frac{a_1a_2 - b_1b_2}{\omega}(e^{a_1\eta}\sin b_1\eta - e^{a_2\eta}\sin b_2\eta)\right.$$

$$+\frac{a_1b_2 + a_2b_1}{\omega}(e^{a_1\eta}\cos b_1\eta - e^{a_2\eta}\cos b_2\eta)$$

$$+\frac{\eta e^{a_1\eta}}{(2a_1+1)^2 + 4b_1^2}(a_1^2 - b_1^2)\,[(2a_1+1)\cos b_1\eta + 2b_1\sin b_1\eta]$$

$$-2a_1b_1[2a_1+1)\sin b_1\eta - 2b\cos b_1\eta]\} + \frac{1}{2\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}}\times$$

$$\{(a_1a_2 - b_1b_2)\,(e^{a_1\eta}\sin b_1\eta - e^{a_2\eta}\sin b_2\eta) - (a_1b_2 + a_2b_1)$$

$$\left.(e^{a_1\eta}\cos b_1\eta - e^{a_2\eta}\cos b_2\eta)\} - \frac{(a_1^2 - b_1^2)e^{a_1\eta}\sin b_1\eta}{R_c + 2\Omega}\right] \quad \ldots(3.56)$$

$$u_1(\eta) = \frac{\varepsilon}{\omega}\,[b_1\,(e^{a_1\eta}\cos b_1\eta - e^{a_2\eta}\cos b_2\eta) + a_1\,(e^{a_1\eta}\sin b_1\eta - e^{a_2\eta}\sin b_2\eta)] \quad \ldots(3.57)$$

$$v_1(\eta) = \frac{\varepsilon}{\omega}\,[b_1(e^{a_1\eta}\sin b_1\eta - e^{a_2\eta}\sin b_2\eta) - a_1\,(e^{a_1\eta}\cos b_1\eta - e^{a_2\eta}\cos b_2\eta)] \quad \ldots (3.58)$$

$$u_2(\eta) = \frac{\varepsilon^2}{\omega^2}\,[(a_1a_2 - b_1b_2)\,(e^{a_2\eta}\cos b_2\eta - e^{a_3\eta}\cos b_3\eta)$$

$$-(a_1b_2 + a_2b_1)\,(e^{a_2\eta}\sin b_2\eta - e^{a_3\eta}\sin b_3\eta) + \frac{a_1^2 - b_1^2}{2}$$

$$(e^{a_3\eta}\cos b_3\eta) - e^{a_1\eta}\cos b_1\eta) - a_1b_1\,(e^{a_3\eta}\sin b_3\eta) - e^{a_1\eta}\sin b_1\eta)] \quad \ldots (3.59)$$

and $$v_2(\eta) = \frac{\varepsilon^2}{\omega^2}\ [(a_1a_2 - b_1b_2)\ (e^{a_2\eta}\sin b_2\eta - e^{a_3\eta}\sin b_3\eta)$$

$$+ (a_1b_2 + a_2b_1)\ (e^{a_2\eta}\cos b_2\eta - e^{a_3\eta}\cos b_3\eta) + \frac{a_1^2 - b_1^2}{2}$$

$$(e^{a_3\eta}\sin b_3\eta) - e^{a_1\eta}\sin b_1\eta) - a_1b_1\ (e^{a_3\eta}\cos b_3\eta) - e^{a_1\eta}\cos b_1\eta)] \quad \ldots(3.60)$$

Table 3. Values of τ_1 and τ_2 for different values of R_c (low) and $\Omega = 5.0$

	τ_1			τ_2		
ω / R_c	0.5	1.0	1.5	0.5	1.0	1.5
1.5	– 2.846	– 2.566	– 2.224	– 1.171	– 0.690	– 0.118
2.0	– 2.863	– 2.529	– 2.255	– 1.139	– 0.614	– 0.353
2.5	– 2.881	– 2.553	– 2.287	– 1.103	– 0.524	– 0.486
3.0	– 2.898	– 2.578	– 2.319	– 1.065	– 0.413	– 0.592
3.5	– 2.917	– 2.604	– 2.350	– 1.024	– 0.252	– 0.682
4.0	– 2.935	– 2.629	– 2.382	– 0.974	– 0.212	– 0.763
4.5	– 2.954	– 2.655	– 2.413	– 0.931	– 0.394	– 0.836
5.0	– 2.973	– 2.681	– 2.443	–0.879	– 0.516	– 0.904
5.5	– 2.993	– 2.707	– 2.474	– 0.822	– 0.616	– 0.967
6.0	– 3.013	– 2.733	– 2.504	– 0.759	– 0.702	– 1.027
6.5	– 3.033	– 2.758	– 2.534	– 0.689	– 0.779	– 1.084
7.0	– 3.053	– 2.784	– 2.563	– 0.610	– 0.850	– 1.138
7.5	– 3.073	– 2.810	– 2.592	– 0.517	– 0.916	– 1.190
8.0	– 3.093	– 2.835	– 2.621	– 0.402	– 0.978	– 1.240
8.5	– 3.114	– 2.860	– 2.650	– 0.233	– 1.036	– 1.288
9.0	– 3.135	– 2.885	– 2.678	– 0.233	– 1.091	– 1.334
9.5	– 3.155	– 2.910	– 2.706	– 0.405	– 1.144	– 1.379
10.0	– 3.176	– 2.934	– 2.733	– 0.524	– 1.195	– 1.423
10.5	– 3.197	– 2.959	– 2.761	– 0.621	– 1.244	– 1.465
11.0	– 3.218	– 2.983	– 2.788	– 0.705	– 1.291	– 1.506
11.5	– 3.239	– 3.007	– 2.814	– 0.781	– 1.337	– 1.546

Analysis of Temperature Field

The energy equation with the help of eqns. (2.5), (2.8), (2.9) and (2.10), takes the form

$$\rho_c \left[\frac{\partial T}{\partial t} - w_0 \left\{1 + \varepsilon\ (e^{i\omega t} + e^{-i\omega t})\frac{\partial T}{\partial Z}\right\}\right]$$

$$= K\frac{\partial^2 T}{\partial z^2} + \mu\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2\right]$$

$$= K\frac{\partial^2 T}{\partial z^2} + \mu\left|\frac{\partial F}{\partial z}\right|^2 + R_c[u^2 + v^2]$$

$$= K\frac{\partial^2 T}{\partial z^2} + \mu\left|\frac{\partial F}{\partial z}\right|^2 + R_c\left|F^2\right| \quad \ldots (4.1)$$

where $F = u + iv$.

The boundary conditions are

$$\left.\begin{array}{l} T = Tw, \text{ at } z = 0 \\ T = T_\infty, \text{ at } z \to \infty \end{array}\right\} \quad \ldots (4.2)$$

Introducing the following non-dimensional quantities in addition to those given in eqn. (3.54)

$$\left.\begin{array}{l} P = \frac{\mu_c}{k}, E = \frac{A^2}{c\left((T_w - T_\infty\right)}, \theta = \frac{T - T_\infty}{T_w - T_\infty} \\ F^* = \frac{F}{A}, t^* = \frac{w_0^2}{v}t, F_0^* = \frac{F_0}{A}, F_n^* = \frac{F_n}{A}, \end{array}\right\} \quad \ldots (4.3)$$

Equation (4.1) reduces to (dropping stars),

$$\frac{\partial^2\theta}{\partial\eta^2} - P\left[\frac{\partial\theta}{\partial t} - \{1 + \varepsilon(e^{i\omega t} + e^{-i\omega t})\}\frac{\partial\theta}{\partial\eta}\right] = -PE\left|\frac{\partial F}{\partial\eta}\right|^2 - R_c[F]^2 \quad \ldots (4.4)$$

The transformed boundary conditions are from eqn. (4.2)

$$\theta(0, t) = 1, \theta(\infty, t) = 0 \quad \ldots (4.5)$$

In order to solve eqn. (4.4), we assume that in the boundary layer, the unsteady temperature in super-imposed on the mean temperature and thus θ can be represented by the following Fourier series:

$$\theta = \theta_0(\eta) + \sum_{n=1}^{\infty} \theta_n(\eta)\, e^{in\omega t} + \sum_{n=1}^{\infty}\overline{\theta}_n(\eta)\, e^{-in\omega t} \quad \ldots (4.6)$$

With the help of equations (3.4), (4.3) and (4.6), eqn. (4.4), on equating the harmonic and non-harmonic terms, yields

$$\theta_0'' + P\theta_0' + \varepsilon P(\overline{\theta}_1' + \theta_1) = -PE\Big[|F_0'|^2 + 2F_1'\overline{F}_1' + 2F_2'\overline{F}_2'$$

$$+ \ldots + 2F_n'\overline{F}_n' + \ldots.] - PE\, R_c\, [|F_0|^2 + 2F_1 - \overline{F}_1$$

$$+ 2F_2 - \overline{F}_2 + \ldots. + 2F_n\, \overline{F}_n + \ldots.] \quad \ldots (4.7)$$

$$\theta_1'' - i\omega P\theta_1 + P\theta_1' + \varepsilon P(\theta_0' + \theta_2') = -PE\Big[\left(F_0' + \overline{F}_0'\right)F_1' + 2F_2'\overline{F}_1'$$

$$+ 2F_3'\overline{F}_2' + \ldots.] + 2F_n'\overline{F}_{n-1}' + \ldots.\ \ldots.]$$

$$- PE\, R_c\, [\bar{F}_0 F_1 + F_0 F_1 + 2F_2 \bar{F}_1 + 2F_3 \bar{F}_2 + \ldots + 2F_n \bar{F}_{n-1} + \ldots], \quad \ldots (4.8)$$

$$\theta_2'' - 2i\omega P\theta_2 + P\theta_2' + \varepsilon P(\theta_1' + \theta_3') = -PE\Big[\big(F_0' + \bar{F}_0'\big)F_2' + 2F_3'\bar{F}_1'$$

$$+ 2F_4'\bar{F}_2' + \ldots.] + 2F_n'\bar{F}_{n-2}' + \ldots.\ \ldots.\ F_1'^2\Big]$$

$$- PE\, R_c\, [(F_0 + \bar{F}_0)\, F_2 + 2F_3 \bar{F}_0 + 2F_4 \bar{F}_2 + \ldots + 2F_n \bar{F}_{n-2} + \ldots. + F_1^2], \quad \ldots (4.9)$$

And similar equations for θ_n, $n > 2$.

The boundary conditions (4.5) are now modified to

$$\theta_0(0) = 1,\ \theta_n(0) = 0,\ n \geq 1$$

$$\theta_0(\infty) = 1,\ \theta_n(\infty) = 0,\ n \geq 1 \quad \ldots (4.10)$$

In order to obtain solutions of eqns. (4.7) – (4.9), we assume the following expansion for θ_n in powers of :

$$\theta_n(\eta) = \sum_{r=0}^{\infty} \theta_{nr}(\eta)\, \varepsilon^r \quad \ldots (4.11)$$

Substituting θ_0 from eqn. (4.11) in eqn. (4.7) and equating the coefficients of like powers of ε, we get (using eqn. (3.10) and (3.3))

$$\theta_{00}'' + P\theta_{00}' = -PE|F_{00}'|^2 + 2F_{10}'\bar{F}_{10}' + 2F_{20}'\bar{F}_{20}'\Big] - PER_c\Big[|F_{00}|^2$$

$$+ 2F_{10}\bar{F}_{10} + 2F_{20}\bar{F}_{20}\Big], \text{(neglecting terms for } n > 2)$$

$$= - PE\ |F_{00}'|^2 - PER_c\Big[|F_{00}|^2\ , \text{[using eqn. (4.25) and (4.27),} \quad \ldots (4.12)$$

$$\theta_{01}'' + P\theta_{01}' + P(\bar{\theta}_{10}' + \theta_{10}') = -PE[F_{01}'\bar{F}_{00}' + F_{00}'\bar{F}_{01}' + 2(F_{10}'\bar{F}_{11}' + F_{11}'\bar{F}_{10}')$$

$$+2'(F_{20}'\bar{F}_{21}' + F_{21}'\bar{F}_{20}')]$$

$$- PE\, R_c\, [F_{01}\bar{F}_{00} + F_{00}\bar{F}_{01} + 2(F_{10}\bar{F}_{11} + F_{11}\bar{F}_{10} + 2\,(F_{20}\bar{F}_{21} + F_{21}\bar{F}_{20}\,)],$$

$$= 0, \text{[using eqns. (3.25), (3.27) and (3.28)]} \quad \ldots (4.13)$$

$$\theta_{02}'' + P\theta_{02}' + P\,(\bar{\theta}_{11}' + \theta_{11}') = -PE[F_{00}'\bar{F}_{02}' + F_{01}'\bar{F}_{01}' + F_{02}'\bar{F}_{00}'$$

$$+ 2\ F_{10}'\bar{F}_{12}' + 2F_{11}'\bar{F}_{11}' + 2F_{12}'\bar{F}_{10}' + 2\,F_{20}'\bar{F}_{22}' + 2F_{21}'\bar{F}_{21}' + 2F_{22}'\bar{F}_{20}'\,]$$

$$- PE\, R_c\, [\,F_{00}'\bar{F}_{02}' + F_{01}'\bar{F}_{01}' + F_{02}'\bar{F}_{00}' + 2F_{10}\bar{F}_{12}$$

$$+ 2\,F_{11}'\bar{F}_{11}' + 2F_{12}'\bar{F}_{10}' + 2F_{20}'\bar{F}_{22}' + 2F_{21}\bar{F}_{21} + 2F_{22}\bar{F}_{20}\,]$$

$$=- PE\,[\,F_{00}'\bar{F}_{02}' + F_{02}'\bar{F}_{00}' + 2F_{11}'\bar{F}_{11}'\,] - PE\, R_c\, [\,F_{00}'\bar{F}_{02}' + F_{02}'\bar{F}_{00}' + 2F_{11}'\bar{F}_{11}'\,],$$

$$\text{[using eqns. (3.25), (3.27), (3.28) and (3.35)].} \quad \ldots (4.14)$$

Similarly substituting θ_1 and θ_2 from eqn. (4.11) in eqns. (4.8) and eqn. (4.9) and equating the coefficients of like powers of ε (till the term involving ε^2), we obtain

$$\theta_{10}'' - i\omega P\theta_{10} + P\theta_{10}' = -PE\,[\,(F_{00}' + \bar{F}_{00}')F_{10}' + 2\bar{F}_{20}'\bar{F}_{10}'\,]$$

$$- PE\, R_c\, [\, \bar{F}_{00} F_{10} + F_{00} F_{10} + 2F_{20} \bar{F}_{10}\,]$$

$$= 0, \text{ [using eqn. (3.25)]}, \qquad \ldots (4.15)$$

$$\theta''_{11} + P\theta'_{11} - i\omega P\theta_{11} + P(\theta'_{00} + \theta'_{20}) = - PE\, [\, (F'_{01} + \bar{F}'_{01}) F'_{10} + (\bar{F}'_{00} + \bar{F}'_{00}) F'_{11}$$
$$+ 2\, F'_{20} \bar{F}'_{11} + 2F'_{21} \bar{F}'_{10}\,] - PE\, R_c\, [\, \bar{F}_{00} F_{11} + \bar{F}_{01} F_{10} + F_{00} F_{11} + F_{01} F_{10}$$
$$+ 2\, F_{20} \bar{F}_{11} + 2F_{21} \bar{F}_{10}\,]$$
$$= - PE\, [\, (F'_{00} + F'_{00})\, F'_{11}) - PER_c [(F_{00} + \bar{F}_{00}) F_{11}]$$
$$[\therefore F_{10} = F_{20} = F_{21} = 0], \qquad \ldots (4.16)$$

$$\theta''_{12} + P\theta'_{12} - i\omega P\theta_{12} + P(\theta'_{01} + \theta'_{21}) = -PE\, [\, (F'_{00} + \bar{F}'_{00})\, F'_{12} + (F'_{01} + \bar{F}'_{01}) F'_{11}$$
$$+ 2\, (F'_{02} + \bar{F}'_{02}) F'_{10} + 2F'_{20} \bar{F}'_{12} + 2F'_{21} \bar{F}'_{11}$$
$$- 2\, F'_{22} \bar{F}'_{10}\,] - PER_c [\bar{F}_{00} F_{12} + \bar{F}_{01} F_{11} + \bar{F}_{02} F_{10}$$
$$+ F_{00} F_{12} + F_{01} F_{11} + F_{02} F_{10} + 2F_{20}\, \bar{F}_{12} + 2F_{21} \bar{F}_{11} + 2\, F_{22} \bar{F}_{10}\,]$$
$$= 0, [\therefore F_{01} = F_{10} = F_{12} = F_{20} = F_{21} = 0], \qquad \ldots (4.17)$$

$$\theta''_{20} + P\theta'_{20} - 2i\omega P\theta_{20} = - PE\, [\, (F'_{00} + \bar{F}'_{00}) F'_{20} + F'^{2}_{10}\,] - PE\, R_c \left[(F'_{00} + \bar{F}'_{00}) F'_{20} + F'^{2}_{10} \right]$$
$$= 0, \quad [\therefore F_{10} = F_{20} = 0] \qquad \ldots (4.18)$$

$$\theta''_{21} + P\theta'_{21} - 2i\omega P\theta_{21} + P\theta'_{10} = - PE\, [\, (F'_{00} + \bar{F}'_{00}) F'_{21} + (F'_{01} + \bar{F}'_{01}) F'_{20}$$
$$+ 2\, F'_{10} F'_{11}\,] - PER_c [(F_{00} \bar{F}_{00}) F_{21} + F_{01} + \bar{F}_{01}) F_{20} + 2F_{10} F_{11}]$$
$$+ 0, \qquad [\therefore F_{10} = F_{20} = F_{21} = 0] \qquad \ldots (4.19)$$

$$\theta''_{22} + P\theta'_{22} - 2i\omega P\theta_{22} + P\theta'_{11} = - PE\, [\, (F'_{00} + \bar{F}'_{00}) F'_{22} + (F'_{01} + \bar{F}'_{01}) F'_{21}$$
$$+\ (F'_{02} + \bar{F}'_{02}) F'_{20} + F'^{2}_{11} + 2F'_{10} F'_{12}\,]$$
$$- PE\, R_c\ [(F_{00} \bar{F}_{00}) F_{22} + (F_{01} + \bar{F}_{01}) F_{21}$$
$$+ (F_{02} + \bar{F}_{02})\, F_{20} + F_{11}{}^{2} + 2F_{10} F_{12}]$$
$$= - PE \left[(F'_{00} + \bar{F}'_{00}) F'_{22} + F'^{2}_{11} \right] + PER_c \left[(F_{00} + \bar{F}'_{00}) F_{22} + F^{2}_{11} \right]$$
$$[F_{01} = F_{21} = F_{20} = F_{10} = F_{12} = 0] \qquad \ldots (4.20)$$

The boundary conditions (4.10) reduce to

$$\theta_{00}\,(0) = 1;\ \theta_{or}\,(0) = 0,\ r \geq 1;\ \theta_{nr}\,(0) = 0, \text{ for all } r,\ n \geq 1$$
$$\theta_{nr}\,(\infty) = 0, \text{ for all } r \text{ and all } n \qquad (4.21)$$

The solution of eqn. (4.12) subject to the boundary conditions (4.21) is given by

$$\theta_{00}\,(\eta) = e^{-p\eta} + \frac{PE\left[a_1^2 + b_1^2 + R_c \right]}{4a_1^2 + 2a_1 p} \left[e^{-p\eta} - e^{2a_1\eta} \right], \qquad \ldots (4.22)$$

where a_1 and b_1 are the non-dimensional form of the real and imaginary parts a_1, b_1 given in eqn. (3.51).

The solution of eqn. (4.15) with the boundary conditions (4.21) is easily found to be

$$\theta_{10}(\eta) = 0 \quad \dots (4.23)$$

Consequently, eqn. (4.13) reduces to

$$\theta''_{01} + P\theta'_{01} = 0,$$

the solution of which subject to boundary conditions (4.21) is given by

$$\theta_{01}(\eta) = 0 \quad \dots (4.24)$$

Similarly with the help of eqn. (4.23), eqn. (4.19) reduces to

$$\theta''_{21} + P\theta'_{21} - 2i\omega P\theta_{21} = 0,$$

and the solution of this equation with boundary conditions (4.21) is

$$\theta_{21}(\eta) = 0 \quad \dots (4.25)$$

Moreover, solution of eqn. (4.18) subject to boundary conditions (4.21) is obtained as

$$\theta_{20}(\eta) = 0 \quad \dots (4.26)$$

Again eqns. (4.24) and (4.25) reduce eqn. (4.17) to the form

$$\theta''_{12} + P\theta'_{12} - i\omega P\theta_{12} = 0,$$

the solution of which under boundary conditions (4.21) is

$$\theta_{12}(\eta) = 0 \quad \dots (4.27)$$

Equation (4.16) with the help of eqns. (4.22), (4.26), (3.23), (3.30) and (3.54) takes the form

$$\theta''_{11} + P\theta'_{11} - i\omega P\theta_{11} = P\left[Pe^{-p\eta} + \frac{PE(a_1^2 + b_1^2 + R_c)}{4a_1^2 + 2a_1 p}\{e^{-p\eta} + 2a_1 e^{2a_1\eta}\}\right.$$
$$- \frac{PER_c}{a}\left[\beta_2 e^{\beta_2\eta} - \alpha_2 e^{\alpha_2\eta}\right]$$
$$\left. + \left[\alpha_2 e^{\alpha_2\eta} + \bar{\alpha}_2 e^{\bar{\alpha}_2\eta}\right] - PER_c\alpha_2\left[e^{\beta_2\eta} - e^{\alpha_2\eta}\right]\left[e^{\alpha_2\eta - \bar{\alpha}_2\eta}\right]\right]$$
$$= \left[1 + \frac{PE(a_1^2 + b_1^2 + R_c)}{4a_1^2 + 2a_1 p}\right]P^2 e^{-p\eta} + \frac{P^2 E(a_1^2 + b_1^2 + R_c)}{2a_1 + p} e^{2a_1\eta}$$
$$- \frac{PER_c}{\omega}\left[(\alpha_2\beta_2 + R_c)e^{(\alpha_2\beta_2)\eta} + (\bar{\alpha}_2\beta_2 + R_c)\times\right.$$
$$\left. e^{(\bar{\alpha}_2\beta_2)\eta} - (\alpha_2^2 + R_c)\, e^{2\alpha_2\eta} - \left(|\alpha_2|^2 + R_c\right)e^{(\bar{\alpha}_2 + \bar{\alpha}_2)\eta}\right] \quad \dots (4.28)$$

The solution of the eqn. (4.28) is given by

$$\theta_{11}(\eta) = A_1 e^{-\eta/2[P - \sqrt{P^2 + 4i\omega P}]} + A_2\ e^{-\eta/2[P + \sqrt{P^2 + 4i\omega P}]}$$

$$-\left[1+\frac{PE(a_1^2+b_1^2+R_c)}{4a_1^2+2a_1P}\right]\frac{Pe^{-P\eta}}{i\omega}+\left[\frac{P^2E(a_1^2+b_1^2+R_c)\,e^{2a_1\eta}}{(2a_1+P)(4a_1^2+2a_1P-i\omega P)}\right]$$

$$-\frac{PEi\alpha_2}{\omega}\Big[\left(\alpha_2\beta_2+R_c\right)\frac{e^{(\alpha_2\beta_2)\eta}}{\left(\alpha_2+\beta_2\right)^2+P\left(\alpha_2+\beta_2\right)-i\omega P}$$

$$+\left(\bar{\alpha}_2\beta_2+R_c\right)\frac{e^{(\bar{\alpha}_2\beta_2)\eta}}{\left(\bar{\alpha}_2+\beta_2\right)^2+P\left(\bar{\alpha}_2+\beta_2\right)-i\omega P}$$

$$-(a_2^2+R_c)\frac{e^{2\alpha_2\eta}}{4a_1^2+2a_1P-i\omega P}-(a_2^2+b_1^2+R_c)\frac{e^{2\alpha_2\eta}}{4a_1^2+2a_1P-i\omega P}\Big].$$

Applying boundary conditions (4.21), it follows that

$$A_1=0,$$

$$A_2=\left[1+\frac{PE(a_1^2+b_1^2+R_c)}{4a_1^2+2a_1P}\right]\frac{P}{i\omega}+\left[\frac{P^2E(a_1^2+b_1^2+R_c)}{\left(2a_1+P\right)(4a_1^2+2a_1P-i\omega P)}\right]$$

$$-\frac{PEi\alpha_2}{\omega}\Bigg[\frac{\alpha_2\beta_2+R_c}{\left(\alpha_2+\beta_2\right)^2+P\left(\alpha_2+\beta_2\right)-i\omega P}$$

$$+\frac{\bar{\alpha}_2\beta_2+R_c}{\left(\bar{\alpha}_2+\beta_2\right)^2+P\left(\bar{\alpha}_2+\beta_2\right)-i\omega P}-\frac{a_2^2+R_c}{4\alpha_{21}^2+2\alpha_2P-i\omega P}$$

$$-\frac{a_2^2+b_1^2+R_c}{4a_1^2+2a_1P-i\omega P}\Bigg]$$

Hence,

$$\theta_{11}(\eta)=\left[1+\frac{X_1}{2a_1(2a_1+P)}\right]\frac{P}{i\omega}\left(e^{-\alpha\eta/2}-e^{-P\eta}\right)+\frac{PX_1}{(2a_1+P)Y_1}\left(e^{2a_1\eta}-e^{-\alpha\eta/2}\right)$$

$$+\frac{PEi\alpha_1}{\omega}\Bigg[\frac{X_2}{Y_2}\left(e^{-\alpha\eta/2}-e^{(\alpha_2+\beta_2)\eta}\right)$$

$$+\frac{X_{31}}{Y_3}\left(e^{-a\eta/2}-e^{(\bar{\alpha}_2+\beta_2)\eta}\right)\frac{X_4}{Y_4}\left(e^{2a_2\eta}-e^{-\alpha\eta/2}\right)$$

$$+\frac{X_1}{PEY_1}\left(e^{2a_1\eta}-e^{-\alpha\eta/2}\right)\Bigg], \qquad \ldots (4.29)$$

Here

$$\alpha = P + \sqrt{(P^2 + 4i\omega P)},$$
$$X_1 = PE\,(a_1{}^2 + b_1{}^2 + R_c),$$
$$X_2 = \alpha_2\beta_2 + R_c,$$
$$X_3 = \bar{\alpha}_2\beta_2 + R_c,$$
$$X_4 = \alpha_2^2 + R_c,$$
$$Y_1 = 4a_1{}^2 + 2a_1P - i\omega P,$$
$$Y_2 = (\alpha_2 + \beta_2)^2 + P\,(\alpha_2 + \beta_2) - i\omega P,$$
$$Y_3 = (\bar{\alpha}_2 + \beta_2)^2 + P(\bar{\alpha}_2 + \beta_2) - i\omega P,$$
$$Y_4 = 4\alpha_2{}^2 + 2\alpha_2 P - \mathrm{i}\omega P.$$

With the help of eqns. (3.23), (3.30), (3.32) and (4.29), the equation (4.14) takes the form

$$\theta''_{02} + P\theta'_{02} = -PE\left[\left\{\frac{Z_1X_1}{PE\omega^2} + \left(\frac{\alpha_2^2}{2\alpha_2+1} - \frac{\bar{\alpha}_2^2}{2\bar{\alpha}_2+1}\right)\frac{i\eta X_1}{PE\omega}\right.\right.$$
$$+ I\,|\alpha_2|^2\left(\frac{\alpha_2}{2\alpha_2+1} - \frac{\bar{\alpha}_2}{2\bar{\alpha}_2+1}\right) + \frac{Z_1X_1}{2\omega PE\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}}$$
$$\left. - \frac{(\alpha_2^2 + \bar{\alpha}_2^2)\,X_1}{2\omega PE\{R_c + 2\Omega\}} + \frac{2|\alpha_2|^2\,X_1}{\omega^2 PE}\right\}e^{2a_1\eta}$$
$$-\frac{\bar{\alpha}_2}{\omega^2}\left(\bar{\beta}_2 + 2\alpha_2\right)\bar{X}_3 e^{\left(\alpha_2+\bar{\beta}_2\right)\eta} - \frac{\alpha_2}{\omega^2}\left(\beta_2 + 2\bar{\alpha}_2\right)X_3 e^{\left(\bar{\alpha}_2+\bar{\beta}_2\right)\eta}$$
$$-\frac{\alpha_2\beta_2X_2e^{(\alpha_2+\beta_2)\eta}}{2\omega\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}} - \frac{\bar{\alpha}_2\bar{\beta}_2\bar{X}_2e^{\left(\bar{\alpha}_2+\bar{\beta}_2\right)\eta}}{2\omega\left\{R_c + 2\Omega + \frac{\omega}{2}\right\}}$$
$$\left.\frac{\alpha_2^2X_4e^{2\alpha_2\eta}}{2\omega\{R_c + 2\Omega\}} + \frac{\bar{\alpha}_2^2\bar{X}_4e^{2\bar{\alpha}_2\eta}}{2\omega\{R_c + 2\Omega\}} + \frac{2|\alpha_2|^2\,Z_2e^{\left(\beta_2+\bar{\beta}_2\right)\eta}}{\omega^2}\right]$$
$$-P\left[\left\{\frac{PX_1\alpha}{2(2a_1+P)Y_1} - \frac{P\alpha}{2i\omega}\left(1 + \frac{X_1}{2a_1(2a_1+P)}\right) + \frac{PEi\alpha_2}{\omega}\right.\right.$$
$$\left.\left(\frac{X_1\alpha}{2PEY_1} + \frac{\alpha X_4}{2Y_4} - \frac{\alpha X_3}{2Y_3} - \frac{\alpha X_2}{2Y_2}\right)\right\}e^{-\alpha\eta/2}\left\{\frac{P\bar{\alpha}}{2i\omega}\left(1 + \frac{X_1}{2a_1\left(2a_1+P\right)}\right)\right.$$
$$\left.+\frac{PX_1\bar{\alpha}}{2\left(2a_1+P\right)Y_1} - \frac{PEi\bar{\alpha}_2}{\omega}\left(\frac{X_1\bar{\alpha}}{2PE\bar{Y}_1} + \frac{\bar{\alpha}X_4}{2\bar{Y}_4} - \frac{\bar{\alpha}\bar{X}_3}{2\bar{Y}_3} - \frac{\bar{\alpha}X_2}{2\bar{Y}_2}\right)\right\}e^{-\bar{\alpha}\eta/2}$$
$$+\left\{\frac{2a_1X_4}{Y_1}\left(\frac{P}{2a_1+P} + \frac{i\alpha_2}{\omega}\right) + \frac{2a_1X_1}{\bar{Y}_1}\left(\frac{P}{2a_1+P} - \frac{i\bar{\alpha}_2}{\omega}\right)\right\}e^{-2a_1\eta}$$

$$-\frac{PEi\alpha_2 X_2}{\omega Y_2}(\alpha_2+\beta_2)e^{(\alpha_2+\beta_2)\eta}+\frac{PEi\bar{\alpha}_2\bar{X}_2}{\omega\bar{Y}_2}(\bar{\alpha}_2+\bar{\beta}_2)e^{(\bar{\alpha}_2+\bar{\beta}_2)\eta}$$

$$\left.-\frac{PEi\alpha_2^2 X_4}{\omega Y_4}e^{2\alpha_2\eta}-\frac{PEi\bar{\alpha}_2^2\bar{X}_4}{\omega\bar{Y}_4}e^{2\bar{\alpha}_2\eta}\right] \qquad \dots (4.30)$$

where $Z_1 = \alpha_2\beta_2 + \bar{\alpha}_2\bar{\beta}_2$,

and $Z_2 = |\beta_2|^2 + R_c$.

The solution of this equation is given by

$$\theta_{02}(\eta) = C_1 + C_2\, e^{-P\eta} + \left[-\frac{Z_1X_1}{\omega^2}+\left(\frac{\bar{\alpha}_2^{2}}{2\bar{\alpha}_2+1}-\frac{\alpha_2^2}{2\bar{\alpha}_2+1}\right)\frac{i\eta X_1}{\omega}\right.$$

$$+\frac{iPE|\alpha_2|^2}{\omega}\left(\frac{\bar{\alpha}_2}{2\bar{\alpha}_2+1}-\frac{\alpha_2}{2\alpha_2+1}\right)-\frac{Z_1X_1}{2\omega\left\{R_c+2\Omega+\frac{\omega}{2}\right\}}$$

$$+\frac{X_1\left(\alpha_2^2+\bar{\alpha}_2^2\right)}{2\omega\{R_c+2\Omega\}}-\frac{2|\alpha_2|^2X_1}{\omega^2}-\frac{2a_1X_1}{Y_1}$$

$$\left.\left(\frac{P^2}{2a_1+P}+\frac{iP\alpha_2}{\omega}\right)\frac{2a_1X_1}{\bar{Y}_1}\left(\frac{P^2}{2a_1+P}-\frac{iP\bar{\alpha}_2}{\omega}\right)\right]\frac{e^{2a_1\eta}}{4a_1^2+2a_1P}$$

$$+\frac{PE\bar{X}_3}{\omega}\left\{\frac{\bar{\alpha}\left(\bar{\beta}_2+2\alpha_2\right)}{\omega}-\frac{Pi\bar{\alpha}_2\left(\alpha_2+\bar{\beta}_2\right)}{\bar{Y}_3}\right.\frac{e^{(\alpha_2+\bar{\beta}_2)\eta}}{\left(\alpha_2+\bar{\beta}_2\right)\left(\alpha_2+\bar{\beta}_2+P\right)}$$

$$+\frac{PE\alpha_2X_3}{\omega}\left\{\frac{(\beta_2+2\bar{\alpha}_2)}{\omega}+\frac{Pi\left(\bar{\alpha}_2+\bar{\beta}_2\right)}{Y_3}\right\}\frac{e^{(\bar{\alpha}_2+\beta_2)\eta}}{(\bar{\alpha}_2+\beta_2)(\bar{\alpha}_2+\beta_2+P)}$$

$$+\frac{PE\alpha_2X_3}{\omega}\left\{\frac{\beta_2}{2\left\{R_c+2\Omega+\frac{\omega}{2}\right\}}-\frac{Pi\left(\bar{\alpha}_2+\bar{\beta}_2\right)}{Y_2}\right\}\frac{e^{(\bar{\alpha}_2+\bar{\beta}_2)\eta}}{\left(\bar{\alpha}_2+\bar{\beta}_2\right)\left(\bar{\alpha}_2+\bar{\beta}_2+P\right)}$$

$$-\frac{PE\alpha_2^2X_4}{\omega}\left\{\frac{1}{2\{R_c+2\Omega\}}+\frac{2Pi}{Y_4}\right\}\frac{e^{2\alpha_2\eta}}{2\alpha_2(2\alpha_2+P)}$$

$$-\frac{PE\bar{\alpha}_2^2\bar{X}_4}{\omega}\left\{\frac{1}{2\{R_c+2\Omega\}}-\frac{2Pi}{\bar{Y}_4}\right\}\frac{e^{2\bar{\alpha}_2\eta}}{2\bar{\alpha}_2(2\bar{\alpha}_2+P)}$$

$$-\frac{2|\alpha_2|^2PEZ_2}{\omega^2}\frac{e^{2\alpha_2\eta}}{2a_2(2a_2+P)}\left\{\frac{P^2\alpha}{2i\omega}\left(1+\frac{X_1}{2a_1(2a_1+P)}\right)\right.$$

$$-\frac{P^2 X_1\alpha}{2(2a_1+P)Y_1}-\frac{P^2 Ei\alpha_2}{\omega}\left(\frac{X_1\alpha}{2PEY_1}+\frac{\alpha X_4}{2Y_4}-\frac{\alpha X_3}{2Y_3}-\frac{\alpha X_2}{2Y_2}\right)\Bigg\}$$

$$\frac{4e^{\alpha_2\eta/2}}{\alpha^2-2P\alpha}-\left\{\frac{P^2\bar{\alpha}}{2i\omega}\left(1+\frac{X_1}{2a_1(2a_1+P)}+\frac{P^2X_1\bar{\alpha}}{2(2a_1+P)\bar{Y}_1}\right)\right.$$

$$\left.-\frac{P^2 Ei\bar{\alpha}_2}{\omega}\left(\frac{X_1\bar{\alpha}}{2PEY_1}+\frac{\bar{\alpha}\bar{X}_4}{2\bar{Y}_4}-\frac{\bar{\alpha}\bar{X}_3}{2\bar{Y}_3}-\frac{\bar{\alpha}\bar{X}_2}{2\bar{Y}_2}\right)\right\}\frac{4e^{-\bar{\alpha}\eta/2}}{\bar{\alpha}^2-2P\bar{\alpha}}.$$

Applying the boundary conditions (4.21), we obtain,

$$\theta_{02}(\eta)=\left[-\frac{Z_1X_1}{\omega^2}+\frac{iPE|\alpha_2|^2}{\omega}\left(\frac{\bar{\alpha}_2}{2\bar{\alpha}_2+1}-\frac{\alpha_2}{2\alpha_2+1}\right)-\frac{Z_1X_1}{\omega Z_3}\right.$$

$$\frac{X_1\left(\alpha_2^2+\bar{\alpha}_2^2\right)}{2\omega\{R_c+2\Omega\}}-\frac{2|\alpha_2|^2X_1}{\omega^2}-\frac{2a_1X_1}{Y_1}\left(\frac{P^2}{2a_1+P}+\frac{iP\alpha_2}{\omega}\right)$$

$$\left.+\frac{2a_1X_1}{\bar{Y}_1}\left(\frac{P^2}{2a_1+P}-\frac{iP\bar{\alpha}_2}{\omega}\right)\right]\frac{e^{2a_1\eta}-e^{-P\eta}}{4a_1^2+2a_1P}+\left(\frac{\bar{\alpha}_2^2}{2\bar{\alpha}_2+1}-\frac{\bar{\alpha}_2^2}{2\bar{\alpha}_2+1}\right)$$

$$\frac{i\eta X_1}{\omega}\frac{e^{2a_1\eta}}{4a_1^2+2a_1P}+\frac{PE\bar{\alpha}_2\bar{X}_3}{\omega}\left\{\frac{\bar{\beta}_2+2\alpha_2}{\omega}-\frac{Pi\left(\alpha_2+\bar{\beta}_2\right)}{\bar{Y}_3}\right\}$$

$$\times\frac{e^{\left(\alpha_2+\bar{\beta}_2\right)\eta}-e^{-P\eta}}{\left(\alpha_2+\bar{\beta}_2\right)\left(\alpha_2+\bar{\beta}_2+P\right)}+\frac{PE\alpha_2X_3}{\omega}\left\{\frac{\beta_2+2\bar{\alpha}_2}{\omega}+\frac{Pi\left(\bar{\alpha}_2+\bar{\beta}_2\right)}{Y_3}\right\}$$

$$+\frac{e^{\left(\bar{\alpha}_2+\bar{\beta}_2\right)\eta}-e^{-P\eta}}{\left(\bar{\alpha}_2+\bar{\beta}_2\right)\left(\bar{\alpha}_2+\bar{\beta}_2+P\right)}-\frac{PE\alpha_2^2X_4}{\omega}\left\{\frac{1}{2\{R_c+2\Omega\}}+\frac{2Pi}{Y_4}\right\}\frac{e^{2\alpha_2\eta}}{2\alpha_2(2\alpha_2+P)}$$

$$-\frac{PE\bar{\alpha}_2^2\bar{X}_4}{\omega}\left\{\frac{1}{2\{R_c+2\Omega\}}-\frac{2Pi}{\bar{Y}_4}\right\}\frac{e^{2\bar{\alpha}_2\eta}}{2\bar{\alpha}_2(2\bar{\alpha}_2+P)}$$

$$-\frac{2|\alpha_2|^2PEZ_2}{\omega^2}\frac{e^{2\alpha_2\eta}-e^{-P\eta}}{2a_2(2a_2+P)}\left\{\frac{P^2\alpha}{2i\omega}\left(1+\frac{X_1}{2a_1(2a_1+P)}\right)\right.$$

$$\left.-\frac{P^2X_1\alpha}{2(2a_1+P)Y_1}-\frac{P^2Ei\alpha_2\alpha}{2\omega}\left(\frac{X_1}{PEY_1}+\frac{X_4}{Y_4}-\frac{X_3}{Y_3}-\frac{X_2}{Y_2}\right)\right\}\frac{4\left(e^{-\alpha\eta/2}-e^{-P\eta}\right)}{\alpha^2-2P\alpha}$$

$$-\left\{\frac{P^2\bar{\alpha}}{2i\omega}\left(1+\frac{X_1}{2a_1(2a_1+P)}+\frac{P^2X_1\bar{\alpha}}{2(2a_1+P)\bar{Y}_1}\right)\right.$$

$$-\frac{P^2 Ei\bar{\alpha}_2\alpha}{2\omega}\left(\frac{X_1}{PE\bar{Y}_1}+\frac{\bar{X}_4}{\bar{Y}_4}-\frac{\bar{X}_3}{\bar{Y}_3}-\frac{\bar{X}_2}{\vec{Y}_2}\right)\Bigg\}\frac{4\left(e^{-\bar{\alpha}\eta/2}-e^{-P\eta}\right)}{\bar{\alpha}^2-2P\bar{\alpha}},$$

where $$Z_3 = 2\left\{R_c + 2\Omega + \frac{\omega}{2}\right\} \tag{4.31}$$

Equation (4.20), with the aid of equations (3.23), (3.30) (3.41) and (4.29) reduces to

$$\theta_{22}'' + P\theta_{22}' - 2i\omega P\theta_{22} = -\frac{PE\alpha_2}{\omega^2}\Bigg[X_2\left\{(\beta_2+2\alpha_2)-\frac{Pi\omega}{Y_2}(\alpha_2+\beta_2)\right\}e^{(\alpha_2+\beta_2)\eta}$$

$$+\frac{\alpha_2-2\beta_2}{2}X_5 e^{(\alpha_2+\gamma_2)\eta}+\frac{\alpha_2-2\beta_2}{2}X_6 e^{(\bar{\alpha}_2+\gamma_2)\eta}$$

$$-\alpha_2 X_4\left(\frac{3}{2}-\frac{2Pi\omega}{Y_4}\right)e^{2\alpha_2\eta}-\alpha_2 X_7 e^{2\beta_2\eta}\Bigg]$$

$$+X_1\left(\frac{\alpha_2^2}{2\omega^2}-\frac{2a_1P}{Y_1}\left\{\frac{P}{2a_1+P}+\frac{i\alpha_2}{\omega}\right\}\right)e^{2a_1\eta}$$

$$+\Bigg\{\frac{P^2\alpha}{2i\omega}\left(1+\frac{X_1}{2a_1(2a_1+P)}\right)-\frac{P^2X_1\alpha}{2(2a_1+P)Y_1}$$

$$+\frac{P^2Ei\alpha_2X_2\alpha}{2\omega Y_2}-\frac{P^2Ei\alpha_2}{\omega}\left(-\frac{\alpha X_3}{2Y_3}+\frac{\alpha X_4}{2Y_4}-\frac{\alpha X_1}{2PEY_1}-\frac{X_2}{Y_2}\right)\Bigg\}$$

$$e^{-\alpha\eta/2}-\frac{P^3}{i\omega}\left(1+\frac{X_1}{2a_1(2a_1+P)}e^{-P\eta}+\frac{P^2Ei\alpha_2X_3}{\omega Y_3}\right)$$

$$\left(\bar{\alpha}_2+\beta_2\right)e^{(\bar{\alpha}_2+\beta_2)\eta}, \tag{4.32}$$

where $X_5 = \alpha_2\gamma_2 + R_c$

$X_6 = \bar{\alpha}_2\,\gamma_2 + R_c$

$X_7 = \beta_2{}^2 + R_c$

The solution of eqn. (4.32) is given by

$$\theta_{22}''(\eta) = C_1\, e^{\left(-P+\sqrt{P^2+8i\omega P}\right)\eta} + C_2\, e^{-\left(P+\sqrt{P^2+8i\omega P}\right)\eta}$$

$$-\frac{PE\alpha_2}{\omega^2}\Big[X_2\left\{\beta_2+2\alpha_2-\frac{Pi\omega(\alpha_2+\beta_2)}{Y_2}\right\}\frac{e^{(\alpha_2+\beta_2)\eta}}{Y_2-i\omega P}$$

$$+ \frac{(\alpha_2 - 2\beta_2) X_5}{2Y_5} e^{(\alpha_2+\gamma_2)\eta} + \frac{(\alpha_2 - 2\beta_2) X_6}{2Y_6} e^{(\bar{\alpha}_2+\gamma_2)\eta}$$

$$- \alpha_2 X_4 \left(\frac{3}{2} - \frac{2Pi\omega}{Y_4}\right) \frac{e^{2\alpha_2\eta}}{Y_4 i\omega P} - \frac{\alpha_2 X_7}{Y_7} e^{2\beta_2\eta}\Bigg]$$

$$+ X_1 \left(\frac{\alpha_2^2}{2\omega^2} - \frac{2a_1 P}{Y_1}\left\{\frac{P}{2a_1 + P} + \frac{i\alpha_2}{\omega}\right\}\right) \frac{e^{2a_1\eta}}{Y_1 - i\omega P}$$

$$+ \left\{\frac{P^2\alpha}{2i\omega}\left(1 + \frac{X_1}{2a_1(2a_1+P)}\right) - \frac{P^2 X_1 \alpha}{2(2a_1+P)Y_1}\right.$$

$$\left. + \frac{P^2 Ei\alpha_2 X_2 \alpha}{2\omega Y_2} + \frac{P^2 Ei\alpha_2}{\omega}\left(\frac{\alpha X_3}{2Y_3} - \frac{\alpha X_4}{2Y_4} - \frac{\alpha X_1}{2PEY_1}\right)\right\} \frac{e^{-\alpha\eta/2}}{Y_8}$$

$$- \frac{P^2}{2\omega^2}\left(1 + \frac{X_1}{2a_1(2a_1+P)} e^{-P\eta}\right) e^{-P\eta}$$

$$+ \frac{P^2 Ei\alpha_2 X_3 (\bar{\alpha}_2 + \beta_2)}{\omega Y_3 (Y_3 - i\omega P)} e^{(\bar{\alpha}_2+\beta_2)\eta},$$

Here $Y_5 = (\alpha_2 + \gamma_2)^2 + P(\alpha_2 + \gamma_2) - 2i\omega P.$

$Y_6 = (\bar{\alpha}_2 + \gamma_2)^2 + P(\bar{\alpha}_2 + \gamma_2) - 2i\omega P.$

$Y_7 = 4\beta_2^2 + 2P\beta_2 - 2i\omega P,$

and $Y_8 = \frac{\alpha_2}{4} - \frac{P\alpha}{2} - 2i\omega P.$

Applying the boundary conditions (4.21) we obtain

$$\theta_{22}(\eta) = \frac{PE\alpha_2}{\omega^2}\Bigg[X_2\left\{\beta_2 + 2\alpha_2 - \frac{Pi\omega(\alpha_2 + \beta_2)}{Y_2}\right\} \frac{e^{-\beta\eta} - e^{(\alpha_2+\beta_2)\eta}}{Y_2 - i\omega P}$$

$$+ \frac{(\alpha_2 - 2\beta_2) X_5}{2Y_5}\left(e^{-\beta\eta} - e^{(\alpha_2+\gamma_2)\eta}\right)$$

$$+ \frac{(\alpha_2 - 2\beta_2) X_6}{2Y_6}\left(e^{-\beta\eta} - e^{(\alpha_2+\gamma_2)\eta}\right) - \alpha_2 X_4\left(\frac{3}{2} - \frac{2Pi\omega}{Y_4}\right)$$

$$\frac{e^{-\beta\eta} - e^{2\alpha_2\eta}}{Y_4 - i\omega P} - \frac{\alpha_2 X_7}{Y_7}\left(e^{-\beta\eta} - e^{2\beta_2\eta}\right)\Bigg] - X_1$$

$$\left(\frac{\alpha_2^2}{2\omega^2}-\frac{2a_1P}{Y_1}\left\{\frac{P}{2a_1+P}+\frac{i\alpha_2}{\omega}\right\}\right)\frac{e^{-\beta\eta}-e^{2\alpha_1\eta}}{Y_1-i\omega P}$$

$$-\left\{\frac{P^2\alpha}{2i\omega}\left(1+\frac{X_1}{2a_1(2a_1+P)}\right)-\frac{P^2X_1\alpha}{2(2a_1+P)Y_1}\right.$$

$$\left.+\frac{P^2Ei\alpha_2}{\omega}\left(\frac{\alpha X_3}{2Y_3}+\frac{\alpha X_4}{2Y_3}-\frac{\alpha X_4}{2Y_4}-\frac{\alpha X_1}{2PEY_1}\right)\right\}$$

$$\frac{e^{-\beta\eta}-e^{-\alpha\eta/2}}{Y_8}+\frac{P^2}{2\omega^2}\left(1+\frac{X_1}{2a_1(2a_1+P)}\right)\left(e^{-P\eta}-e^{-P\eta}\right)$$

$$-\frac{iP^2E\alpha_2X_3(\bar{\alpha}_2+\beta_2)}{\omega Y_3(Y_3-i\omega P)}\left(e^{-\beta\eta}-e^{(\bar{\alpha}_2+\beta_2)\eta}\right), \qquad \ldots (4.33)$$

where $\beta = P+\sqrt{P^2+8i\omega P}.$

Combining equations (4.11), (4.22) – (4.27), (4.29), (4.31) and (4.33) we get

$$\theta_0(\eta)=e^{-P\eta}+\frac{X_1}{2a_1(2a_1+P)}\left[e^{-P\eta}-e^{2a_1\eta}\right]$$

$$+\varepsilon^2\left\{\left[-\frac{Z_1X_1}{\omega^2}+\frac{PEb_1(a_1^2+b_1^2)}{2\omega\left(a_1^2+b_1^2+a_1+\frac{1}{4}\right)}+\frac{X_1(a_1^2+b_1^2)}{\omega\left\{\frac{mM}{1+m^2}+2\Omega\right\}}\right.\right.$$

$$\left.-\frac{Z_1X_1}{\omega Z_3}-\frac{2X_1(a_1^2+b_1^2)}{\omega^2}-\frac{4a_1X_1}{Y_{1r}^2+Y_{1i}^2}\left\{Y_{1r}\left(\frac{P^2}{2a_1+P}-\frac{Pb_1}{\omega}\right)-Y_{1i}\frac{Pa_1}{\omega}\right\}\right]$$

$$\frac{e^{2a_1\eta}-e^{-P\eta}}{2a_1(2a_1+P)}+\frac{b_1\eta X_1(a_1^2+b_1^2+a_1)}{\omega\left(a_1^2+b_1^2+a_1+\frac{1}{4}\right)}\frac{e^{2a_1\eta}}{2a_1(2a_1+P)}$$

$$+2\,(T_5T_6+U_5U_6-T_6\,e^{-P\eta})+2\,(T_7T_{11}-U_7U_{11}-T_{11}e^{-P\eta})$$

$$+2\,(T_{13}T_{16}-U_{13}\,U_{16}-T_{16}e^{-P\eta})-\frac{PE(a_1^2+b_1^2)Z_2}{\omega^2}\frac{e^{2a_1\eta}-e^{-P\eta}}{a_2(2a_2+P)}$$

$$\left.+2\,(T_{19}T_{20}+U_{19}U_{20}-T_{20}e^{-P\eta})\right\}, \qquad \ldots (4.34)$$

$$\theta_1(\eta)=\varepsilon\left[\frac{P}{i\omega}\left(1+\frac{X_1}{2a_1(2a_1+P)}\right)\left(e^{-\alpha\eta/2}-e^{-P\eta}\right)+\frac{PX_1}{(2a_1+P)Y_1}\left(e^{2a_1\eta}-e^{-\alpha\eta/2}\right)\right.$$

$$+\frac{PEi\alpha_2}{\omega}\left\{\frac{X_2}{Y_2}\left(e^{-\alpha\eta/2}-e^{(\alpha_2+\beta_2)\eta}\right)\right.$$

$$+\frac{X_3}{Y_3}\left(e^{-\alpha\eta/2}-e^{(\bar{\alpha}_2+\beta_2)\eta}\right)\frac{X_4}{Y_4}\left(e^{\alpha_2\eta}-e^{-\alpha\eta/2}\right)$$

$$\left.\left.+\frac{X_1}{PEY_1}\left(e^{\alpha_1\eta}-e^{-\alpha\eta/2}\right)\right\}\right] \quad \ldots (4.35)$$

$$\theta_2(\eta)=\varepsilon^2\frac{PE\alpha_2}{\omega^2}\left[X_2\left\{\beta_2+2\alpha_2-\frac{Pi\omega(\alpha_2+\beta_2)}{Y_2}\frac{e^{-\beta\eta}-e^{(\alpha_2+\beta_2)\eta}}{Y_2-i\omega P}\right.\right.$$

$$+\frac{(\alpha_2-2\beta_2)}{2Y_5}\left\{\frac{X_5}{Y_5}\left(e^{-\beta\eta}-e^{(\alpha_2+\gamma_2)\eta}\right)+\frac{X_6}{Y_6}\left(e^{-\beta\eta}-e^{(\bar{\alpha}_2+\gamma_2)\eta}\right)\right\}$$

$$\left.\left.-\alpha_2X_4\left(\frac{3}{2}-\frac{2Pi\omega}{Y_4}\right)\frac{e^{-\beta\eta}-e^{2\alpha_2\eta}}{Y_4-i\omega P}-\frac{\alpha_2X_7}{Y_7}\left(e^{-\beta\eta}-e^{2\beta_2\eta}\right)\right)\right]$$

$$-X_1\left(\frac{\alpha_2^2}{2\omega^2}-\frac{2a_1P}{Y_1}\left\{\frac{P}{2a_1+P}+\frac{i\alpha_2}{\omega}\right\}\right)\frac{e^{-\beta\eta}-e^{2\alpha_1\eta}}{Y_1-i\omega P}$$

$$-\left\{\frac{P^2\alpha}{2i\omega}\left(1+\frac{X_1}{2a_1(2a_1+P)}\right)-\frac{P^2X_1\alpha}{2(2a_1+P)Y_1}\right.$$

$$\left.+\frac{P^2Ei\alpha_2}{\omega}\left(\frac{X_2}{Y_2}+\frac{X_3}{Y_3}-\frac{X_4}{Y_4}-\frac{X_1}{PEY_1}\right)\right\}\frac{e^{-\beta\eta}-e^{-\alpha\eta/2}}{Y_8}$$

$$+\frac{P^2}{2\omega^2}\left(1+\frac{X_1}{2a_1(2a_1+P)}\right)\left(e^{-P\eta}-e^{-P\eta}\right)$$

$$-\frac{iP^2E\alpha_2X_3(\bar{\alpha}_2+\beta_2)}{\omega Y_3(Y_3-i\omega P)}\left(e^{-\beta\eta}-e^{(\bar{\alpha}_2+\beta_2)\eta}\right), \quad \ldots (4.36)$$

where $Y_1 = Y_{1r} - iY_{1i}$,

$X_3 = X_{3r} + iX_{3i}$,

$Y_3 = Y_{3r} + iY_{3i}$

$T_1 = a_1 X_{3r} - b_1 X_{3i}$

$U_1 = a_1X_{3i} + b_1X_{3r}$

$$T_2 = \frac{2a_1 + a_2}{\omega} + \frac{P}{Y_{3r}^2 + Y_{3i}^2} \{Y_{3i}(a_1 + a_2) + Y_{3r}(b_1 - b_2)\},$$

Table 4. Values of τ_1 and τ_2 for different values of Ω, when R_c = 4.0

	τ_1			τ_2		
Ω_η/Ω	**5.0**	**10.0**	**20.0**	**5.0**	**10.0**	**20.0**
1.5	– 2.846	– 2.862	– 2.883	– 1.171	– 1.176	– 1.192
2.0	– 2.863	– 2.875	– 2.891	– 1.139	– 1.142	– 1.155
2.5	– 2.881	– 2.890	– 2.902	– 1.103	– 1.106	– 1.117
3.0	– 2.898	– 2.906	– 2.916	– 1.065	– 1.067	– 1.077
3.5	– 2.917	– 2.924	– 2.931	– 1.024	– 1.025	– 1.034
4.0	– 2.935	– 2.941	– 2.948	– 0.974	– 0.981	– 0.989
4.5	– 2.954	– 2.958	– 2.965	– 0.931	– 0.932	– 0.940
5.0	– 2.973	– 2.977	– 2.983	– 0.879	– 0.380	– 0.887
5.5	– 2.993	– 2.996	– 3.001	– 0.822	– 0.823	– 0.829
6.0	– 3.013	– 3.015	– 3.020	– 0.759	– 0.760	– 0.766
6.5	– 3.033	– 3.035	– 3.040	– 0.689	– 0.690	– 0.695
7.0	– 3.053	– 3.053	– 3.059	– 0.610	– 0.611	– 0.616
7.5	– 3.073	– 3.075	– 3.079	– 0.517	– 0.518	– 0.523
8.0	– 3.093	– 3.095	– 3.099	– 0.402	– 0.403	– 0.407
8.5	– 3.114	– 3.116	– 3.119	– 0.233	– 0.234	– 0.238
9.0	– 3.135	– 3.137	– 3.140	– 0.233	– 0.234	– 0.237
9.5	– 3.155	– 3.157	– 3.161	– 0.405	– 0.406	– 0.409
10.0	– 3.176	– 3.178	– 3.182	– 0.524	– 0.525	– 0.528
10.5	–3.197	–3.199	–3.203	–0.621	–0.622	–0.626
11.0	–3.218	–3.220	–3.223	–0.705	–0.707	–0.710
11.5	– 3.239	– 3.241	– 3.244	– 0.781	– 0.783	– 0.786

$$U_2 = \frac{2b_1 - b_2}{\omega} + \frac{P}{Y_{3r}^2 + Y_{3i}^2} \{Y_{3r}(a_1 + a_2) - Y_{3i}(b_1 - b_2)\},$$

$$T_3 = \frac{PE}{\omega} \{T_1 T_2 + U_1 U_2),$$

$$U_3 = \frac{PE}{\omega} \{T_1 U_2 - U_1 U_2),$$

$$T_4 = (a_1 + a_2)^2 - (b_1 - b_2)^2 + P(a_1 + a_2),$$

$$U_4 = 2(a_1 + a_2)(b_1 - b_2) + P(b_1 - b_2),$$

$$T_5 = e^{(a_1 + a_2)\eta} \sin(b_1 - b_2)\eta,$$

$$T_6 = \frac{T_3T_4 + U_3U_4}{T_4^2 + U_4^2}$$

$$U_6 = \frac{T_3U_4 - U_3T_4}{T_4^2 + U_4^2}$$

$$T_7 = e^{(a_1+a_2)\eta} \cos (b_1 + b_2)\, \eta,$$

$$U_7 = e^{(a_1+a_2)\eta} \sin (b_1 + b_2)\, \eta,$$

$$T_8 = (a_1 + a_2)^2 - (b_1 + b_2)^2 + P\,(a_1 + a_2),$$

$$U_8 = 2\,(a_1 + a_2)\,(b_1 + b_2) + P\,(b_1 + b_2),$$

$$X_2 = X_{2r} + iX_{2i},$$

$$T_9 = a_1X_{2r} - b_1X_{2i}$$

$$U_9 = a_1X_{2i} + b_1X_{2r},$$

$$Y_2 = Y_{2r} + iY_{2i},$$

$$T_{10} = \frac{a_2}{Z_3} + \frac{P}{Y_{2r}^2 + Y_{2i}^2}\,\{(a_1 + a_2)\,Y_{2i} - (b_1 + b_2)\,Y_{2r}\},$$

$$U_{10} = \frac{b_2}{Z_3} + \frac{P}{Y_{2r}^2 + Y_{2i}^2}\,\{(a_1 + a_2)\,Y_{2r} + (b_1 + b_2)\,Y_{2i}\},$$

$$T_{11} = \frac{PE}{\omega(T_8^2 + U_8^2)}\,\{T_8\,(T_9T_{10} - U_9U_{10}) + U_8\,(T_9U_{10} + U_9T_{10})\},$$

$$U_{11} = \frac{PE}{\omega(T_8^2 + U_8^2)}\,\{T_8\,(T_9T_{10} + U_9T_{10}) - U_8\,(T_9T_{10} - U_9U_{10})\},$$

$$X_4 = X_{4r} + iX_{4i},$$

$$X_4 = Y_{4r} + iY_{4i},$$

$$T_{12} = \frac{1}{2\,(R_c + 2\Omega)} + \frac{2PY_{4i}}{Y_{4r}^2 + Y_{4i}^2},$$

$$U_{12} = \frac{2PY_{4r}}{Y_{4r}^2 + Y_{4i}^2}$$

$$T_{13} = e^{2a_1\eta} \cos (2b_1\eta),$$

$$U_{13} = e^{2a_1\eta} \sin (2b_1\eta),$$

$$T_{14} = 4\,(a_1^{\,2} - b_1^{\,2}) + 2Pa_1$$

$$U_{14} = 8a_1b_1 + 2Pb_1,$$

$$T_{15} = -\frac{PE}{\omega}\,\{a_1^{\,2} - U_1^{\,2}) - 2a_1b_1X_{4i}\},$$

$$U_{15} = -\frac{PE}{\omega}\ \{a_1 b_1 X_{4r} + (2a_1{}^2 - b_1{}^2)\, X_{4i}\},$$

$$T_{16} = \frac{T_{14}(T_{15}T_{12} - U_{15}U_{12}) + U_{14}(T_{15}U_{12} + U_{15}T_{12})}{T_{14}^2 + U_{14}^2},$$

$$U_{16} = \frac{T_{14}(T_{15}U_{12} + U_{15}T_{12}) - U_{14}(T_{15}T_{12} - U_{15}T_{12})}{T_{14}^2 + U_{14}^2},$$

$$\alpha = \alpha_r + i\alpha_I,$$

$$T_{17} = \alpha_r^2 - \alpha_i^2 - 2P\alpha_r,$$

$$U_{17} = 2\alpha_r\alpha_i - 2P\alpha_i,$$

$$P_{16} = -\frac{P^2}{2\omega}\left(1 + \frac{X_1}{2a_1(2a_1 + P)}\right)\alpha_i,$$

$$Q_{16} = \frac{P^2}{2\omega}\left(1 + \frac{X_1}{2a_1(2a_1 + P)}\right)\alpha_i,$$

$$P_{17} = \frac{P^2 X_1}{2(2a_1 + P)}\left(\frac{\alpha_r Y_{1r} - \alpha_i Y_{1i}}{Y_{1r}^2 + Y_{1i}^2}\right),$$

$$Q_{17} = \frac{P^2 X_1(\alpha_r Y_{1r} + \alpha_i Y_{1r})}{2(2a_1 + P)(Y_{1r}^2 + Y_{1i}^2)},$$

$$P_{18} = \frac{X_{2r}Y_{2r} + X_{2i}Y_{2i}}{Y_{2r}^2 + Y_{2i}^2} + \frac{X_{3r}Y_{3r} + X_{3i}Y_{3i}}{Y_{3r}^2 + Y_{3i}^2} - \frac{X_{4r}Y_{4r} + X_{4i}Y_{4i}}{Y_{4r}^2 + Y_{4i}^2} - \frac{X_1}{PE}\frac{Y_{1r}}{Y_{1r}^2 + Y_{1i}^2},$$

$$Q_{18} = \frac{X_{2i}Y_{2r} - X_{2r}Y_{2i}}{Y_{2r}^2 + Y_{2i}^2} + \frac{X_{3i}Y_{3r} - X_{3r}Y_{3i}}{Y_{3r}^2 + Y_{3i}^2} - \frac{X_{4i}Y_{4r} - X_{4r}Y_{4i}}{Y_{4r}^2 + Y_{4i}^2} - \frac{X_1}{PE}\frac{Y_{1i}}{Y_{1r}^2 + Y_{1i}^2},$$

$$P_{19} = \frac{P^2 E}{2\omega}\ (b_1\alpha_r + a_1\alpha_i),$$

$$Q_{19} = \frac{P^2 E}{2\omega}\ (b_1\alpha_i - a_1\alpha_r),$$

$$T_{18} = P_{16} + P_{17} + P_{18}P_{19} - Q_{19}Q_{19},$$

$$U_{18} = Q_{16} + Q_{17} + P_{19}\, Q_{18} + Q_{19}P_{18},$$

$$T_{19} = e^{-\alpha_r \frac{n}{2}} \cos \frac{\alpha_i \eta}{2}$$

$$U_{19} = e^{-\alpha_r \frac{\eta}{2}} \sin \frac{\alpha_i \eta}{2},$$

$$T_{20} = -\frac{4}{T_{14}^2 + U_{17}^2} \ (T_{17}T_{18} + U_{17}U_{18})$$

$$U_{20} = -\frac{4}{T_{14}^2 + U_{17}^2} \ (T_{17}T_{18} - U_{17}U_{18}).$$

In equation (4.6), truncating the series for θ after $\eta = 2$, we obtain,

$$\theta\,(\eta, t) = \theta_0\,(\eta) + (\theta_1\,(\eta)\, e^{i\omega t} + \overline{\theta}_1\,(\eta)\, e^{-i\omega t}) + (\theta_2\,(\eta)\, e^{12\omega t} + \overline{\theta}_2\,(\eta)\, e^{-12\omega t})$$

$$= \theta_0\,(\eta) + 2(\theta_{1r}\,(\eta) \cos \omega t - \theta_{1i}\,(\eta) \sin \omega t)$$

$$+ 2\,(\theta_{2r}\,(\eta) \cos 2\omega t - \theta_{2i}\,(\eta) \sin 2\omega t) \qquad \ldots (4.37)$$

where $\theta_1\,(\eta) = \theta_{1r}\,(\eta) + i\omega_{1i}\,(\eta)$,

$\theta_2\,(\eta) = \theta_{2r}\,(\eta) + i\omega_{2i}\,(\eta)$,

Eqns. (4.34) – (4.37) yield the value of the temperature $\theta\,(\eta, t)$, which is clearly a real valued function.

SHEARING STRESS AT THE WALL

The sharing stress at the wall, along x-axis is given by

$$\tau_{xz}\Big]_{z=0} = \mu \left[\frac{\partial u}{\partial z} \right]_{z=0},$$

Which with the help of the non-dimensional variables given in eqn. (4.54) takes the form

$$\tau_1 = \left[\frac{\partial u}{\partial z} \right]_{\eta=0}, \qquad \ldots (5.1)$$

where

$$u\,(\eta, t) = u_0\,(\eta) + 2\,(u_1\,(\eta) \cos \omega t - v_1\,(\eta) \sin \omega t) + 2\,(u_2\,(\eta) \cos 2\omega t\ v_2\,(\eta) \sin 2\omega t)$$

is the non-dimensional form of $u\ (z)$ given in eqn. (3.52), truncated after $n = 2$. Equations (3.55), (3.57) – (3.60) and (5.1) yield

$$\tau_1 = a_1 + \frac{\varepsilon^2}{\omega} \left[\frac{a_1a_2 - b_1b_2}{\omega}(a_1 - a_2) - \frac{a_1b_2 + a_2b_1}{\omega}(b_1 - b_2) \right.$$

$$- \frac{2b_1(a_1^2 + b_1^2 + a_1)}{(2a_1 + 1)^2 + 4b_1} + \frac{(a_1a_2 - b_1b_2)(a_1 - a_2) + (a_1b_2 + a_2b_1)(b_1 + b_2)}{z_3}$$

$$- \frac{2a_1 b_1^2}{R_c + 2\Omega} + \frac{2\varepsilon}{\omega}\left[\{b_1 (a_1 - a_2) a_1 (b_1 - b_2)\}\cos \omega t\right.$$

$$- \{b_1 (b_1 - b_2) - a_1 (a_1 - a_2)\} \sin \omega t]$$

$$+ 2 \frac{\varepsilon^2}{\omega^2} [\{(a_1 a_2 - b_1 b_2)(a_2 - a_3) - (a_1 b_2 + a_2 b_1)(b_2 - b_3)$$

$$+ \frac{a_1^2 - b_1^2}{2}(a_3 - a_1) - a_1 b_1 (b_3 - b_1)\} \cos 2\omega t - \{(a_1 a_2 - b_1 b_2)(b_2 - b_3)$$

$$+ (a_1 b_2 + a_2 b_1) a_2 - a_3\} + \frac{a_1^2 - b_1^2}{2} (b_3 - b_1)$$

$$+ a_1 b_1 (a_3 - a_1)\} \sin 2\omega t]. \qquad \ldots (5.2)$$

Similarly the shearing stress at the wall along y-axis is given by

$$\tau_{yz}\Big]_{z=0} = \left[\frac{\partial y}{\partial z}\right]_{z=0},$$

Which takes the non-dimensional form

$$\tau_2 = \left[\frac{\partial v}{\partial \eta}\right]_{\eta=0}, \qquad \ldots (5.3)$$

where $v(\eta, t) = v_0(\eta)$

is the non-dimensional form of $v(z)$ given in eqn. (3.53). Eqns. (3.56) and (5.3) give

$$\tau_2 = b_1 + \frac{\varepsilon^2}{\omega}\left[\frac{a_1 a_2 - b_1 b_2}{\omega}(b_1 - b_2) - \frac{a_1 b_2 + a_2 b_1}{\omega}(a_1 - a_2)\right.$$

$$- \frac{(a_1^2 - b_1^2)(2a_1 + 1) + 4a_1 b_1^2}{(2a_1 + 1)^2 + 4b_1^2} + \frac{(a_1 a_2 - b_1 b_2)(b_1 + b_2) - (a_1 b_2 + a_2 b_1)(a_1 - a_2)}{z_3}$$

$$\left. - \frac{b_1 (a_1^2 - b_1^2)}{R_c + 2\Omega}\right]. \qquad \ldots (.5.4)$$

Conclusions

In this paper the flow of an incompressible, visco-elastic fluid past a porous plate with time dependent suction has been studied, when the plate and the fluid are in solid body rotation. The plate moves with a constant velocity, in a direction parallel to itself. The flow field is characterized by the parameters : R_c (Non-Newtonian parameter), Ω and ω (the frequency parameters) and P (Prandtl number). The effect of these parameters on the velocity, temperature and shearing stresses have been shown through several graphs and tables.

Fig. 1 shows $u(\eta, t)$, the velocity component along x-axis, for different values of R_c. It is observed that decreases near the plate as R_c increases, but an opposite effect is noticed away from the plate.

Fig. 2 presents u (η, t), for various values of Ω and ω it is seen that u decreases near the plate, when Ω increases but u increases with Ω, away from the plate. The effect of ω on u is opposite to that of Ω on u.

Fig. 3 illustrates the nature of v, the velocity component, along y-axis, for different values of R_c. It is observed that an increase in R_c retards v, near the plate and the effect is reversed away from the plate.

Fig. 4 presents v, for several values of Ω and ω. It is seen that v decreases near the plate as Ω increases and away from the plate v increases with Ω. The effect of ω on v is the same as that of Ω v.

Fig. 5 shows the temperature θ, for different values of R_c. It is observed that the temperature rises with an increase in R_c, near the plate, but it falls as R_c increases, away from the plate.

Fig. 6 presents θ, for various values of ω and Ω. It is seen that the temperature at any point of the fluid decreases as Ω increases. Moreover, the effect of Ω on θ is seen to be the same as that of M on it.

Fig. 7 illustrates θ, for several values of the Prandtl number P. It is observed that an increase in P reduces the temperature at any point of the fluid.

Table 1 shows θ, for different values of ωt. It is noticed that θ increases, as Ωt increases in $0 < \eta < 1$. In $1 \le \eta < z$, θ decreases as ωt increases from $\frac{\pi}{4}$ to $\frac{\pi}{2}$ and at $\eta = 2$, θ increases as Ωt increase from $\frac{\pi}{4}$ to $\frac{\pi}{2}$. Similarly in $1.2 \le \eta \le 2.0$, θ decreases as ωt increases from 0 to $\frac{\pi}{4}$.

Table 2 presents τ_1 and τ_2, the shearing stresses at the wall, for various values of M. It is seen that τ_1 decreases as R_c increases, whereas τ_2 decreases near the line ω. 1.5, as R_c increases and increases with R_c far away from this line.

Table 3 illustrates τ_1 and τ_2, for several values of R_c. It is observed that τ_1 increases as R_c increases.

Table 4 shows τ_1 and τ_2 for different values of Ω. It is seen that Ω has a reparatory effect on both τ and τ_2, that is any one of τ_1 and τ_2 decreases with an increases in Ω.

References

1. Miles, J.W., *J. Fluid Mech.,* **17**, 1, 75 (1963).
2. Greenspan, H.P. and Howard, L.N., *J. Fluid Mech.,* **17**, 385 (1963).
3. Batchelor, G.K., *An Introduction to Fluid Dynamics,* Cambridge University Press (1967).
4. Chawla, S.S., *J. Phys. Soc. Japan*, **23**, 663 (1967).
5. Singh, M.P. and Santhi, H.L., *J. Math. Mech.,* **18**, 193 (1968).
6. Thornley, C., *Qrly. J.M.A.M.,* **21**, 451 (1968).
7. Pop, I., *ZAMM*, **48**, 69 (1969).
8. Dordjevic, V.D., *ZAMM*, **52**, 353 (1972).
9. of. Cit
10. Debnath, L. and Mukherjee, S., Phys. *Fluids,* **16**, 1418 (1973).
11. Pop, I. and Soundalgekar , V.M., *J. Inst. Math. Appl.*, **15**, 343 (1975).
12. Soundalgekar, V.M., Martin, B.W., Gupta, S.K. and Pop, I., Publication *de L' Institute Mathematique Nourelle Science*, **20**, (34), 215 (1975).

13. Kelley, R.E., *Qrly. J. Mech. Appl. Math.,* **18,** 287 (1965).
14. Biswal, S., *Acta Ciencia Indica*, **35P**, 2, 211 (2009).
15. Biswal, S., Roy, G.S. and Mishra, A., *Int. J. Sci. Engy. Res.,* **2**, 9 (2011).
16. Dash, S.K. and Paikaray, P., *Ultra Scientist of Physical Sciences,* 25 (3) B, 369 (2013).

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