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SINGLE INJECTION SCL CURRENT FLOW IN INSULATOR CONTAINING DISTRIBUTED TRAPS OPERATING UNDER CDDM REGIME

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The steady state space-charge-limited (SCL) single injection current flow in insulator containing two sets of distributed traps operating under CDDM regime has been studied with the help of regional approximation method. The complete current-voltage characteristics are studied in the full variation of applied voltage across the insulator. It is observed that the complete current-voltage characteristic is started by pure Ohm's law which finally merges into cube power law for the dependence of current on applied voltage, after passing through the four transition current-voltage regimes.

KEYWORDS : Current Injection, Insulators, Trapping states, Nonconstant mobility and CDDM regime.

INTRODUCTION

The space-charge-limited single injection current flow in insulator has been studied in various trap configurations and carrier mobility regimes (Helfrich, 1967). In low mobility insulators (Wintle, 1972), it is observed that the carrier mobility is directly proportional to the carrier concentration. The electrical properties of insulator are usually influenced by the presence of trapping states (Sharma, 1974). The physical parameters characterizing the distributed trapping states in energy are estimated with the help of current injection technique. The important informations are obtained about the distribution of trapping states and their effects on electrical transport properties under CDDM regime. The regional approximation method (Lampert and Mark, 1972) is applied to solve the complicated problems. The non-constant mobility regime is considered for the investigation in low mobility insulators.

GENERAL EQUATIONS

Let us consider a low mobility insulator containing a significant density of distributed trapping states with the energy separation of two sets of the traps too small to give rise to a *J-V* characteristic with maximum structure under carrier density dependent mobility regime. The general equations characterizing the current flow and Poisson's law are given by (Sharma, Sharma and Raghav, 1983)

$$J = e \mu n(x) E(x) \qquad \ldots (1)$$

$$\frac{\varepsilon}{e} \cdot \frac{dE}{dx} = (n - n_0) + (n_t - n_{t,\, o}) \qquad \ldots (2)$$

where J is the current density, e is the magnitude of electronic charge, μ is the carrier mobility, $n\,(x)$ is the concentration of free current carriers at position x, $E\,(x)$ is the electric field strength at position x, ε is the permittivity of insulator, x is one dimensional planer co-ordinate for the current flow for a point from the cathode inside the insulator, n_0 is the concentration of thermal free carriers, $n_t\,(x)$ is the concentration of trapped current carriers at position x, $n_{t,\, o}$ is the thermal-equilibrium value of $n_t\,(x)$.

The general equations described above are usually subjected to the boundary condition for ohmic contact as

$$E\,(O) = 0 \qquad \ldots (3)$$

which is generally applied to the single injection current theories (Lampert and Mark, 1972). In low mobility insulator (Wintle, 1972), some trapping states are permanently occupied by the electrons so that there is no delay to the moving electrons. It gives rise the carrier density' dependent mobility regime in such materials. The carrier mobility relationship in such low mobility insulator for planer current flow is given by

$$\mu = h\, n\,(x) \qquad \ldots (4)$$

where h is the proportionality constant. The full applied voltage across the insulator is given by

$$V = \int_0^L E\,(x)\; dx \qquad \ldots (5)$$

where L is the device length.

A new trap distribution function is suggested by the workers to explain the situation which is quite amenable to usual simple analysis and broadly possess the features of the Gaussian's distribution of the trapping states. The proposed trap distribution function is given by (Kumar, Vashistha and Sharma, 2007)

$$H\,(E) = \frac{N_0}{kT_n} \cdot \frac{\exp(E - E_n)/kT_n]}{\{\exp[(E - E_n)/kT_n] + 1\}^2} \qquad \ldots (6)$$

where N_n is the total concentration of traps, k is the Boltzmann's constant, T is the characteristic temperature whose magnitude depends on the width of the trap distribution, E is the energy level around which the distribution of the trapping states occurs and E is the energy of the electron. The proposed trap distribution function is suitable for the application of regional approximation method in the current injection problems.

The problem is complicated due to the presence of distributed traps. The general equations characterizing the current flow and Poisson's law in the different regions of the insulator operating under carrier density dependent mobility regime are given as the sets of different equations with the help of regional approximation method as (Sharma, 1974).

Region I $(0 \le x \le x_t)$: Perfect Insulator Region

$$[E_c \ge F(x) \ge E_c - \frac{kT}{\ln(N_c / N_t)} \text{ and}$$

$$n_t\,(x) = n\,(x) - n_0 = n\,(x) >> n_t\,(x) >> n_0$$

$$J = ehn^2 E \qquad \ldots (7)$$

$$\frac{\varepsilon}{e}, \frac{dE}{dx} = n \qquad \ldots (8)$$

$$n\,(x_t) = N_1 + N_2 = N_t \qquad \ldots (9)$$

Region IIa $(x_t \le x \le x_{ab})$: Trapped Charge Region a

$$\left[E_c - kT \ln\left(\frac{N_c}{N_t}\right) \ge E_2 \right]$$

$$n_t\,(x) = N_1 + N_2 = N_t$$

$$J = ehn^2 E \qquad \ldots (10)$$

$$\frac{\varepsilon}{e}, \frac{dE}{dx} = N_t \qquad \ldots (11)$$

$$n\,(x_{ab}) = N_t \qquad \ldots (12)$$

Region II b $(x_{ab} \le x \le x_{be})$; Trapped Charge Region b

$$[E_2 \ge F(x) \ge E_t]$$

$$n_t\,(X) = N_1 + N_2 \left[\frac{n\,(x)}{N}\right]^{1/t} = N_2 \left[\frac{n\,(x)}{N}\right]^{1/t}$$

$$J = ehn^2 E \qquad \ldots (13)$$

$$\frac{\varepsilon}{e}, \frac{dE}{dx} = N_2 \left[\frac{n\,(x)}{N}\right]^{1/t} \qquad \ldots (14)$$

$$n\,(x_{ac}) = \left[\frac{N_1 (N)^{1/t}}{N_1\,(M)^{1/t}}\right]^{\frac{ml}{(m-l)}} \qquad \ldots (15)$$

Region II $(x_{bc} \le x \le x_{bc})$; Trapped Charge Region e

$$[E_1 \ge F\,(x) \ge F_0 + 0.7\,\mathrm{kT}]$$

$$n_1(x) = N_1 \left[\frac{n\,(x)}{M}\right]^{1/m}$$

$$J = ehn^2 E \qquad \ldots (16)$$

$$\frac{\varepsilon}{e}, \frac{dE}{dx} = N_t \left[\frac{n\,(x)}{M}\right]^{1/m} \qquad \ldots (17)$$

$$n\,(x_2) = n_0 \qquad \ldots (18)$$

Region III $(x_2 \le x \le L)$; Ohmic Region

$$[F_0 + 0.7\ kT \geq F(x) \geq F_0]$$

$$n(x) = n_0$$

$$J = ehn_0^2 E \quad \dots (19)$$

$$\left(\frac{\varepsilon}{e}\right)\frac{dE}{dx} = 0 \quad \dots (20)$$

where the imaginary transition planes x_1, x_{ab}, x_{bc} and x_2 are shifted towards the anode with the increase in injection level of currents.

Distribution of electric field strength

The continuity of the electric field strength inside the low mobility insulator is valid at the four imaginary transition planes. The electric field strength expression in the five regions is derived as follows:-

REGION I : The electric field strength at position x in the region I is derived from equations (7) and (8) as

$$E(x) = \left[\frac{9eJ}{4\varepsilon^2 h}\right]^{1/2} .x^{2/3} \quad \dots (21)$$

The transition plane x_i is evaluated from equations (9) and (21) as

$$x_1 = \frac{2\varepsilon J}{3e^2 h N_e^3} \quad \dots (22)$$

REGION IIa : The electric field strength in the region II a is derived from. Equations (9) -(11) as

$$E(x) = \frac{eN_t x}{\varepsilon} + \frac{J}{3ehN_t^2} \quad \dots (23)$$

The position of imaginary transition plane x_{ab} in the insulator is evaluated from the equations (12), (13) and (23) as

$$x_{ab} = \frac{\varepsilon JP}{e^2 h} \quad \dots (24)$$

where $$P = \frac{1}{N_t N^2}\left[1 - \frac{N^2}{3N_t^3}\right] \quad \dots (25)$$

REGION IIb : The expression for the electric field strength in the region II b is derived from the equations (12) - (14), (24) and (25) as

$$E(x) = [P_i x + Q_1]^{t_2\, e/(2e+1)} \quad \dots (26)$$

where $$P_1 = \left[\frac{2l+1}{2l}\right]\frac{eN_2}{\varepsilon}\left[\frac{J}{ehN^2}\right]^{1/2l} \quad \dots (27)$$

$$Q_1 = \left[\frac{J}{ehN^2}\right]^{\frac{(2l+1)}{2l}} \left[1 - \left(\frac{2l+1}{2l}\right)\right]$$

$$= -\frac{1}{2l}\left(\frac{J}{ehN^2}\right)^{(2l+1)/2l}$$

The last equality in the equation (28) is valid for the situation $N >> N_1$ so that

$$N_t = N_1 + N_2 = N_2 \text{ and } N << N_t$$

The expression for the position of the imaginary transition plane x_{ab} is derived from the equations [13], [15], [26]–[28] as

$$x_{bc} + \frac{1}{(2l+1)} \cdot \frac{\varepsilon J}{e^2 hN^2 N_2}\left[2l(\lambda^m, B)^{\frac{(2l+1)}{(m-t)}} + 1\right] \qquad \dots (29)$$

where $$A = \frac{N_2}{N_1} \text{ and } B = \frac{M}{N} \qquad \dots (30)$$

REGION IIc : The electric field strength in the region lIc is evaluated from the equations (13), (15), (17), (26-30) as

$$E(x) = [P_2 x + Q_2]^{2m/(2m+1)} \qquad \dots (31)$$

where $$P_2 = \left[\frac{2m+1}{2m}\right] \cdot \frac{eN_1}{\varepsilon}\left[\frac{J}{ehM^2}\right]^{1/2m} \qquad \dots (32)$$

$$Q_2 = \frac{1}{2n(2l+1)}\left(\frac{J}{ehM^2}\right)^{\frac{(2m+1)}{2m}} \cdot \frac{B^2}{A}\left[2(m-1)(A^{\frac{(2l+1)}{(m-1)}} - (2m+1)\right] \qquad \dots (33)$$

The imaginary transition plane x_2 is obtained from equations (16) - (20) as

$$X_2 \cong \frac{m}{(2m+1)} \cdot \frac{2\varepsilon J}{e^2 hN_1 n_0^2}\left[\frac{M}{n_0}\right]^{1/m} \qquad \dots (34)$$

REGION III : The electric field strength in the region III is obtained from the equation (I 9) as

$$E = \frac{J}{ehn_0^2} \qquad \dots (35)$$

which is a constant value throughout the region.

COMPLETE CURRENT-VOLTAGE CHARACTERISTIC

The complete current-voltage characteristic of the steady state space-charge-limited single injection current flow in low mobility insulator with very small energy separation for the two sets of distributed trapping states operating in CDDM regime is divided into six regimes in order as follows:

4.1. True Ohm's Regime ($J << J_{er}$) :

All the five regions are present in the insulator. But the injection level is initially very small and all the transition planes are very close to the cathode. Therefore, the contributions of the regions I and II are negligibly small and the region III is extended from cathode to anode. The current-voltage characteristic of true Ohm's regime is derived from equations (5) and (35) as

$$J = \frac{ehn_0^2}{L}.V \qquad \dots (36)$$

which shows a pure Ohm's law ($J \propto V$). The pure Ohm's regime is converted into the ohmic regime with the increase of injection level.

4.2 Ohmic Regime ($J < J_{cr,\,1}$) :

All the five regions are present in the insulator. The current flow is the mixture of ohmic, trapping and space charge contributions. The total voltage developed across the insulator is obtained from equations (5), (7)-(8) and (21) - (35) as

$$V = V_1 + V_{IIa} + V_{IIb} + V_{IIc} = a_1 J^2 + b_I J \qquad \dots (37)$$

where
$$a_1 = \frac{\varepsilon}{e^3 h^2 N_1 M^4}\left[\frac{1}{2(4l+1)}.\frac{B^4}{A} + \frac{2(l-m)}{(4l+1)(4m+1)}(A^l - B)^{\frac{(2m+l)}{(m-l)}} - \frac{4m^2}{(4m+1)(2m+1)}\left[\frac{M}{n_0}\right]^{\frac{(4m+1)}{m}}\right] \qquad \dots (38)$$

and
$$b_1 = \frac{L}{ehn_0^2} \qquad \dots (39)$$

The ohmic regime is terminated at the first critical current $J_{cr,\,1}$ and first critical voltage $V_{cr.I}$ derived from the equations (31) - (34) with $x_2 = L$ as

$$J_{cr,1} = \frac{(2m+1)}{2m}.\frac{e^2 L N_1 n_0^2}{\varepsilon}\left[\frac{n_0}{M}\right]^{1/m} \qquad \dots (40)$$

$$V_{cr,\,1} = \frac{(2m+1)^2}{2m(4m+1)}.\frac{eL^2 N_1}{\varepsilon}\left[\frac{n_0}{M}\right]^{1/m} \qquad \dots (41)$$

4.3. Trap-Filled-Limit Regime ($J_{crI} \leq J \leq J_{cr.\,ab}$) :

The injection level of current increases sufficiently high so that the imaginary transition plane x_2 leaves the insulator. All the trapping states are gradually filled with electrons to give a trap dominated current-voltage regime. The current-voltage characteristic of this regime is derived from equations (5) and (29) - (33) as

$$V = V_1 + V_{IIa} + V_{IIb} + V_{IIc} = a_1 J + b_2 j^{1/(2m+I)} \qquad \dots (42)$$

where $$a_2 = \frac{\varepsilon}{e^3h^2N_1M^4}\left[\frac{1}{2(4l+1)}.\frac{B^4}{A}+\frac{2(l-m)}{(4l+1)(4m+1)}(A^l-B)^{\frac{(4m+1)}{(m-l)}}\right] \quad \text{… (43)}$$

$$b_2 = \frac{2m}{(4m+1)}.\frac{\varepsilon}{e^3h^2N_1M^4}\left[\frac{(2m+1)}{2m}.\frac{e^2hN_1M^4}{E}\right]^{\frac{(4m+1)}{(2m+1)}} \quad \text{… (44)}$$

The first trap-filled-limit regime is terminated from the insulator at the critical current $J_{cr.ab}$ and critical voltage $V_{cr.ab}$ derived from equations (24) - (30) as

$$J_{cr,\,ab} = (2l+1)\frac{e^2hN^2N_2}{\varepsilon}\left[2l\,(A^m.B)^{\frac{(2l+1)}{(m-l)}}+1\right]^{-1} \quad \text{… (45)}$$

and $$V_{cr,\,ab} = \frac{(2l+1)^2}{(4l+1)}.\frac{eL^2N_2}{\varepsilon}\frac{\left[\frac{1}{2}+2l\,(A^m-B)^{\frac{(4l+1)}{(m-l)}}\right]}{\left[1+2l\,(A^m-B)^{\frac{(4l+1)}{(m-l)}}\right]^2} \quad \text{… (46)}$$

where the parameters A and B are given by the equation (30).

4.4. Second Trap-Filled-Limit Regime ($J_{cr,ab} \leq J \leq J_{cr,\,bc}$)

The injection level of current increases to such a level that the transition planes x_1 and x_{ab} are present in the insulator. The trapping effect increases and the total applied voltage across the insulator is derived from the equations (5) and (21)-(30) as

$$V - V_1 + V_{IIa} + V_{IIb} + V_{IIc} = a_3J^2 + b_3J^{1/(2l+1)} \quad \text{… (47)}$$

where $$a_3 = \frac{1}{2(2l+1)}.\frac{\varepsilon}{e^3h^2N_2N^4} \quad \text{… (48)}$$

$$b_3 = \frac{1}{(4l+1)}.\frac{\varepsilon}{e^3h^2N_2N^4}\left\{\frac{(2l+1)}{2l}.\frac{e^3hLN_2N^2}{\varepsilon}\right\}^{\frac{(4l+1)}{(2l+1)}} \quad \text{… (49)}$$

The equations (21) - (25) are used to evaluate the critical current $J_{cr,bc}$ and critical voltage $J_{cr,bc}$ as

$$J_{cc,\,bc} = \frac{e^2hL}{\varepsilon}.\frac{N_1N^2}{\left[1-{}^{N^2}3N_{l^2}\right]}-\frac{e^3hN_1N^2}{\varepsilon} \quad \text{… (50)}$$

$$V_{cc,bc} = \frac{eL^2N_1}{2\varepsilon} \quad \text{… (51)}$$

4.5. Third Trap-Filled-Limit Regime ($J_{cr,\,bc} < J < J_{cr,\,2}$) :

The transition plane x_1 is only present in the insulator and the total voltage is contributed by the regions I and IIa as

$$V = V_I + V_{IIa} = a_4 J^2 + b_4 J + c_4 \quad \ldots (52)$$

where
$$a_4 = \frac{2\varepsilon}{5c^3 h^2 N_1^5} \quad \ldots (53)$$

$$b_4 = \frac{2\varepsilon}{5chN_1^2} \quad \ldots (54)$$

$$c_4 = \frac{\varepsilon L^2 N_1}{2\varepsilon} \quad \ldots (55)$$

and the equations (22) - (25) are used. The critical values are obtained from the equations (22) and (23) as

$$J_{cr,2} = \frac{3e^2 hLN_0^3}{2\varepsilon} \quad \ldots (56)$$

$$V_{cr,\,2} = \frac{9eL^2 N\grave{}}{2\varepsilon} \quad \ldots (57)$$

4.6 Trap-Free Regime ($J_{cr,bc} \leq J \leq J_{cr,2}$)

There is no imaginary transition plane present in the insulator and all the trapping states are completely filled with electrons. The current-voltage characteristic of space-charge-limited trap-free regime is derived from equations (5) and (21) as

$$J = \frac{500}{243} \cdot \frac{\varepsilon^2 h}{e} \cdot \frac{V^3}{L^5} \quad \ldots (58)$$

which represents the cube power law dependence for the current on applied voltage.

Discussion and conclusions

The complete current-voltage characteristic of steady state space-charge-limited single injection current flow in low mobility insulator with the energy separation of two sets of traps too small to give rise to a current-voltage characteristic with maximum structure under CDDM regime is studied with the help of regional approximation method. The results may be verified for the samples of amorphous selinium and polyethylene with the help of experiemental techniques described elsewhere [1-4, 6]. The complete current-voltage characteristic is started from the pure Ohm's law which finally merges to cube power law after passing through four transition regimes *i.e.* Ohmic, first trap-filled-limit, second trap-filled-limit and third trap-filled-limit regimes. The complete current-voltage characteristic is thus complex contributions of the six current-voltage regimes. The lengths and locations of the transition current-voltage regimes depend mainly on the concentration of the distributed traps and the depth of the trapping states localized in the forbidden gap of insulator from the lowest energy level of the conduction band.

The effect of distributed trapping states is mainly present in the four transition regimes. It is negligibly small in true Ohm's and trap-free regimes. The important feature of the complete current-voltage characteristic is that it is always started from the pure Ohm's law and finally merges into cube power law in the CDDM regime irrespective of the type of trap configuration present in the solid.

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