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STUDY OF UNSTEADY INCOMPRESSIBLE FLUID THROUGH CYLINDRICAL DUCTS

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The general equations of motion of viscous flow through a rigid porous medium and modification of Lorentz force are used to solve the unsteady flow of an incompressible conducting fluid through cylindrical porous ducts with elliptic sections. The exact solution of the fluid equation for constant pressure distribution has been found. Some observations about the vorticity of the flow have been made.

INTRODUCTION

The subject of homogenous flow through porous media has many technical and engineering applications in fields such as Petroleum industry and surface water hydrology. Muskat [9], Dickey and Brydon [4] have discussed the flow through porous media in connection with filteration. Ahmadi and Manvi [1] have derived the general equation of motion for the flow of a viscous fluid through a porous medium on the principle suggested by Eringen. Gulab Ram and Mishra [5] studied MHD flow of conducting fluid through porous medium. Varshney [16] studied unsteady MHD flow through a porous medium in a circular pipe. Gupta [6] studied the unsteady flow through porous media in a channel of circular cross-section. Narshima Murthy [10] have dicussed the influence of magnetic field on the veliocity of a conducting fluid in a porous media. Kumar *et al*. [8] have given a theoretical analysis of an unsteady laminar flow of various incompressible and electricity conducting fluid through a porous medium in a channel in the presence of radial magnetic field and time-dependent pressure gradient.

In this paper, we have discussed the influence of an applied magnetic field on the unsteady flow of an incompressible conducting fluid through cylindrical porous ducts with elliptic section while constant pressure is applied. In this case we have derived closed from solution of the governing equation, and the effect of uniform applied magnetic field is indicated. Some observation have been made about the velocity and vorticity of the flow.

FORMULATION OF THE PROBLEM

Let us consider the motion of an incompressible, viscous, electrically conducting fluid, permeated by an applied magnetic filed in an isotropic porous media. The equations governing the motion are [following Ahmadi and Manvi [1]],

$$\frac{\partial p}{\partial t} + \nabla(\rho\bar{v}) = 0 \qquad \dots (1)$$

$$\rho\frac{\partial \bar{v}}{\partial t} = -\nabla p - \frac{\mu\bar{v}}{k} + \mu\nabla^2\bar{v} + \bar{j} \times \bar{B} \qquad \dots (2)$$

where $\rho, \bar{v}, p, \mu, k, \bar{j}$ and $\bar{B}$ are the density, velocity vector, pressure, fluid viscosity, permeability, current density vector and magnetic induction.

Let us consider the flow in a cylindrical porous tube with elliptic cross-section. The three impervious surface are given by

ξ = constant – elliptic cylinder

$$\frac{x^2}{\xi^2 - 1} + \frac{y^2}{\xi^2} = a^2$$

η = constant – hyperbolic cylinder

$$\frac{y^2}{\eta^2} - \frac{x^2}{1-\eta^2} = a^2 \qquad \dots (3)$$

Z = Constant – planes normal to the cylindrical axes.

Let ξ denotes the coordinate parallel to the flow direction, η the coordinate perpendicular to the flow and z the coordinate perpendicular to both ξ and η coordinates.

The applied magnetic field B_0 is uniform and is transverse to the flow (in the direction of η axis). Let us consider the uniform unsteady motion of an incompressible fluid through the cylindrical porous tube with elliptic cross-section. Let μ_0 represents the suction velocity at the axis of the tube, then from equation of continuity $\frac{\partial v}{\partial z} = 0.$ This with the condition that at $z = 0,\ v = u_0$ leads to the result that $v = u_0$ everywhere. From the symmetry of the problem all physical variables will be functions of z only. Also, let the pressure p be constant. For simplicity we assume that R_m, the magnetic Reynold's number is small, thereby rendering Maxwell's equations redundant.

Now, the equation of motion thus reduces to

$$\frac{\partial v}{\partial t} - \frac{u}{\rho}\frac{\partial^2 v}{\partial z^2} + \left(\frac{\mu}{k\rho} + \frac{\sigma B_0^2}{\rho}\right)v = 0 \qquad \dots (4)$$

Let $$\frac{\mu}{\rho} = A \text{ and } \left(\frac{\mu}{k\rho} + \frac{\sigma B_0^2}{\rho}\right) = B \qquad \dots (5)$$

then the equation (4) becomes

$$\frac{\partial v}{\partial t} - A\frac{\partial^2 v}{\partial z^2} + Bv = 0 \qquad \dots (6)$$

The boundary conditions of the problem are :

$$\left.\begin{aligned} v(z,0) &= v_0 \\ v_x(0,\, t) &= 0 \\ v(a,\, t) &= 0 \end{aligned}\right\} \qquad \ldots (7)$$

Applying Laplace transform on each term of equation (4), we get

$$p\bar{v} - v(z,0) - A\frac{d^2\bar{v}}{dz^2} + B\bar{v} = 0$$

by equation (7) this reduces to

$$A\frac{d^2\bar{v}}{dz} - (B+p)\bar{v} = -v_0 \qquad \ldots (8)$$

Also, on applying Laplace transform on conditions given by (7), we get

$$\bar{v}_x\ (0, p) = 0$$

$$\bar{v}\ (a, p) = 0 \qquad \ldots (9)$$

The solution of equation (8) will be

$$\bar{v} = c_1 \cosh\sqrt{\frac{B+p}{A}}z + C_2 \sinh\sqrt{\frac{B+p}{A}}z + \frac{v_0}{(B+p)} \qquad \ldots (10)$$

by applying conditions (9) we get

$$C_1 = -\frac{v_0 /(B+p)}{\cosh\sqrt{\frac{B+p}{A}}} \qquad \ldots (11)$$

and $$C_2 = 0$$

Hence solution (10) becomes $\bar{v} = \dfrac{v_0}{(B+p)} - \dfrac{v_0\, A\, \cosh\sqrt{\frac{B+p}{A}}z}{\sqrt{\frac{B+p}{A}}\cosh\sqrt{\frac{B+p}{A}}a}$

where $$\bar{v} = \int_0^\infty e^{-pT} v dT$$

Using Laplace inversion theorem, the velocity distribution v^* is given by

$$v^* = \exp\left\{-\frac{\mu}{\rho}\left(\frac{1}{k^*} + M^2\right)t\right\} - \frac{\mu}{\rho}\exp\left\{-\left(\frac{1}{k^*} + M^2\right)t\right\}.$$

$$\left[1 + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n-1)}\exp\left\{\frac{-(2n-1)^2\pi^2\mu t}{4a^2\rho}\right\},\ \cos\frac{(2n-1)\pi z}{2a}\right] \qquad \ldots (12)$$

where $$k^* = \frac{k}{a^2},\ M = aB_0, \sqrt{\frac{\sigma}{\rho v}} \qquad \ldots (13)$$

$$v^* = \frac{v}{v_0}$$

From equation (12), we can very easily deduce the vorticity of the flow as,

$$\zeta^* = \frac{2\mu}{\rho} \exp\left\{-\left(\frac{1}{k^*} + M^2\right)t\right\}. \qquad \dots (14)$$

where $$\zeta^* = \frac{a\zeta}{v_0} \qquad \dots (15)$$

numerical calculations have been carried out for liquid.

Table 1. $k^* = 0.1$, $t = 0.1$

	z	**0.0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
$M = 0$	v^*	– 0.1403	– 0.0652	0.0097	0.0848	0.1592	0.2348
$M = 0.5$	v^*	– 0.1383	– 0.0651	0.0081	0.0812	0.1544	0.2276
$M = 2$	v^*	– 0.1095	– 0.0593	-0.0089	0.0413	0.0916	0.1418
$M = 5$	v^*	– 0.0212	– 0.0151	-0.0089	– 0.0027	0.0034	0.0095

Table 2. $k^* = 0.2$, $t = 0.1$

	z	0.0	0.2	0.4	0.6	0.8	1.0
$M = 0$	v^*	– 0.1790	– 0.0553	0.0683	0.1920	0.3157	0.4394
$M = 0.5$	v^*	– 0.1773	– 0.0566	0.0639	0.1846	0.3053	0.4259
$M = 2$	v^*	– 0.1483	– 0.0654	0.0175	0.1004	0.1833	0.2662
$M = 5$	v^*	– 0.0324	– 0.0222	– 0.0121	0.0019	0.0082	0.0183

Table 3. $k^* = 0.1$, $t = 2$

	z	0.0	0.2	0.4	0.6	0.8	1.0
$M = 0$	v^*	-0.2435×10^{-8}	-0.2434×10^{-8}	-0.2432×10^{-8}	-0.2431×10^{-8}	-0.2430×10^{-8}	-0.2428×10^{-8}
$M = 0.5$	v^*	-0.1479×10^{-8}	-0.1478×10^{-8}	-0.1477×10^{-8}	-0.1479×10^{-8}	-0.1475×10^{-8}	-0.1474×10^{-8}
$M = 2$	v^*	-0.8272×10^{-12}	-0.8267×10^{-12}	-0.8262×10^{-12}	-0.8257×10^{-12}	-0.8252×10^{-12}	-0.8247×10^{-12}
$M = 5$	v^*	-0.4770×10^{-30}	-0.4767×10^{-30}	-0.4765×10^{-30}	-0.4762×10^{-30}	-0.4759×10^{-30}	-0.4756×10^{-30}

Table 4. $k^* = 0.2$, $t = 2$

	z	0.0	0.2	0.4	0.6	0.8	1.0
$M = 0$	v^*	-0.4833×10^{-4}	-0.4830×10^{-4}	-0.4827×10^{-4}	-0.4824×10^{-4}	-0.4820×10^{-4}	-0.4818×10^{-4}
$M = 0.5$	v^*	-0.2967×10^{-4}	-0.2965×10^{-4}	0.2963×10^{-4}	-0.2961×10^{-4}	-0.2959×10^{-4}	-0.2957×10^{-4}
$M = 2$	v^*	-0.1786×10^{-7}	-0.1785×10^{-7}	0.1783×10^{-7}	-0.1782×10^{-7}	-0.1781×10^{-7}	-0.1780×10^{-7}
$M = 5$	v^*	-0.1051×10^{-25}	-0.105×10^{-25}	-0.1049×10^{-25}	-0.1048×10^{-25}	-0.1048×10^{-25}	-0.1047×10^{-25}

Table – 5. $k^* = 0.1,\ t = 0.1$

	z	0.0	0.2	0.4	0.6	0.8	1.0
$M = 0$	ζ^*	0.0000	– 0.1707	– 0.3414	– 0.5121	– 0.6828	– 0.8535
$M = 0.5$	ζ^*	0.0000	– 0.1665	– 0.3330	– 0.4994	– 0.6659	– 0.8324
$M = 2$	ζ^*	0.0000	– 0.1144	– 0.2288	– 0.3433	– 0.4577	– 0.5721
$M = 5$	ζ^*	0.0000	– 0.0140	– 0.0280	– 0.0420	– 0.0560	– 0.0701

Table 6. $k^* = 0.2,\ t = 0.1$

	z	0.0	0.2	0.4	0.6	0.8	1.0
$M = 0$	ζ^*	0.0000	– 0.2814	– 0.5628	– 0.8443	– 1.1257	– 1.4071
$M = 0.5$	ζ^*	0.0000	– 0.2745	– 0.5489	– 0.8225	– 1.0979	– 1.3724
$M = 2$	ζ^*	0.0000	– 0.1886	– 0.3773	– 0.5659	– 0.7546	– 0.9432
$M = 5$	ζ^*	0.0000	– 0.0231	– 0.0462	– 0.0693	– 0.0924	– 0.1155

Table 7. $k^* = 0.1,\ t = 0.2$

	z	0.0	0.2	0.4	0.6	0.8	1.0
$M = 0$	ζ^*	0.0000	-0.3505×10^{-11}	-0.7010×10^{-11}	-1.0516×10^{-11}	-1.4021×10^{-11}	-1.7526×10^{-11}
$M = 0.5$	ζ^*	0.0000	-0.2126×10^{-11}	-0.4251×10^{-11}	-0.6378×10^{-11}	– 0.8504	-1.0603×10^{-11}
$M = 2$	ζ^*	0.0000	-0.1156×10^{-14}	-0.2352×10^{-14}	-0.3527×10^{-14}	-0.4703×10^{-14}	-0.5879×10^{-14}
$M = 5$	ζ^*	0.0000	-0.6761×10^{-33}	-1.3521×10^{-33}	-2.0282×10^{-33}	-2.7043×10^{-33}	-3.3804×10^{-33}

Table 8. $k^* = 0.2,\ t = 0.2$

	z	0.0	0.2	0.4	0.6	0.8	1.0
$M = 0$	ζ^*	0.0000	-0.7721×10^{-7}	-1.5442×10^{-7}	-2.3163×10^{-7}	-3.0884×10^{-7}	-3.8605×10^{-7}
$M = 0.5$	ζ^*	0.0000	-0.4683×10^{-7}	-0.9366×10^{-7}	-1.4049×10^{-7}	-1.8732×10^{-7}	-2.3415×10^{-7}
$M = 2$	ζ^*	0.0000	-0.2590×10^{-10}	-0.5180×10^{-10}	-0.7770×10^{-10}	-1.0360×10^{-10}	-1.295×10^{-10}
$M = 5$	ζ^*	0.0000	-0.1489×10^{-28}	-0.2978×10^{-28}	-0.4467×10^{-28}	-0.5957×10^{-28}	-0.7446×10^{-28}

Discussion

From table 1 to 4 we see that:

1. As we move away from the axis of the tube the velocity decreases. Somewhere near the axis the velocity of flow becomes zero and then it again increases continuously. The region of zero velocity exist slightly away from the axis. As the value of M increases the rate of increases of velocity is almost constant. It is noticeable that at the moment when velocity is zero, vorticity does not vanish.
2. As the value of magnetic parameter M increases the velocity of flow decreases throughout.

3. For small values of *t*, with increases in magnetic parameter *M*, the velocity decreases slowly.
4. The effect of the permeability parameter k^* is to increase the velocity of the fluid.
5. For large values oft, the velocity of fluid decrease sharply.

From table 5 to 8 we conclude that:

1. Vorticity is zero at the axis of the cylindrical tube (with elliptic section) *i.e*. the flow is irritation at the axis of the tube and as we move away from the axis of the tube vorticity comes into picture and increases with the increases in distance from the axis of the tube.
2. For fixed *t* and k^* with increase in magnetic parameter *M*, the vorticity decreases slowly.
3. For fixed *t* and *M*, the vorticity increases with increases in k^*, the permeability parameter.
4. For fixed k^*, with increase in time *t*, the vorticity decreases sharply.
5. As the value of *t* and k^* increase, value of vorticity although increased but its rate of increase decrease continuously. But for increased k^* and *t* the role of vorticity does not remain predominant and the flow remain almost irrotational.

REFERENCES

1. Ahmadi, G. and Manvi, R., *Indian J. Tech*., **9**, 441 (1971).
2. Sallabh, R., *Proc. Bemares Math. Soc.* (N.S.), IT 69 (1940).
3. Ballabh, R., Hydrodynamic Superposability, *U.P. Scientific Research Committee Monograph*, 27 (1968).
4. Brydon, C.L. and Dickey, G.D., *Theory and Practice of Filteration*, Reinhold, New York (1946).
5. Ram, Gulab and Mishra, *Indian Jour. of Pune and Appl. Maths*., Vol. **8**, No. **6**, 637 (1977).
6. Gupta, P.C., *Ph.D. Thesis*, Agra University, Agra, Ch-2, p-l00 (1982).
7. Iwamura, N. and Yamamoto, K., *J. Engng. Math*, **10**, 41 (1976).
8. Kumar, K. and Gupta, P.C., *Indian J. of Theo. Phys*., Vol. **33**, No. **2**, 101-105 (1982).
9. Muskat, M., *Physical Principles of Oil Production*, McGraw Hill Book Co., New York (1947).
10. Murthy, Narshima, *Proc. Mat. Acad. Sci. India*, **51(A)**, **III**, 369-375 (1981).
11. Ong, RS. and Nicholls, J.A., *Aero-Space Sci*., **26**, 313 (1959).
12. Rayleigh, Lord, *Phil Mag*., **21**, 697 (1911).
13. Singh, A.K., *Indian J. of Theoretical Physics*, Vol. **31**, No. **1** (1988).
14. Stokes, G.G., *Math Phy Papers*, **3**, **1** (1901).
15. Stuart, J.T., *Proc. Ray, Soc. A*., **313**, 16 (1965).
16. Varshney, *Acta Ciencia Indica*, Vol. **V** M, No. **4**, 190 (1979).

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