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HEAT AND MASS TRANSFER IN THE UNSTEADY COUETTE FLOW OF OLDROYD LIQUID BETWEEN TWO HORIZONTAL PARALLEL POROUS PLATES WITH HEAT SOURCES, DARCY'S DISSIPATION AND CHEMICAL REACTION

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This paper deals with heat and mass transfer in the unsteady Couette flow of Oldroyd liquid between two horizontal parallel porous plates with heat sources and Darcy's dissipation. Galerkin technique has been applied to solve the constitutive equations of momentum, heat and mass transfer. Nature of the non-Newtonian flow, heat and mass transfer has been studied through graphs and tables. It is observed that the porosity of the plate, source parameter and dissipation terms affect the flow and heat transfer patterns appreciably as like the varied species concentration.

KEYWORDS : Heat and mass transfer, Oldroyd liquid, heat sources, Darcy's dissipation and chemical reaction.

Introduction

The study of Couette flow problems has attracted the attention of many researchers in recent years.

The problem of Couette flow of an incompressible viscous liquid between two plates has already been studied by Pai [1]. The same flow through a porous channel has been investigated by Nanda [2], while Katagiri [3] and Muhuri [4] have analysed the same problem independently, taking into consideration the imposition of magnetic field on the field of flow. Rath *et al* [5] have discussed the heat transfer problem in case of unsteady Couette flow between two parallel walls maintained at different temperatures. Mishra [6], Dutta [7] and Kaloni [8] have analysed the plane Couette flow of an Oldroyd liquid with different physical conditions. Mishra [9] has also analysed the generalized plane Couette flow of an Oldroyd fluid with either suction or injection at the stationary wall. Further Bhatnagar [10] has discussed the plane Couette flow of Rivlin-Fricksen higher order fluid with uniform suction at the stationary plate. The plane Couette flow of Walters' B' liquid with equal rate of injection at one wall and suction at the other moving wall has been studied by Soundalgekar [11]. Moreover Mishra and Mohapatra [12] have investigated the problem of flow formation in Couette motion between two walls in case of Riener-Rivlin fluid imposing magnetic field. The commencement of unsteady Couette flow in case of second order liquid has been analysed by

Padhy [13]. Dash and Biswal [14] have investigated the problem of commencement of Couette flow in Oldroyd liquid through a porous channel in the presence of heat sources.

Biswal and Mahalik [15] have analysed the unsteady free convection flow and heat transfer of a viscoelastic fluid past on impulsively started porous flat plate with heat sources/sinks. Same researchers have investigated heat transfer in the free convection flow of a visco elastic fluid inside a porous vertical channel with constant suction and heat sources [16]. Biswal [17] alone has studied heat and mass transfer effects of oscillatory hydromagnetic free convective flow of a viscoelastic fluid past an infinite vertical porous flat plate in the presence of Hall current. Further, Biswal [18] has analysed the unsteady free convection flow and heat transfer of a viscoelstic fluid past an impulsively started porous wall. Muduli, Jena and Biswal [19] have analysed the effect of mass transfer on magnetohydrodynamic free convective flow of water at 4°C through a porous medium with Darcy's dissipation.

From technological view point, the study of both Newtonian and Non-Newtonian Couette flow problems in the presence of porous media is very important. Consequently, the literature is replete with copious instances of such investigations on Couette flows, through porous channel.

In the present problem, our aim is to study the commencement of Couette flow in Oldroyd liquid between two horizontal parallel porous plate, with heat sources, under the following physical situation *i.e.*

When the lower wall suddenly starts moving with time varying velocity A t'^n, where n is positive. The present investigation in the further generalization of previous cases of Padhy [13] and Biswal [14].

FORMULATION OF THE PROBLEM

Let X'-axis be chosen along the lower wall and y'-axis be normal to it. The upper plane be specified by the equation $y' = L$, where the symbol L will be defined later. It is also supposed that the walls extend to infinity in both sides of the X'-axis and the walls are porous. The suction and injection velocity V' at the walls is considered to be a constant. Now, the velocity components u' and v' at any point (X', Y') in the flow field compatible with the equation of continuity can be given by

$$U' = U'(y,t) \quad \dots (2.1)$$

Following the stress-strain rate relation, the stress components are given by

$$P^{x'x'} = 2K_0\left(\frac{\partial u'}{\partial y'}\right)^2 \quad \dots (2.2)$$

$$P^{x'y'} = \eta_0\left(\frac{\partial u'}{\partial y'}\right) - K_0\left(V\frac{\partial u'}{\partial y'\partial t'}\right) \quad \dots (2.3)$$

$$P^{y'y'} = 0 \quad \dots (2.4)$$

where $K_0 = \eta_0 (\lambda_1 - \lambda_2)$

Since the motion in both the cases is due to the shearing action of the fluid layers,

$$\frac{\partial P'}{\partial y'} = 0 \qquad \dots (2.5)$$

Thus the equations of motion and energy including viscous dissipation and heat sources are given below following the visco-elastic fluid model of Oldroyd's B' liquid [15].

Equation of motion

$$\rho\left(\frac{\partial u'}{\partial t'} + V\frac{\partial u'}{\partial y'}\right) = \eta_0 \frac{\partial^2 u'}{\partial y'^2} - K_0\left(\frac{\partial^3 u'}{\partial y'^2 \partial t'} + V\frac{\partial^3 u'}{\partial y'^3}\right) - \frac{\eta_0}{K'}u' \qquad \dots (2.6)$$

Equation of energy

$$\left(\frac{\partial \theta'}{\partial t'} + V\frac{\partial \theta'}{\partial y'}\right) = \frac{K}{\rho C_P}\frac{\partial^2 \theta'}{\partial y'^2} + \frac{\eta_0}{\rho C_P}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{\nu}{K' C_P}\left(u'\right)^2$$

$$-\frac{K_0}{\rho C_P}\left[\frac{\partial^2 u'}{\partial y' \partial t'}\frac{\partial u'}{\partial y'} + V\frac{\partial u'}{\partial y'}.\frac{\partial^2 u'}{\partial y'^2}\right] + S'(\theta' - \theta_L) \qquad \dots (2.7)$$

Equation of continuity

$$\frac{\partial C'}{\partial t'} + V\frac{\partial C'}{\partial y'} = D_1\frac{\partial^2 C'}{\partial y'^2} + \lambda', \qquad (2.7a)$$

where $\lambda' = -K''\left(C' - C_L'\right)^n$,

K'' is the reaction rate constant and n is the order of the reaction followed from Aris [20].

FORMATION OF THE EQUATIONS

The relevant boundary conditions to which equation (2.6) is subjected to are

$$\left.\begin{array}{l} t' = 0 : \ \theta' = 0,\ c' = 0 \ \text{ for all } \ y' \\ t' > 0 : \ u' = At'^{/n} \ \text{ for all } \ y' = 0 \\ \theta' = \theta_L, \ \text{ for } \ y' = L \end{array}\right\} \qquad \dots (3.1)$$

We introduce here the following non-dimensional parameters:

$$Y = \frac{y'}{\sqrt{\nu_1 . T'}},\ t = \frac{t'}{T'},\ u = \frac{u'}{AT''}$$

$R = \dfrac{V\sqrt{T}}{\sqrt{\nu_1}}$, the suction parameter,

$R_c = \dfrac{\lambda_1 - \lambda_2}{T}$, the elastic parameter,

$$P_r = \frac{v_1 \rho C_P}{\varepsilon K}$$, the Prandtl number

$$E = \frac{A^2 T^{2n}}{C_P \theta_L}$$, the Eckert number,

$$\theta = \frac{\theta' - \theta_L}{\theta_L}$$, θ_L being the temperature of the upper plate

$$S = \frac{4S' v_1}{V^2}$$, the source parameter

$$K^* = \frac{K' u^2}{v^2}$$, the non-dimensional permeability factor of the porous medium,

$$K_1 = \frac{\nu K''}{V^2}$$, non-dimensional chemical reaction parameter,

where T is some reference time. Here the maximum value of $\varepsilon = 0.8$ or $\varepsilon < 0.7$ when decreased rapidly

$$v_1 = \frac{\eta_0}{\rho}$$, the Kinematic viscosity

$$L = \sqrt{v_1 T}$$, the distance between the two walls of the channel

and $K_0 = \eta_0 (\lambda_1 - \lambda_2)$ is the volume co-efficient of elasticity of the fluid. With the help of the above non-dimensional parameters, the equations (2.6) and (2.7) are now reduced to their dimensionless forms as follows:

$$\frac{\partial u}{\partial t} + R\frac{\partial u}{\partial y} - \frac{\partial^2 u}{\partial y^2} + RR_c\frac{\partial^3 u}{\partial y^3} + R_c\frac{\partial^3 u}{\partial y^2 \partial t} + \frac{1}{K^*}u = 0 \quad \ldots (3.2)$$

nd

$$\frac{\partial \theta}{\partial t} + R\frac{\partial \theta}{\partial y} - \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + R_c E\left[\frac{\partial^3 u}{\partial y^2}\frac{\partial u}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y \partial t}\right] - \frac{\in E}{K^*}(u)^2 - E\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{4}R^2 S\theta = 0 \quad \ldots (3.3)$$

and

$$\frac{\partial C}{\partial t} + R\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - K_1 C \quad \ldots (3.3a)$$

The modified boundary conditions are

$$\left.\begin{array}{l} t = 0: \; u = 0 \text{ for all } y \\ t > 0: \; u = t^n \text{ for all } y = 0 \\ u = 0: \text{ for } y = 1 \end{array}\right\} \quad \ldots (3.4)$$

and

$$\left.\begin{array}{l} t = 0: \quad \theta = 0 \quad c = 0 \quad \text{for all } y \\ t > 0: \dfrac{\partial \theta}{\partial y} = 0 \quad \dfrac{\partial c}{\partial y} = 0, \text{ for all } y = 0 \\ \theta = 0 \quad c = 0 \quad \text{for all } y = 1 \end{array}\right\} \qquad \ldots (3.5)$$

Solution of the equations

Now equation (3.1) is a third order differential equation, which requires three boundary conditions for its solution. But the present problem provides only two boundary conditions. To overcome this difficulty, we follow small parameter perturbation technique given by Beard and Walters [21] to obtain the approximate solution of equation (3.2) and hence expand u in powers of R_c for $R_c << 1$. Thus we write

$$u = \sum_{i=0}^{\infty} R_c^i u_i \qquad \ldots (4.1)$$

where $i = 0, 1, 2, \ldots\ldots\ldots\ldots$ etc.

Substituting (4.1) in (3.2) and equating the co-efficient of R_c^0 and R_c^1, while neglecting those of R_c^2, R_c^3 ….. etc, we obtain, zeroth order equation :

$$\frac{\partial u_0}{\partial t} + R\frac{\partial u_0}{\partial y} - \frac{\partial^3 u_0}{\partial y^2} + \frac{1}{K^*}u_0 = 0 \qquad \ldots (4.2)$$

and first order equation

$$\frac{\partial u_1}{\partial t} + R\left(\frac{\partial u_1}{\partial y} - \frac{\partial^2 u_0}{\partial y^3}\right) - \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^3 u_0}{\partial y^2 \partial t} + \frac{1}{K^*}u_1 = 0 \qquad (4.3)$$

The boundary conditions (3.4) are further modified as

$$\left.\begin{array}{l} t = 0: \; u_0 = 0, \; u_1 = 0 \;\; \text{for all } y \\ t > 0: \; u_0 = t^n, \; u_1 = 0 \;\; \text{for all } y = 0 \\ u_0 = 0, \; u_1 = 0, \qquad \text{for all } y = 1 \end{array}\right\} \qquad \ldots (4.4)$$

In order to solve equations (4.2), (4.3) and (3.2) by Galerkin technique subjected to the boundary conditions (4.4) and (3.4), we choose the following approximate infinite expressions for u_0, u_1 and θ

$$u_0 \approx t^n (1 - y) + a_1 t y (1 - y) + a_2 t^2 y^2 (1 - y)^2 + a_3 t^3 y^3 (1 - y)^3 \qquad \ldots (4.5)$$

$$u_1 \approx b_1 t y (1 - y) + b_2 t^2 y^2 (1 - y)^2 + b_3 t^3 y^3 (1 - y)^3 \qquad (4.6)$$

$$\theta \approx c_1 t (1 - y^2) + c_2 t^2 y (1 - y^2)^2 + c_3 t^3 y^3 (1 - y^2)^3 \qquad \ldots (4.7)$$

where a_j, b_j and c_j ($i = 1, 2, 3\ldots$) are arbitrary constants to be determined later.

The equation of concentration is solved by small parameter regular perturbation technique taking

$$C = C_0 + \varepsilon_0 e^{iwt} C_1 \qquad \ldots(4.7a)$$

Solution of zeroth order equation

Substituting eqn. (4.5), in eqn. (4.2) the defect function Du_0 is obtained as

$$\begin{aligned} Du_0 &= -Rt^n + (1-y)\left(\frac{1}{K^*}t^n + nt^{n-1}\right) \\ &+ a_1\left[y(1-y) + Rt(1-2y) + 2t + \frac{1}{K^*}ty(1-y)\right] \\ &+ 2a_2\,[ty^2(1-y)^2 + Rt^2y(1-y)(1-2y) \\ &-t^2\left(6y^2-6y+1\right) + \frac{\frac{1}{K^*}}{2}t^2y^2(1-y)^2] \\ &+ 3a_3\,[t^2y^2(1-y)^3 + Rt^3y^2(1-y)^2(1-2y) \\ &-2t^3y(1-y)(5y^2-5y+1) + \frac{\frac{1}{K^*}}{2}t^3y^3(1-y)^3], \end{aligned} \qquad \ldots(4.8)$$

The defect function Du_0 is then minimized by Galerkin technique of orthogonalisation leading to the following three double integrals.

$$\int_0^1 \int_0^1 Du_0\, t^j y^j (1-y)^j\, dt\, dy = 0 \qquad \ldots(4.9)$$

where $j = 1, 2, 3$

It is note worthy here that $t \,\varepsilon\, [0, 1]$, since t is not large

After performing the integrations of equation (4.9) we arrive at the following three algebraic equations involving the parametric constants $a_{j\,=\,1,2,3}$ (a_1, a_2, a_3) *as*

$$a_1\left(\frac{23}{180} + \frac{\frac{1}{K^*}}{90}\right) + a_2\left(\frac{3}{140} + \frac{\frac{1}{K^*}}{560}\right) + a_3\left(\frac{17}{4200} + \frac{\frac{1}{K^*}}{3150}\right)$$

$$= \frac{R}{6(n+2)} - \frac{1}{12}\left(\frac{\frac{1}{K^*}}{n+2} + \frac{n}{n+1}\right) \ldots \qquad (4.10)$$

$$a_1\left(\frac{2}{105} + \frac{\frac{1}{K^*}}{560}\right) + a_2\left(\frac{29}{6300} + \frac{\frac{1}{K^*}}{3150}\right) + a_3\left(\frac{1}{990} + \frac{\frac{1}{K^*}}{16632}\right)$$

$$= \frac{R}{30(n+3)} - \frac{1}{60}\left(\frac{\frac{1}{K^*}}{n+3} + \frac{n}{n+2}\right) \qquad \ldots(4.11)$$

$$a_1\left(\frac{41}{1260} + \frac{\frac{1}{K^*}}{12600}\right) + a_2\left(\frac{13}{13860} + \frac{\frac{1}{K^*}}{16632}\right) + a_3\left(\frac{191}{840840} + \frac{\frac{1}{K^*}}{84084}\right)$$

$$= \frac{R}{140(n+4)} - \frac{1}{12}\left(\frac{\frac{1}{K^*}}{n+4} + \frac{n}{n+3}\right) \quad \ldots (4.12)$$

Eqns. (4.10), (4.11) and (4.12) can be written as

$$\left.\begin{aligned} A_1a_1 + A_2a_2 + A_3a_3 &= d_1 \\ B_1a_1 + B_2a_2 + B_3a_3 &= d_2 \\ C_1a_1 + C_2a_2 + C_3a_3 &= d_3 \end{aligned}\right\} \quad \ldots (4.13)$$

where $A_1 = \frac{23}{180} + \frac{\frac{1}{K^*}}{90}$, $A_2 = \frac{3}{140} + \frac{\frac{1}{K^*}}{560}$, $A_3 = \frac{17}{4200} + \frac{\frac{1}{K^*}}{3150}$

$B_1 = \frac{23}{105} + \frac{\frac{1}{K^*}}{560}$, $B_2 = \frac{29}{6300} + \frac{\frac{1}{K^*}}{3150}$, $B_3 = \frac{1}{990} + \frac{\frac{1}{K^*}}{16632}$

$C_1 = \frac{41}{1260} + \frac{\frac{1}{K^*}}{12600}$, $C_2 = \frac{13}{13860} + \frac{\frac{1}{K^*}}{16632}$, $C_3 = \frac{191}{840840} + \frac{\frac{1}{K^*}}{84084}$

$$d_1 = \frac{R}{6(n+2)} - \frac{1}{12}\left(\frac{\frac{1}{K^*}}{n+2} + \frac{n}{(n+1)}\right)$$

$$d_2 = \frac{R}{30(n+3)} - \frac{1}{60}\left(\frac{\frac{1}{K^*}}{n+3} + \frac{n}{(n+2)}\right)$$

$$d_3 = \frac{R}{140(n+4)} - \frac{1}{280}\left(\frac{\frac{1}{K^*}}{n+4} + \frac{n}{(n+3)}\right)$$

The three linear equations in (4.13) are solved by Cramer's rule to give a_1, a_2, a_3 as a_1 as Equns. (4.11), (4.12) and (4.13) are solved by Cramer's rule to determine a_1, a_2, and a_3 as

$$a_1 = \frac{\begin{bmatrix} d_1 & A_2 & A_3 \\ d_2 & B_2 & B_3 \\ d_3 & C_2 & C_3 \end{bmatrix}}{A} = \frac{d_1\left(B_2C_3 - B_3C_2\right) + A_2\left(B_3d_3 - C_3d_2\right) + A_3\left(C_2d_3 - B_2d_3\right)}{A} \quad \ldots (4.14)$$

$$a_2 = \frac{\begin{bmatrix} A_1 & d_1 & A_3 \\ B_1 & d_2 & B_3 \\ C_1 & d_3 & C_3 \end{bmatrix}}{A} = \frac{A_1\left(C_3d_2 - B_3d_3\right) + d_1\left(C_1B_3 - B_1C_3\right) + A_3\left(B_1d_3 - C_1d_2\right)}{A} \quad \ldots (4.15)$$

$$a_3 = \frac{\begin{bmatrix} A_1 & A_1 & d_1 \\ B_1 & B_2 & d_2 \\ C_1 & C_2 & d_3 \end{bmatrix}}{A} = \frac{A_1\left(B_2 d_3 - C_2 d_3\right) + A_2\left(C_1 d_2 - B_1 d_3\right) + d_1\left(B_1 C_2 - C_1 B_2\right)}{A} \quad \ldots(4.16)$$

where $\Delta A_1\,(B_2C_3 - B_3C_2) + A_2\,(B_3C_1 - B_1C_3) + A_3\,(B_1C_2 - C_1B_2)$

Now the values of a_1, a_2 and a_3 are put in equation (4.5) to get u_0

Solution of first order equation

The defect function Du_1 is obtained from equation (4.3) with the help of (4.6) and (4.5) as

$$\begin{aligned} Du_1 = & -2a_1 + 4a_2 t\,[(1-y)^2 - 4y\,(1-y) + 2y^2 - 3Rt\,(1-2y)] \\ & + 6a_3 t^2\,[(1-y)^3\,(3y + Rt) - 9y\,(1-y)^2\,(y + Rt) \\ & + 3y^2\,(1-y)\,(y + 3Rt) - Rty^3] \\ & + b_1\,[y\,(1-y) + Rt\,(1-2y) + \frac{1}{K^*}\,ty\,(1-y) + 2t] \\ & + 2b_2\,[ty^2\,(1-y)^2 + Rt^2 y\,(1-y)\,(1-2y) \\ & - t^2\,\{(1-y)^2 - 2y\,(1-y) + y^2 - 2y\} \\ & + \frac{1/K^*}{2}\,t^2 y^2\,(1-y)^2] \end{aligned} \quad \ldots(4.17)$$

which is minimized by Galerkin technique of orthogonalization resulting the following three double integrals started as

$$\int_0^1 \int_0^1 Du_1\, t^j\, y^j\, (1-y)^j\, dt\, dy = 0 \quad \ldots(4.18)$$

where $j = 1, 2, 3$.

Performing the above integrations, we obtain the following three algebraic equations involving the constants b_1, b_2 and b_3 as

$$b_1\left(\frac{23}{180} + \frac{\frac{1}{K^*}}{90}\right) + b_2\left(\frac{1}{14} + \frac{\frac{1}{K^*}}{560}\right) + b_3\left(\frac{113}{4200} + \frac{\frac{1}{K^*}}{3150}\right) = \frac{a_1}{6} - \frac{a_2}{45} + \frac{3a_3}{280} \quad \ldots(4.19)$$

$$b_1\left(\frac{2}{105} + \frac{\frac{1}{K^*}}{560}\right) + b_2\left[\frac{11}{900} + \frac{\frac{1}{K^*}}{3150}\right] + b_3\left[\frac{23}{4620} + \frac{\frac{1}{K^*}}{16632}\right] = \frac{a_1}{45} + (0 \times a_2) + \frac{a_3}{350} \quad \ldots(4.20)$$

$$b_1\left(\frac{41}{12600} + \frac{\frac{1}{K^*}}{3150}\right) + b_2\left(\frac{47}{20790} + \frac{\frac{1}{K^*}}{16632}\right) + b_3\left(\frac{163}{168168} + \frac{\frac{1}{K^*}}{84084}\right) \quad (4.21)$$

These equations (4.19 – 4.21) can be put in the following form

$$\left.\begin{aligned} A_1' b_1 + A_2' b_2 + A_3' b_3 = d_1' \\ B_1' b_1 + B_2' b_2 + B_3' b_3 = d_2' \\ C_1' b_1 + C_2' b_2 + C_3' b_3 = d_3' \end{aligned}\right\} \quad (4.19)$$

where $A_1' = \dfrac{23}{180} + \dfrac{\frac{1}{K^*}}{90}$, $A_2' = \dfrac{1}{14} + \dfrac{\frac{1}{K^*}}{560}$, $A_3' = \dfrac{113}{4200} + \dfrac{\frac{1}{K^*}}{3150}$

$$B_1' = \frac{2}{105} + \frac{\frac{1}{K^*}}{560},\ B_2' = \frac{11}{900} + \frac{\frac{1}{K^*}}{3150},\ B_3' = \frac{23}{4620} + \frac{\frac{1}{K^*}}{16632}$$

$$C_1' = \frac{41}{1260} + \frac{\frac{1}{K^*}}{3150},\ C_2' = \frac{47}{20,790} + \frac{\frac{1}{K^*}}{16632},\ C_3' = \frac{163}{168168} + \frac{\frac{1}{K^*}}{84084}$$

$$d_1' = \frac{a_1}{6} - \frac{a_2}{45} + \frac{3a_3}{280},\ d_2' = \frac{a_1}{45} + (0 \times a_2) + \frac{a_3}{350},\ d_3' = \frac{a_1}{280} + \frac{a_2}{3150} + \frac{3a_3}{1540},$$

The three equations in (4.19) – (4.21) are solved by Cramer's rule to give b_1, b_2, b_3

$$b_1 = \frac{\begin{bmatrix} d_1' & A_2' & A_3' \\ d_2' & B_2' & B_3' \\ d_3' & C_2' & C_3' \end{bmatrix}}{A} = \frac{d_1'\left(B_2'C_3' - B_3'C_2'\right) + A_2'\left(B_3'd_3' - C_3'd_2'\right) + A_3'\left(C_2'd_3' - B_2'd_3'\right)}{A'} \quad \ldots(4.23)$$

$$b_2 = \frac{\begin{bmatrix} A_1' & d_1' & A_3' \\ B_1' & d_2' & B_3' \\ C_1' & d_3' & C_3' \end{bmatrix}}{A'} = \frac{A_1'\left(C_3'd_2' - B_3'd_3'\right) + d_1'\left(C_1'B_3' - B_1'C_3'\right) + A_3'\left(B_1'd_3' - C_1'd_2'\right)}{A'} \quad \ldots(4.24)$$

$$b_3 = \frac{\begin{bmatrix} A_1' & A_1' & d_1' \\ B_1' & B_2' & d_2' \\ C_1' & C_2' & d_3' \end{bmatrix}}{A'} = \frac{A_1'\left(B_2'd_3' - C_2'd_3'\right) + A_2'\left(C_1'd_2' - B_1'd_3'\right) + d_1'\left(B_1'C_2' - C_1'B_2'\right)}{A'} \quad \ldots(4.25)$$

where $\Delta' = A_1'\left(B_2'C_3' - B_3'C_2'\right) + A_2'\left(B_3'C_1' - B_1'C_3'\right) + A_3'\left(B_1'C_2' - C_1'B_2'\right)$

Now the values of b_1, b_2 and b_3 are put in (4.6) to get u_1

Consequently, the expression for velocity ($u = u_0 + R_c u_1$) becomes

$$u = t^n (1 - y) + a_1 ty (1 - y) + a_2 t^2 y^2 (1 - y)^2 + a_3 t^3 y^3 (1 - y)^3$$
$$+ R_c b_1 ty (1 - y) + R_c b_2 t^2 y^2 (1 - y^2) + R_c b_3 t^3 y^3 (1 - y)^3 \quad \ldots (4.26)$$

Solution of equation of energy:

The defect function $D\theta$ is obtained from equation (3.3) with the help of equation (4.7) and (4.26) as

$$D\theta = e_1 \left[(1 - y^2) - 2Rty + \frac{2}{P^r}\, t - R^2 st (1 - y^2)\right]$$
$$+ c_2 \left[2ty (1 - y^2)^2 + Rt^2 (1 - 6y^2 + 5y^4) + \frac{4}{P^r}\, t^3 (3y - 5y^3)\right)$$

$$- \frac{1}{4} R^2 S t^2 y (1 - y^2)] + c_3 [3t^2 y^2 (1 - y^2)^3 + 2Rt^3 (y - 6y^2 + 9y^5 - 4y^7)$$

$$- \frac{2t^3}{P_r} (1 - 18y^2 + 45y^4 - 28y^6) - \frac{1}{4} R^2 S t^3 y^2 (1 - y^2)^3]$$

$$+ ER_c n t^{2n-1} - En^{2n} Et^n R_c nt (1 + n) (1 - 2y) + 2Er^{n+1}$$

$$[R_c R a_1 - a_2 R_c (2 + n) (y - 3y^2 + 2y^3) + (a_1 + R_c b_1) (1 - 2y)]$$

$$+ Et^{n+2} [4(a_2 + R_c b_2) (y - 3y^2 + 2y^3) - 3a_3 R_c (3 + n)$$

$$(y^2 - 4y^3 + 5y^4 - 2y^5) - 2RR_c a_2 (1 - 6y + 6y^2)]$$

$$Et^{n+3} [6(a_3 + R_c b_3) (y^2 - 4y^3 + 5y^4 - 2y^5) - 6RR_c a_3 (y - 6y^2 + 10y^3 - 5y^4)]$$

$$+ Eta_1^3 R_c (1 - 2y)^2 + Et^2 [6R_c a_1 a_2 (y - 5y^2 + 8y^3 - 4y^4)$$

$$- 2RR_c a_{1+3} (1 - 2y) - (a^3 + 2R_c a_1 b_1) (1 - 2y)^2]$$

$$- Et^3 [2RR_c a_1 a_2 (1 - 10y + 24y^2 - 16y^3) + 12R_c a_1 a_3 (y^2 - 6y^3 + 13y^4 - 12y^5 + 4y^6)$$

$$+ 8R_c a_2^3 (y^3 - 6y^3 + 13y^4 + 12y^5 + 4y^6) + (R_c a_1 b_2 + R_c a_2 b_1 + a_1 a_2)$$

$$(y - 5y^2 + 8y^3 - 4y^4)] + Et^4 [(6RR_c a_1 a_3 + 4RR_c a_2^2) (y - 9y^2 + 26y^3 - 30y^4 + 13y^2)$$

$$+30R_c a_2 a_3 (y^3 - 7y^4 + 19y^5 - 25y^6 + 16y^7 - 4y^8)$$

$$- 4a_2 (a_2 + 2R_c b_2) (Y - 3y^2 + 2y^3)^2 - 6 (R_c a_1 b_3 + R_c a_3 b_1 + a_1 a_3)$$

$$(y^3 + 6y^2 + 13y^4 + 12y^5 + 4y^6)]$$

$$+ Et^5 [6RR_c a_2 a_3 (3y^2 - 28y^3 + 95y^4 - 150y^5 + 112y^6 - 32y^7)$$

$$+27R_c a_3^2 (y^4 - 8y^5 + 26y^5 - 44y^7 + 41y^8 - 20y^9 + 4y^{10})$$

$$- 12 (R_c a_3 b_2 + R_c a_2 b_3 + a_2 a_3) y^3 - 7y^4 + 19y^5 - 25y^6 + 16y^7 - 4y^8)]$$

$$Et^6 [18RR_c a_3^2 (y^3 - 10y^4 + 37y^5 - 77y^6 + 82y^7 - 48y^8 + 10y^9)$$

$$- 9 (a_3^2 + 2R_c a_3 b_3 (y^4 - 8y^5 + 26y^6 - 44y^7 + 41y^8 - 20y^9 - 4y^{10})]$$

$$+ \frac{\in E}{K^*} [t^{2n} (1 - y)^2 + a_1^2 t^2 y^2 (1 - y)^2 + 2a_1 t^{n+1} y (1 - y)^2$$

$$+ a_1^2 t^4 y^4 (1 - y)^4 + a_3^2 t^6 y^6 (1 - y)^6 + 2a_2 a_3 t^5 y^5 (1 - y)^5$$

$$+ R_c^2 b_1^2 t^2 y^2 (1 - y)^2 + R_c^2 b_2^2 t^4 y^4 (1 - y)^4 + R_c^2 b_3^2 t^6 y^6 (1 - y)^6$$

$$+ 2R_c^2 b_1 b_2 t^3 y^3 (1 - y)^3 + 2R_c^2 b_2 b_3 t^5 y^5 (1 - y)^5$$

$$+ 2R_c^2 b_1 b_3 t^4 y^4 (1 - y)^4 + 2a_2 t^{n+2} y^2 (1 - y)^3$$

$$+ 2a_1 a_2 t^3 y^3 (- y)^3 + 2a_3 t^{n+3} y^3 (1 - y)^4$$

$$+ 2a_1 a_3 t^4 y^4 (1 - y)^4 + 2R_c a_2 b_1 t^3 y^3 (1 - y)^3$$

$$+ 2R_c a_3 b_1 t^4 y^4 (1 - y)^4 + 2R_c a_2 b_2 t^4 y^4 (1 - y)^4$$

$$+ 2R_c a_3 b_2 t^5 y^5 (1 - y)^5 + 2R_c a_2 b_3 t^5 y^5 (1 - y)^5$$

$$+ 2R_c a a_3 b_3 t^6 y^6 (1 - y)^6 + 2R_c b_1 t^{n+1} (1 - y)^2$$

$$+ 2R_c a_1 b_1 t^2 y^2 (1-y)^2 + 2R_c a_1 b_2 t^3 y^3 (1-y)^3$$
$$+ 2R_c a_1 b_3 t^4 y^4 (1-y)^4], \qquad (4.27)$$

which is minimized by Galerkin technique of orthogonalization, resulting the following three double integrals as

$$\int_0^1 \int_0^1 D\,\theta\, t^1 y^{j-1} (1-y^2)^1 \, dt\, dy = 0 \qquad \ldots (4.28)$$

where $j = 1, 2, 3$.

The above integrations are then carried out and solved by Cramer's rule to give the constants c_1, c_2 and c_3 as

$$c_1 = \frac{\begin{bmatrix} d_1'' & A_2'' & A_3'' \\ d_2'' & B_2'' & B_3'' \\ d_3'' & C_2'' & C_3'' \end{bmatrix}}{\Delta''} = \frac{d_1''\left(B_2''C_3'' - B_3''C_2''\right) + A_2''\left(B_3''d_3'' - C_3''d_2''\right) + A_3''\left(C_2''d_3'' - B_2''d_3''\right)}{\Delta''} \qquad \ldots(4.29)$$

$$c_2 = \frac{\begin{bmatrix} A_1'' & d_1'' & A_3'' \\ B_1'' & d_2'' & B_3'' \\ C_1'' & d_3'' & C_3'' \end{bmatrix}}{\Delta''} = \frac{A_1''\left(C_3''d_2'' - B_3''d_3''\right) + d_1''\left(C_1''B_3'' - B_1''C_3''\right) + A_3''\left(B_1''d_3'' - C_1''d_2''\right)}{\Delta''} \qquad \ldots(4.30)$$

$$c_3 = \frac{\begin{bmatrix} A_1'' & A_1'' & d_1'' \\ B_1'' & B_2'' & d_2'' \\ C_1'' & C_2'' & d_3'' \end{bmatrix}}{\Delta''} = \frac{A_1''\left(B_2''d_3'' - C_2''d_3''\right) + A_2''\left(C_1''d_2'' - B_1''d_3''\right) + d_1''\left(B_1''C_2'' - C_1''B_2''\right)}{\Delta''} \qquad \ldots(4.31)$$

where $\Delta'' = A_1''\left(B_2''C_3'' - B_3''C_2''\right) + A_2''\left(B_3''C_1'' - B_1''C_3''\right) + A_3''\left(B_1''C_2'' - C_2'' - C_1''B_2''\right)$

$$A_1'' = \frac{4}{15} + \frac{R}{6} + \frac{4}{9P_r} - \frac{2R^2S}{45}$$

$$A_2'' = \frac{4}{12} + \frac{4R}{105} + \frac{1}{9P_r} - \frac{R^2S}{35}$$

$$A_3'' = \frac{32}{1155} - \frac{11R}{100} + \frac{32}{1575P_r} - \frac{32R^2S}{17325}$$

$$B_1'' = \frac{1}{24} - \frac{4R}{105} + \frac{1}{12P_r} - \frac{R^2S}{128}$$

$$B_2'' = \frac{64}{3465} + \frac{128}{1575P_r} - \frac{32R^2S}{17325}$$

$$B_3'' = \frac{1}{140} - \frac{17077R}{540540} + \frac{1}{45P_r} - \frac{R^2S}{2016}$$

$$C_1'' = \frac{32}{3465} - \frac{R}{100} + \frac{32}{1575P_r} - \frac{32R^2S}{17325}$$

$$C_2'' = \frac{1}{210} - \frac{5R}{16632} + \frac{1}{45P_r} - \frac{R^2S}{2016}$$

$$C_3'' = \frac{512}{255255} - \frac{82R}{2156} + \frac{512}{63063P_r} - \frac{256R^2S}{1786785}$$

$$d_1'' = \frac{2ER_cn}{3(2n+1)} - \frac{2E}{3(2n+2)} - \frac{ER_cc_1(1+n)}{6(n+2)}$$

$$+\frac{2E}{n+3}\left[\frac{a_2R_c(2+n)}{60} - \frac{2RR_ca_1}{3} - \frac{(a_1+R_cb_1)}{6}\right]$$

$$+\frac{E}{n+4}\left[\frac{a_3R_c(3+n)}{140} - \frac{(a_2+R_cb_2)}{15} - \frac{RR_ca_2}{15}\right]$$

$$+\frac{E}{70(n+5)}\left[-(a_3+R_cb_3) - RR_ca_3\right] - \frac{Ea_1^2R_c}{15}$$

$$+\frac{E}{4}\left[\frac{RR_ca_1^2}{3} + \frac{(a_1^2+2R_ca_1b_1)}{5} - \frac{9R_ca_1a_2}{70}\right]$$

$$+\frac{E}{5}\left[\frac{3(R_ca_1b_2+R_ca_2b_1+a_1b_2)}{3} - + \frac{RR_ca_1a_2}{15} - \frac{(12R_ca_1a_3+8R_ca_2^2)}{315}\right]$$

$$+\frac{E}{6}\left[\frac{6(R_ca_1b_3+R_ca_3b_1+a_1a_2)}{315} - \frac{(6RR_ca_1a_3+4RR_ca_2^2)}{420} - \frac{30R_ca_2a_3}{1848}\right]$$

$$+\frac{E}{7}\left[\frac{(R_ca_3b_2+R_ca_2b_1+a_2a_3)}{154} - \frac{RR_ca_2a_3}{210} - \frac{27R_ca_3^2}{10010}\right]$$

$$+\frac{E}{8}\left[\frac{1679}{1540}RR_ca_3^2 + \frac{9}{10010}\left(a_3^2+2R_ca_3b_3\right)\right]$$

$$d_2'' = \frac{ER_cn}{12(1+n)} - \frac{E}{6(2n+3)} - \frac{ER_ca_1(1+n)}{70(n+3)}$$

$$+\frac{2E}{n+4}\left[\frac{a_2R_c(2+n)}{504} - \frac{RR_ca_1}{6} - \frac{(a_1+R_cb_1)}{70}\right]$$

$$+\frac{E}{n+5}\left[\frac{3a_3R_c(3+n)}{3080} - \frac{2(a_2+R_cb_2)}{35} - \frac{17RR_ca_2}{210}\right]$$

$$+\frac{E}{n+6}\left[-\frac{6RR_ca_3}{315}-\frac{6(a_2+R_cb_2)}{3080}-\frac{Ea_1^2R_c}{140}\right]$$

$$+\frac{E}{5}\left[\frac{RR_ca_1^2}{35}+\frac{(a_1^2+2R_ca_1b_2)}{35}-\frac{11}{420}R_ca_1a_2\right]$$

$$+\frac{E}{6}\left[\frac{RR_ca_1a_2}{315}+\frac{11}{630}(R_ca_1b_2+R_ca_2b_1+a_1a_2)-\frac{(12R_ca_1a_3+8R_ca_2^2)}{1320}\right]$$

$$+\frac{E}{7}\left[\frac{(R_ca_1a_3+R_ca_3b_1+a_1a_3)}{220}-\frac{(6RR_ca_1a_3+4RR_ca_2^2)}{27720}-\frac{17}{4004}R_ca_2a_3\right]$$

$$+\frac{E}{8}\left[\frac{RR_ca_2a_3}{4004}+\frac{3R_ca_3^2}{4004}+\frac{17(R_ca_3b_2+R_ca_2b_3+a_2a_3)}{10010}\right]$$

$$+\frac{E}{9}\left[\frac{5147RR_ca_3^2}{20020}+\frac{(a_3^2+2R_ca_3b_3)}{4004}\right]$$

and

$$d_3''=\frac{16ER_cn}{315(2n+3)}-\frac{8E}{315(n+2)}-\frac{ER_ca_1(1+n)}{1260(n+4)}$$

$$+\frac{2E}{n+5}\left[\frac{a_2R_c(2+n)}{9240}-\frac{16RR_ca_1}{315}-\frac{(a_1+R_cb_1)}{1260}\right]$$

$$+\frac{E}{n+6}\left[\frac{a_3R_c(3+n)}{20020}-\frac{(a_2+R_cb_2)}{2310}-\frac{223RR_ca_2}{6930}\right]$$

$$+\frac{E}{n+7}\left[\frac{-(a_3+R_cb_3)}{10010}-\frac{513RR_ca_3}{60060}\right]-\frac{43}{34650}Ea_1^2R_c$$

$$+\frac{E}{6}\left[\frac{RR_ca_1^2}{630}+\frac{43}{6930}(a_1^2+2R_ca_1b_2)-\frac{131}{20020}R_ca_1a_2\right]$$

$$+\frac{E}{7}\left[\frac{RR_ca_1a_2}{6930}+\frac{131}{30030}(R_ca_1b_2+R_ca_2b_1+a_1a_2)-\frac{37}{180180}(12R_ca_1a_3+8R_ca_2^2)\right]$$

$$+\frac{E}{8}\left[\frac{37(R_ca_1a_3+R_ca_3b_1+a_1a_3)}{30030}-\frac{(6RR_ca_1a_3+4RR_ca_2^2)}{120120}-\frac{930R_ca_2a_3}{765765}\right]$$

$$+\frac{E}{9}\left[\frac{124(R_ca_3b_2+R_ca_2b_3+a_2a_3)}{255255}-\frac{944R_ca_3^2}{3233230}-\frac{RR_ca_2a_3}{30030}\right]$$

$$+\frac{E}{10}\left[\frac{2419RR_ca_3^2}{34034}+\frac{3147(a_3^2+2R_ca_3b_3)}{3233230}\right]$$

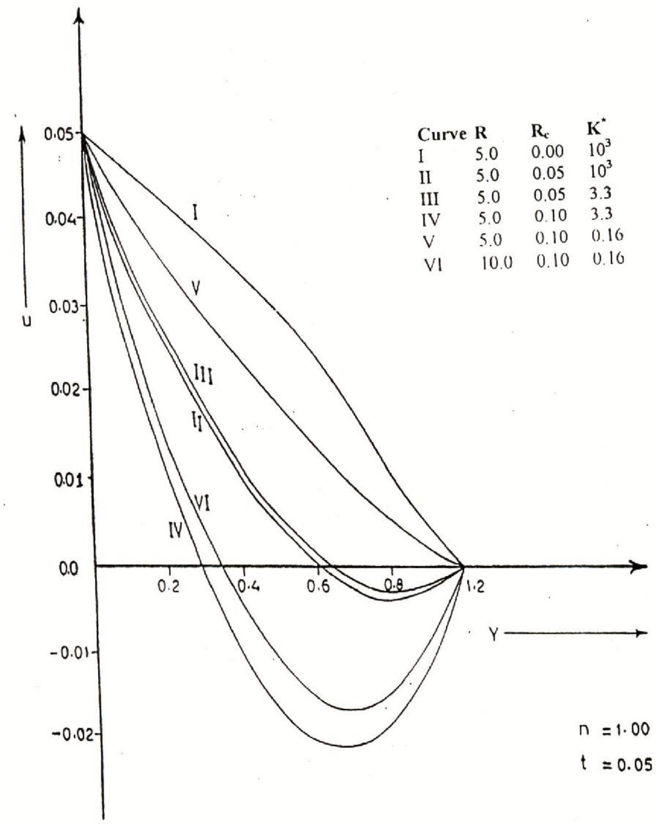

Fig. 1 : Effect of R, R_c and K^* on velocity field.

Thus the values of c_1, c_2 and c_3, the expression for temperature θ becomes

$$\theta = c_1 t\,(1 - y^2) + c_2 t^2 y\,(1 - y^2)^2 + c_3 t^3 y^2\,(1 - y^2)^3 \qquad \ldots (4.32)$$

The non-dimensional skin friction is given by

$$\tau_{xy} = \frac{\partial u}{\partial y} - R_c \left\{ R\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial y dt} \right\} \qquad \ldots (4.33)$$

The skin friction at the lower and upper plate are calculated from equation (4.33) taking $y = 0$ and $y = 1$ respectively

$$\tau_0 = \tau_{xy}\,|_{y=0}$$

$$= -t^n + t\,(a_1 + R_c b_1) - R_c\,\{-2Rta_1 + 2Ra_2t^2 - nt^{n-1} + a_1\} \qquad \ldots (4.34)$$

and

$$\tau_1 = \tau_{xy}\,|_{y=0} \qquad \ldots (4.35)$$

$$= -t^n - (a_1 + R_c b_1) - R_c\,\{-2Ra_2t^2 - nt^{n-1} - a_1\}$$

Further, the rate of heat transfer at the lower plate is given by

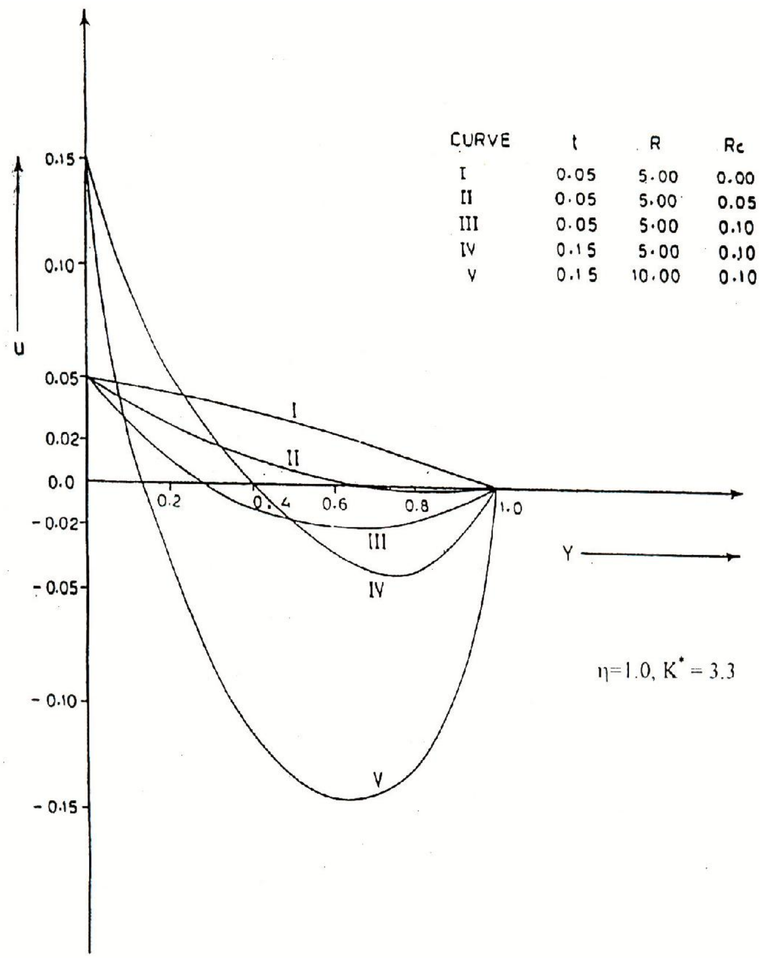

Fig. 2 : Effect of t, R and R_c on velocity field.

$$NU_0 = -\left.\frac{\partial\theta}{\partial y}\right|_{y=0} = -c_2 t^2 \qquad \ldots (4.36)$$

and that at the upper plate is

$$NU_1 = -\left.\frac{\partial\theta}{\partial y}\right|_{y=1} = 2c_1 t \qquad \ldots (4.37)$$

Solution of concentration equation (3.3a)

Solving equn. (3.3a), we get

$$C_0 = e^{-RScy,} \qquad \ldots (4.38)$$

and

$$C_1 = e^{\frac{1}{2}\left[S_c R + \sqrt{R^2 S_c^2 + 4iw\left(S_c + K_1\right)}\right]} = e^{-a_2 y} \qquad \ldots (4.39)$$

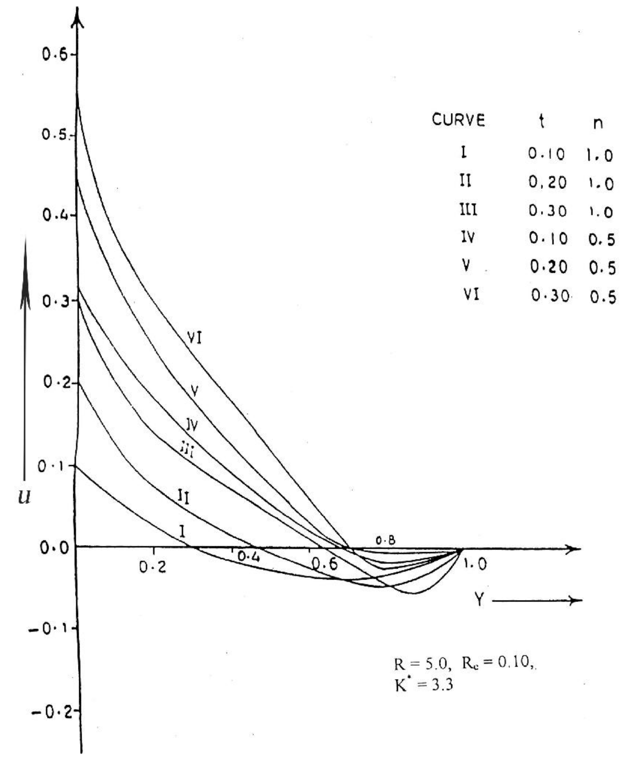

Fig. 3 : Effect of t and n on velocity field.

where $$a_2 = \frac{1}{2}\left[S_c R + \sqrt{R^2 S_c^2 + 4iw\left(S_c + K_1\right)}\right] \qquad \ldots (4.40)$$

It is observed that the final equation for concentration contains imaginary terms alongwith real parts. In order to separate the real and imaginary parts, we can express C as

$$C = C_0 + \varepsilon_0 \left[(C_r \text{ Cos } wt - C_i \text{ Sin } wt) + I\,(C_r \text{ Sin } wt + C_i \text{ Cos } wt)\right] \qquad \ldots (4.41)$$

Taking only the real part, we have

$$C = C_0 + \varepsilon_0 \,(C_r \text{ Cos } wt - C_i \text{ Sin } wt) \qquad \ldots (4.42)$$

When $wt = \pi/2$, we obtain $C = C_0 - \varepsilon_0 C_i$, … (4.43)

RESULTS AND DISCUSSIONS OF CASE (I)

In this case, the effects of various parameters on the flow behaviour of viscoelastic fluid flowing through a medium have been studied with the help of graphs and tables. The study is carried out for two positive values of n, *i.e.*,

(i) $n = 1$, constant acceleration

(ii) $n = \frac{1}{2}$, variable acceleration

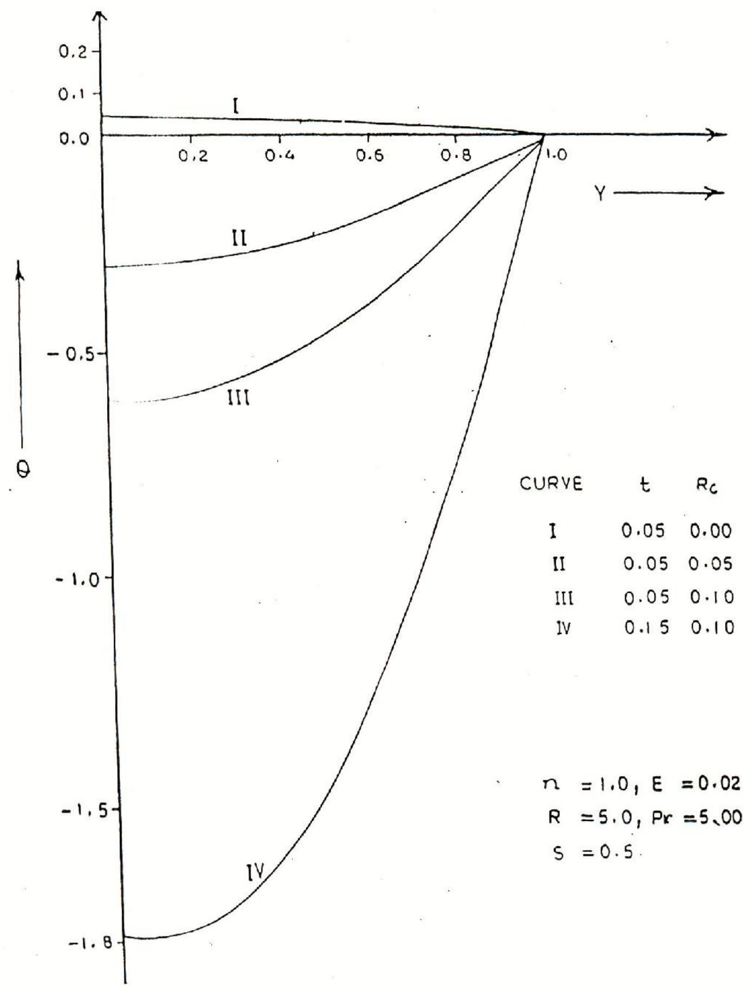

Fig. 4 : Effect of t and R_c on temperature field.

The effects of R, R_c and K^* on velocity field are exhibited by the curves of Fig. 1. In the absence of magnetic field, it is observed that the velocity decreases with R_c attaining negative values between $y = 0.6$ and $y = 1.0$ (curves I and II). The velocity of the fluid decreases rapidly with R_c (curves III and IV), the velocity increases having all positive values. This behaviour is ascertained from the curves IV and V. The effect of Reynolds number (R) on the velocity profiles are shown in the curves V and VI, which reveal that the velocity decreases as R increases keeping the permeability of the porous medium constant.

Fig. 2 exhibits the influence of t, R and R_c on the velocity field. It is remarked from the curves I, II and III that the increase in the elasticity of the fluid decreases the fluid velocity. For Newtonian fluid ($R_c = 0$), the velocity decreases sharply to zero from lower plate to upper plate, while for non-Newtonian fluid it tends to be reversed at the middle of the channel. The rise in the value of t as well as Reynolds number R, cause a flow reversal in the channel.

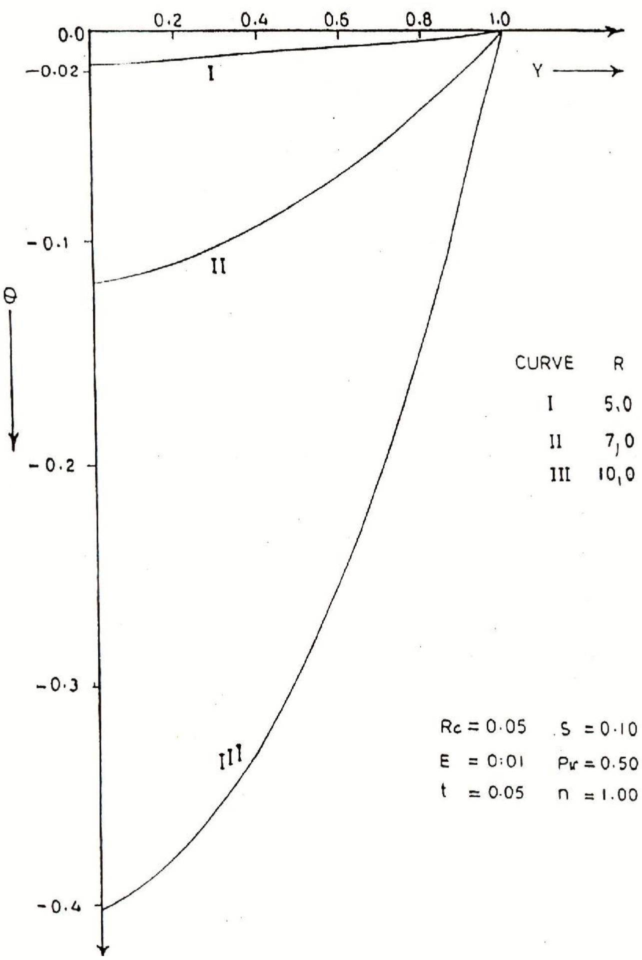

Fig. 5 : Effect of R on temperature field.

Fig. 3 explains the effects of t and n on the velocity field. When 't' increases, velocity also increases near the lower plate but tends to decelerate towards the middle of the channel and becomes zero at the upper plate. However a reverse effect is noticed for the rise of n.

The effects of t and R_c on temperature field are shown in Fig. 4 while the effect of R on temperature is shown in Fig. 5. It is concluded that the rise in the elastic property of the fluid reduces the temperature of the fluid. For Newtonian fluid, the temperature is positive while for non-Newtonian fluid it has a negative value, which increase with the increase of R_c. Further, the temperature falls, with the rise in t. When Reynolds number increases, the temperature near the lower wall falls suddenly attaining negative values but gains slowly towards upper plate. Fig. 6 depicts the influence of R and P_r on the temperature field. The temperature which is negative at the lower plate increases with decrease in P_r. Further, the rise in R increases the temperature at a high rate near the lower plate.

The effect of source parameter S on temperature field is illustrated in Fig. 7, where the negative values of S are meant for sink strength and positive values of S for source strength. As the sink strength rises from – 0.10 to – 0.50, the temperature of the fluid rises sharply (curves I and II). But, the temperature decreases with the further increase of sink strength which is marked from curves II and III. In the absence of source or sink, the temperature of the fluid is low having negative values. This clearly indicates that cooling effect is produced while the visco-elastic fluid exhibits Couette flow under the influence of an external uniform transverse magnetic field. In the presence of an internal heat generating source in the fluid, the temperature rises attaining positive values, when the source strength is low (S = 0.1). Further increase in S decreases the temperature (Curve VI). However, a deviation is marked in case of S = 1.0.

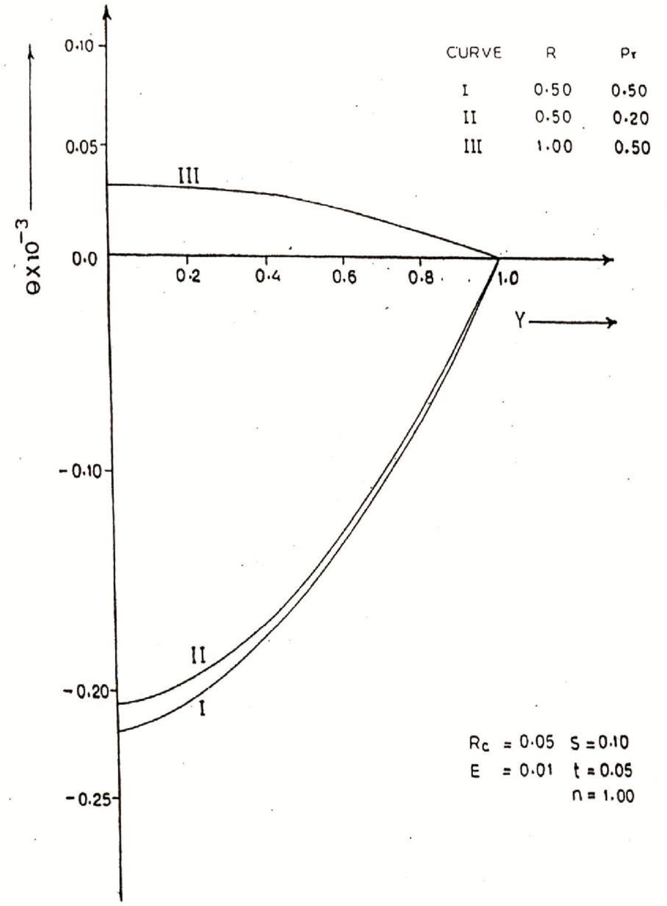

Fig. 6 : Effect of R and P_r on temperature field.

Fig. 8 explains the influence of Prandtl number and Eckert number on the temperature profiles. Curves I and II exhibit the effects of low Prandtl number rise and reveal the fact that the temperature rises with the rise of P_r form 0.1 to 0.2 within the boundary conditions imposed. Further, it is observed that the temperature falls with the rise of Prandtl number (P_r = 5.0). For higher values of Prandtl number, the fall in temperature is somewhat slower (Curve IV). The effect of Eckert number on the temperature field shown in the curves IV, V and VI unveil the fact that the rise in E produces a sharp fall in temperature.

Fig. 9 exhibits the behaviour of concentration with the variation of reaction parameter (K_1) and Schmidt number (S_c). It is seen that the concentration reduces with the rise of S_c as well as K_1.

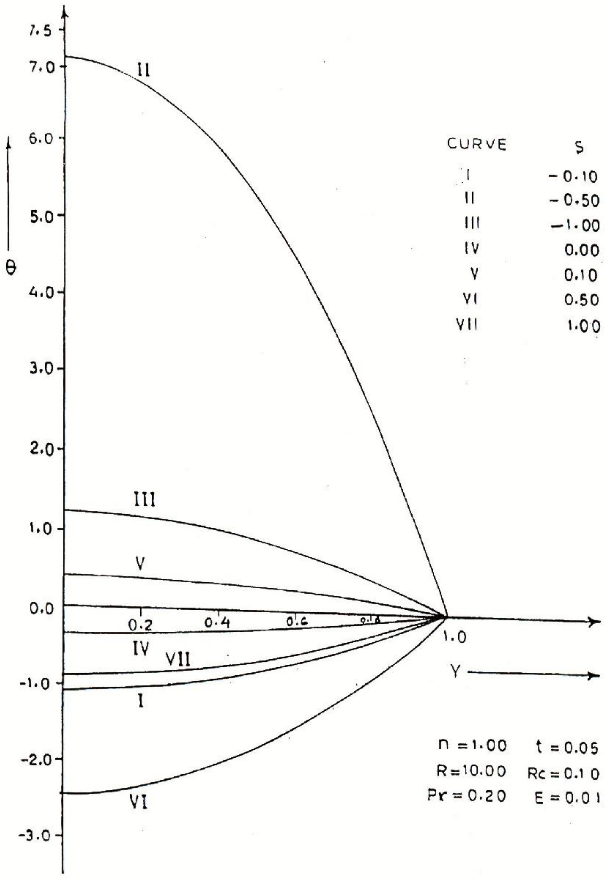

Fig. 7 : Effect of S on temperature field.

The values of shear stresses for different values of R, R_c and K^* are entered in Table 1 keeping all other parameters fixed.

Table : 1. Effects of R, R_c and K^* on skin friction for n = 1.0, t = 0.05, S = 0.05, P_r = 5.0 and E = 0.02

K^*	R_c	0.0		0.05		0.10	
	R	SKF1	SKF2	SKF1	SKF2	SKF1	SKF2
3.3	5.0	$-.482503 \times 10^{-1}$	$-.517497 \times 10^{-1}$	-133580	0.648524×10^{-1}	$-.218910$	0.181455
	10.0	$-.459776 \times 10^{-1}$	$-.540224 \times 10^{-1}$	$-.378443$	0.682500×10^{-1}	$-.710909$	0.190522
	15.0	$-.437049 \times 10^{-1}$	$-.562951 \times 10^{-1}$	$-.709676$	$-.147212 \times 10^{-1}$	$-.137565 \times 10^{-1}$	$.268527 \times 10^{-1}$
	5.0	$-.496650 \times 10^{-1}$	$-.503350 \times 10^{-1}$	$-.227148 \times 10^{-1}$	0.968847×10^{-2}	0.423549×10^{-1}	0.697119×10^{-1}
0.16	10.0	$-.478722 \times 10^{-1}$	$-.521278 \times 10^{-1}$	$-.160166$	0.160444×10^{-1}	$-.272459$	0.842167×10^{-1}
	15.0	$-.460794 \times 10^{-1}$	$-.539206 \times 10^{-1}$	-3.56650	$-.667221$	$-.193478 \times 10^{-1}$	-0.193478×10^{-1}

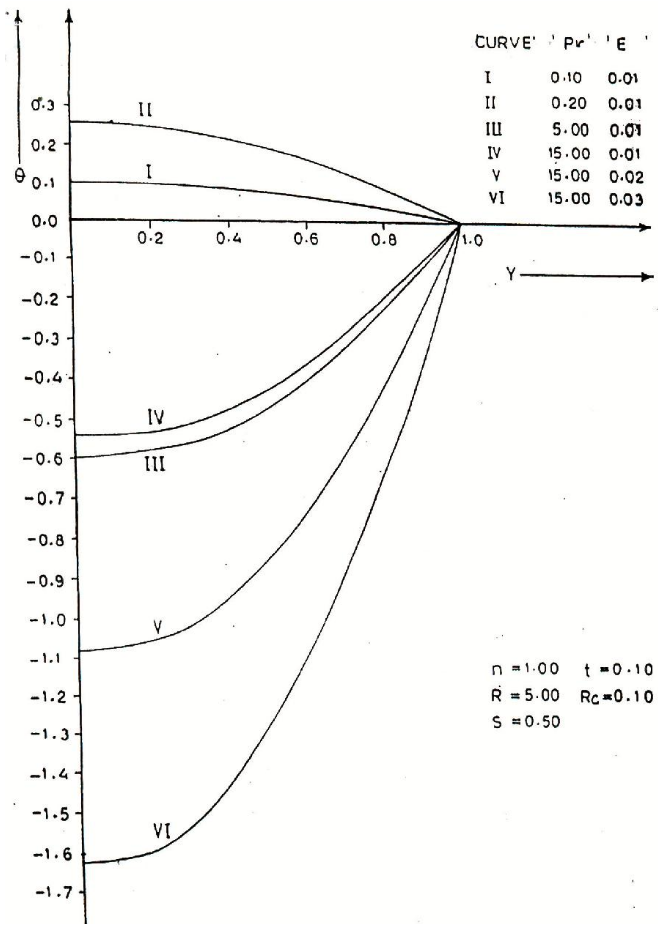

Fig. 8 : Effect of P_r and E on temperature field.

It is observed that the increase in Reynolds number (R) causes the value of shear stress at the lower plate to be more and more negative for visco-elastic fluid while for viscous fluid the negativeness of the shear stress at the lower plate decreases. It is further noticed that in case of viscous fluid (R_c = 0), the value of shear stress at the upper plate decreases as R increases. Interestingly, the skin friction at the upper plate first increases and then decreases with the continuous rise of R for non-Newtonian fluid ($R_c > 0$). The increase in the elasticity of the fluid decreases the value of skin friction at the lower plate and this effect is reversed for the upper plate. From the reading of this table, it is also observed that the value of skin friction at the lower plate increases for visco-elastic fluid and decreases for viscous fluid, with the rise of permeability factor (K^*), while this effect is reversed for upper plate.

The table 2 shows the dependence of rates of heat transfer at both the plates on t, n, R and R_c, all other variables remaining fixed. It is noticed that when t increases, rates of heat transfer also increases at both the plates for n = 0.5, but decreases for n = 1.0 in case of Newtonian and non-Newtonian fluids. The increase in the value of 'n' results in the reduction of rates of heat transfer at both the plates and for both viscous and visco-elastic fluids. It is observed that the value of Nusselts number at lower plate decreases while that at the upper plate increases with the rise of Reynolds number (R) in case of $R_c > 0$. When R_c = 0 the value of Nusselts number falls at both lower and upper plates with the rise of *R*. Finally, as the elastic property of the fluid increases, the rates of heat transfer at both the plates also increases. All the above conclusions are drawn in the presence of a porous medium *i.e.* $\frac{1}{K^*} > 0$.

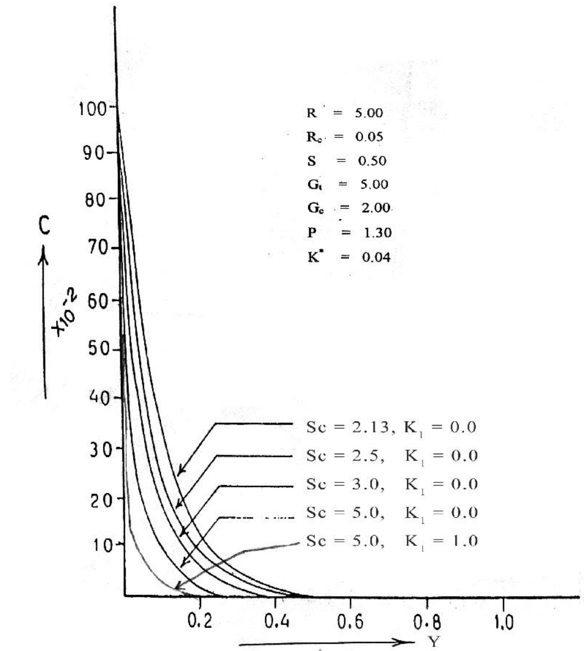

Fig. 9 : Effect of K_1 and S_c on Concentration

Table : 2. Effects of *t*, *n*, *R*, R_c on the rates of heat transfer for $S = 0.1$, $K^* = 3.3$, $Pr = 0.1$ and $E = 0.01$.

R	t	n/R_c	0.5		1.0	
			NU_0	NU_1	NU_0	NU_1
0.5	0.05	0.0	$-.571016c10^{-6}$	$.297534x10^{-4}$	$-.238178x10^{-4}$	$-.639748x10^{-4}$
		0.05	$.101282x10^{-5}$	$.180371x10^{-3}$	$-.230699x10^{-4}$	$-.477565x10^{-4}$
		0.10	$-.59665x10^{5}$	$.170988x10^{-3}$	$-.223221x10^{-4}$	$-.315381x10^{-4}$
	0.10	0.0	$-.228406x10^{5}$	$.595068x10^{-4}$	$-.952711x10^{-4}$	$-.127950x10^{-4}$
		0.05	$.405128x10^{5}$	$.200741x10^{-4}$	$-.922797x10^{4}$	$-.955130x10^{-4}$
		.10	$.103866x10^{-4}$	$.341976x10^{-4}$	$-.892884x10^{-4}$	$-.630762x10^{-4}$
5.0	0.05	0.0	$-.117952x10^{-5}$	$-.648397x10^{-4}$	$-.494657x10^{-4}$	$-.284936x10^{-4}$
		0.5	$-.269204x10^{-2}$	$.753349x10^{-4}$	$-.124071x10^{-4}$	$.351382x10^{-4}$
		0.10	$-.526613x10^{-2}$	$.157154x10^{0}$	$-.243195x10^{-2}$	$.73125x10^{-4}$

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