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PHOTON FISHY STORY

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This paper gives a simple model for the emission of a particle like a photon. It is assumed that the emitted particle has a typical quantum wavelength λ that is large compared to the typical size *R* of the atom or nucleus that does the emitting. The purpose of the model is to show that in that case, the particle will very likely come out with zero orbital angular momentum but has some probability of nonzero angular momentum.

INTRODUCTION

First, photon wave functions are messy and not that easy to make sense of electron wave function. The photon would be much simpler if it did not have spin and was non relativistic. A reasonable wave function for a hypothetical spinless non relativistic photon coming out of the center of the emitter with typical wave length λ would be

$$\psi = \frac{1}{\lambda^{3/2}} f\left(\frac{r^2}{\lambda^2}\right)$$

where *r* is the distance from the center. (The various factors λ have been added to make the function *f* independent of the photon wave length λ despite the corresponding spatial scale and the normalization requirement.)

Table 1. The first few spherical harmonics.

$Y_0^0 = \sqrt{\frac{1}{4\pi}}$	$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos(\theta)$ $Y_1^1 = \sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$ $Y_1^{-1} = \sqrt{\frac{3}{8\pi}}\sin\theta\, e^{-i\phi}$	$Y_2^0 = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$ $Y_2^1 = \sqrt{\frac{15}{16\pi}}\sin\theta\cos\theta\, e^{i\phi}$ $Y_2^{-1} = \sqrt{\frac{15}{16\pi}}\sin\theta\cos\theta\, e^{-i\phi}$ $Y_2^2 = \sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{i\phi}$ $Y_2^{-2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{-2i\phi}$

The above wave function has no preferred direction in the emission, making it spherically symmetric. It depends only on the distance *r* from the center of the emitter. That means that the wave function has zero orbital angular momentum. Recall that zero angular momentum corresponds to the spherical harmonic Y_0^0, which is independent of the angular position, Table 1.

There are various reasons to give why you would want the wave function of a particle coming out of the origin to have zero angular momentum. For one, since it comes out of a featureless point, there should not be a preferred direction. Or in terms of classical physics, if it had angular momentum then it would have to have infinite velocity at the origin. The similar quantum idea is that the relevant wave functions for a particle moving away from the origin, the Hankel functions of the first kind, blow up very strongly at the origin if they have angular momentum. But it is really better to describe the emitted particle in terms of the Bessel functions of the first kind. These have zero probability of the particle being at the origin if the angular momentum is not zero. And a particle should not be created at a point where it has zero probability of being.

Of course, a spherically symmetric quantum wave function also means that the particle is moving away from the emitter equally in all directions. Following the stated ideas of quantum mechanics, this will be true until the position of the particle is measured. Any macroscopic surroundings cannot reasonably remain uncommitted to exactly where the outgoing particle is for very long.

Now consider the same sort of emission, but from a point in the emitter a bit away from the center. For simplicity, assume the emission point to be at $R\hat{k}$, where *R* is the typical size of the emitter and $\hat{k}$ is the unit vector along the chosen *z*-axis. In that case the wave function is

$$\psi = \frac{1}{\lambda^{3/2}} f\left(\frac{(\vec{r} - R\hat{k})}{\lambda^2}\right)$$

Using Taylor series expansion, that becomes

$$\psi = \frac{1}{\lambda^{3/2}} f\left(\frac{r^2}{\lambda^2}\right) - \frac{R}{\lambda}.\frac{1}{\lambda^{3/2}} f'\left(\frac{r^2}{\lambda^2}\right) 2\frac{r}{\lambda}.\frac{z}{r} + \ldots$$

In the second term, *z*/*r* is the spherical harmonic Y_1^0, Table 2. This term has angular momentum quantum number $l = 1$. So there is now uncertainty in momentum. And following the stated ideas of quantum mechanics, the probability for $l = 1$ is given by the square magnitude of the coefficient of the (normalized) eigenfunction.

Table 2: The first few spherical harmonics rewritten.

$Y_0^0 = \sqrt{\frac{1}{4\pi}}$	$rY_1^0 = \sqrt{\frac{3}{4\pi}}\, z$	$r^2 Y_2^0 = \sqrt{\frac{5}{16\pi}}(2z^2 - x^2 - y^2)$
	$rY_1^1 = -\sqrt{\frac{3}{8\pi}}(x + iy)$	$r^2 Y_2^1 = -\sqrt{\frac{15}{16\pi}}\, z\,(x + iy)$
	$rY_1^{-1} = \sqrt{\frac{3}{8\pi}}(x - iy)$	$r^2 Y_2^{-1} = \sqrt{\frac{15}{16\pi}}\, z\,(x - iy)$

		$r^2Y_2^2 = \sqrt{\frac{15}{32\pi}}(x-iy)^2$ $r^2Y_2^{-2} = \sqrt{\frac{15}{32\pi}}(x-iy)^2$

That makes the probability for $l = 1$ proportional to $(R/\lambda)^2$. If you carried out the Taylor series to the next order, you would end up with a $(z/r)^2$ term, which, combined with a spherically symmetric contribution, makes up the spherical harmonic Y_2^0. It then follows that the probability for $l = 2$ is of order $(R/\lambda)^4$, and so on. Under the assumed condition that the emitter size R is much less than the quantum wave length λ of the emitted particle, the probabilities for non zero angular momentum are small and decrease rapidly even further with increasing l.

References

1. Bethe, H.A., *Intermediate Quantum Mechanics.* W.A. Benjamin (1964).
2. Yariv, A., *Theory and Applications of Quantum Mechanics*. Wiley & Sons (1982).
3. Rajput, B. S., *Advanced Quantum Mechanics*, Pragati Prakashan, Third Edition (1995).
4. Griffiths, D J., *Introduction to Quantum Mechanics*. Pearson Prentice-Hall, Second Edition (2005).

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