ONE CARRIER CURRENT FLOW IN INSULATOR WITH DISTRIBUTED TRAPS UNDER CDDM REGIME AND DIMENSIONLESS VARIABLES

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> The current injection technique in insulator is useful to explain the electrical transport properties of insulator. The trapping states are usually present in the forbidden gap of the insulator. These states affect the complete currentvoltage characteristic of the insulator.

> Applying the regional approximation method and three dimensionless variables, the theoretical current-voltage characteristics have been derived for two sets of distributed traps under different starting positions of the thermal-equilibrium Fermi level in the forbidden gap and carrier density dependent mobility regime. The cube power law for the dependence of current on voltage is obtained for the pure space-charge-limited current-voltage regime.

> **KEYWORDS :** Distributed traps, carrier density dependent mobility, current injection, thermal free carrier and traps, and dimensionless variables.

INTRODUCTION

The current injection in insulator is well known from several decades [1-9]. The presence of distributed traps in the forbidden gap of the insulator changes the form and magnitude of the current flow by trapping the current carriers. The carrier mobility affects the injection of current in insulators. The effects of distributed traps and carrier mobility on the complete current-voltage characteristics have been given with the help of regional approximation method [3, 5-9] and there dimensionless variables.

THEORETICAL FORMULATION

Let us consider an insulator with the lower trap distribution larger or equal to one of the upper trap distribution under carrier density dependent mobility regime [7]. The two sets of trapping states are distributed around the two energy levels E_1 and E_2 . The present paper is given for the behaviour of steady state space-charge-limited single injection current flow in low mobility insulators along with the conditions where the thermodynamical Fermi level F_0 lies below both the trap energy levels E_1 and E_2 . The total concentrations of two sets of electron trapping states distributed in energy around levels E_1 and E_2 are N_1 and N_2 , respectively.

The Gaussian distribution of traps around a single energy E_t would be appropriate to characterize the broadening of trapping levels [4, 5]. The direct current-voltage relations is obtained with the help of trap distribution function expressed as⁵

$$H(E) = \frac{N_t}{kT_t} \frac{exp\left[\frac{(E-E_t)}{kT_t}\right]}{\left\{exp\left[\frac{(E-E_t)}{kT_t}\right] + 1\right\}^2} \dots (1)$$

where T_t is the characteristic temperature. The problem is simplified by the following assumptions as

(i) The diffusion currents are neglected, and

(ii) The injecting electrode is taken as the infinite reservoir of the electrons available for carrier injection.

- (iii) The carrier mobility is field independent, and
- (iv) The statistics of trap occupancy are all assumed valid.

The general equations of the problem are given by

$$J = e \mu n(x) E(x)$$
 ... (2)

$$\frac{\epsilon}{e}\frac{dE}{dx} = (n - n_o) + (n_t - n_{t,o}) \qquad \dots (3)$$

$$\mu = h n(x) \qquad \dots (4)$$

where x denotes the distance from the cathode, ϵ is static dielectric constant, μ is the free carrier mobility, E(x) is the position dependent electric field strength, n is free carrier concentration and n_t is trapped charge concentration at position x, n_0 is concentration of thermal free carriers, $n_{t,0}$ is the thermal-equilibrium value of n_t and h is the proportionality constant.

The regional approximation method [5-7] is used to derive the *J*-*V* characteristics. The insulator is divided into different regions with the help of this method [5, 7]. The number of regions to be taken into account will depend on the initial position of the Fermi level with respect to the trap levels and the conduction band as well as on the number of trapping centers present in the solid [5]. The equations are expressed in terms of three dimensionless variables [5, 7] as

$$u(x)\frac{n_o}{n(x)} = n_o \sqrt{\frac{he E(x)}{J}} \qquad \dots (5)$$

$$w(x) = \frac{e n_o^2 x}{\epsilon} \sqrt{\frac{h e}{J E(x)}} \qquad \dots (6)$$

$$v(x) = \frac{e^3 n_o^3 h V(x)}{\epsilon I E(x)} \qquad \dots (7)$$

where *u*, *v* and *w* are the three dimensionless variables, respectively.

THE REGIONAL APPROXIMATION SCHEME FOR THE ENERGY LEVEL

$$E_2 > E_1 > F_0$$

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In the regional approximation method, the insulator between the electrodes may be divided into three regions :

(1) Perfect insulator region where the injected free charge $n_i(x) = n(x) - n_0 \approx n(x)$ dominates. It extends from the injecting contact (x = 0) up to a plane x_{TF} where the total excess trapped charge equals to the free carrier concentration.

(2) Trapped charge region : In this region, the excess trapped charge dominates both the injected free carrier and the free carriers in thermal-equilibrium.

(3) Ohmic region : In this region, the thermal-equilibrium carriers n_0 are dominant.

Region I : Perfect insulator region : $0 \le x \le x_{TF}$

$$E_c \ge F(x) \ge E_c kT \ln \frac{N_c}{N_2}$$
$$n_i(x) = n(x) - n_o \approx n(x) \ge n_t(x) \ge n_o$$
$$n(x_{TF}) = n_t (x_{TF}) = N_2 + N_1 \approx N_2$$

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The equations (3), (5) and (6) give the dimensionless Poisson's equation for the region I as

$$\frac{\epsilon}{e} \frac{dE}{dx} = n(x)$$

$$duw = \frac{h e^2 n_o^3}{\epsilon J} dx$$

$$\frac{duw}{du} = 2u^2$$
....(8)

The integration of above equations and the equations (5)-(7) give the three dimensionless variables as

$$w = \frac{2}{3} u^{2} \qquad \dots (9)$$
$$v = \frac{1}{u^{2}} \int u^{2} duw$$
$$v = \frac{2}{5} u^{3} \qquad \dots (10)$$

Region II : Trapped – charge region II 'a' : $(x_{TF} \le x \le x_{\Omega})$

$$n_t(x) \ge n(x) \ge n_o$$

$$\frac{\epsilon}{e} \frac{dE}{dx} = n_t(x) \qquad \dots (11)$$

The trapped charge region is divided into four parts [3, 5-7] as

Trapped charge region II 'a' : $x_{TF} \le x \le x_1$

$$E_c - kT \ln \frac{N_c}{N_2} \ge F(x) \ge E_2$$
$$n_t(x) = N_2 + N_1 \approx N_2$$

The values of three dimensionless variables at the imaginary transition plane x_{TF} are given by

$$u(x_{TH}) = \frac{n_o}{n(x)} = \frac{n_o}{N_2} = P$$

$$W(X_{TF}) = \frac{2}{3}P^2$$

$$W(X_{TF}) = \frac{2}{5}P^2$$
...(12)

The Poisson's equation (11) is modified with the help of dimensionless variables (12) as

$$\frac{duw}{du} = 2uP \qquad \dots (13)$$

The integration of equation (13) gives

$$w = Pu - \frac{P^2}{3} \qquad ... (14)$$

$$v = \frac{P u^2}{2} - \frac{P^5}{2u^2} + \frac{2}{5} P^3 \qquad \dots (15)$$

Trapped charge region II 'b' :

$$x_{1} \leq x \leq x_{2}$$

$$E_{2} \geq F(x) \geq E_{2} kT_{2} \ln A \qquad \text{where } A = \frac{N_{2}}{N_{1}}$$

$$n_{t}(x) = N_{2} \left[\frac{n(x)}{N}\right]^{\frac{1}{l}} + N_{1} \approx N_{2} \left(\frac{n(x)}{N}\right)^{\frac{1}{l}}$$

The values of three dimensionless variables at the imaginary transition plane x_1 are obtained from the equations (5) – (7), (9) and (10) as

$$(u(x_1) = \frac{n_0}{N} = B$$

$$w(x_1) = PB - \frac{P^2}{3} = PB \left[1 - \frac{R}{3} \right] \quad \text{where } R = \frac{P}{B}$$

$$w(x_1) \frac{PB^2}{2} - \frac{P^5}{2B^2} + \frac{2}{5} P^3 = \frac{PB^2}{2} \left[1 - R^4 + \frac{4}{5} R^2 \right] \quad (16)$$

The Poisson's equation for region II 'b' is derived from equation (11) and (16) as

$$\frac{duw}{du} = \frac{2P}{B^{\frac{1}{l}}} u^{\left(\frac{l+1}{l}\right)}(17)$$

The integration of equation (16) gives

$$w = \frac{PB}{2l+1} \left[2l \left(\frac{u}{b}\right)^{\frac{l+1}{l}} + \left\{ 1 - \frac{(2l+1)R}{3} \right\} \right] \qquad \dots (18)$$

$$v = \frac{PB^2}{4l+1} \left[\left(\frac{u}{B}\right)^{\frac{2l+1}{l}} 2l - \frac{2B^2l}{u^2} + \frac{4l+1}{2} \left(1 - R^4 + \frac{4}{5}R^2\right) \right] \dots (19)$$

Trapped charge region II 'c' : $x_2 \le x \le x_3$

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$$E_2 - kT_2 \ln A \ge F(x) \ge E_1$$
$$n_t(x) = N_1$$

The value of three dimensionless variables at the imaginary transition plane x_2 are obtained from the equations (5)-(7), (9) and (10) as

$$u(x_{2}) = BA^{l}$$

$$w(x_{2}) = \frac{PB}{2l+1} \left[lA^{l+1} + \left\{ 1 - \frac{(2l+1)R}{3} \right\} \right]$$

$$w(x_{2}) = \frac{2lPBA^{l+1}}{2l+1} \left[1 + \frac{A^{-(l+1)}}{2l} \left\{ 1 - \frac{(2l+1)R}{3} \right\} \right]$$

$$w(x_{2}) = \frac{2lPB^{2}A^{2l+1}}{4l+1} \left[1 - A^{-(4l+1)} + \frac{4l+1}{4l} A^{-(2l+1)} \left(1 - R^{4} + \frac{4}{5} R^{2} \right) \right] \right\} \dots (20)$$

The Poisson's equation for the region II 'c' is given by the equation (11) as

$$\frac{duw}{du} = 2Qu \qquad \text{where } Q = \frac{n_o}{N_1} \qquad \dots (21)$$

The integration of equation (20) gives :

$$w = Qu - \frac{QBA^{l}}{2l+1} \left[2l+1 - \left(\frac{1}{A}\right)^{l} 2l \left\{ 1 - \frac{(2l+1)R}{3} - A^{l+1} \right\} \right] \dots (22)$$
$$v = \frac{Qu^{2}}{2} - \frac{QB^{4} A^{4l}}{2(4l+1)} \left[\frac{4l+1}{u^{2}} - \frac{4l RA^{-2l+1}}{B^{2}} \left\{ 1 - A^{-(4l+1)} + \frac{4l+1}{4l} A^{-(2l+1)} \left[1 - R^{4} + \frac{4}{5} R^{2} \right] \right\} \right] \dots (23)$$

Trapped charge region II 'd': $x_3 \le x \le x_{\Omega}$

$$E_1 \ge F(x) \ge F_0 + 0.7 kT$$
$$n_t(x) = N_1 \left[\frac{n(x)}{M}\right]^{\frac{1}{m}}$$

The value of three dimensionless variables at the imaginary transition plane x_3 are obtained from the equations (5) – (7), (9) and (10) as

$$u(x_{3}) = D$$

$$w(x_{3}) = QD \left[1 - \frac{A^{l}}{2l+1} \left(\frac{B}{D} \right) \left\{ 2l+1 + \left(\frac{1}{A} \right)^{l} 2l \left(1 - \frac{(2l+1)R}{3} - A^{l+1} \right) \right\} \right]$$

$$v(x_{3}) = \frac{QD^{2}}{2} \left[1 - \frac{A^{4l}}{4l+1} \left(\frac{B^{2}}{D} \right)^{2} \left\{ \frac{4l+1}{D^{2}} - \frac{4lRA^{-2l+1}}{B^{2}} \left\{ 1 - A^{-(4l+1)} + \frac{4l+1}{4l} A^{-(2l+1)} \left[1 - R^{4} + \frac{4}{5} R^{2} \right] \right\} \right\} \right] \qquad (24)$$

The Poisson's equation for the region II 'd' is given by equation (11) as

$$\frac{duw}{du} = \frac{2Q}{D^{\frac{1}{m}}} u^{\frac{m+1}{m}} \dots (25)$$

The integration of equation (24) gives

$$w = \frac{QD}{2m+1} \left[2m \left(\frac{u}{D}\right)^{\frac{m+1}{m}} + \left\{ 1 - \left(\frac{2m+1}{2l+1}\right) A^{l} \left(\frac{B}{D}\right) \left[2l+1 - \left(\frac{1}{A}\right)^{l} 2l \left(1 - \frac{(2l+1)R}{3} - A^{l+1}\right) \right] \right\} \right] \dots (26)$$

$$v = \frac{QD^{2}}{4m+1} \left[2m \left(\frac{u}{D}\right)^{\frac{2m+1}{m}} + (4m+1) \left\{ \frac{1}{2} - \frac{A^{4l}}{4l+1} \left(\frac{B^{2}}{D}\right)^{2} \left(\frac{4l+1}{D^{2}} - \frac{4lRA^{-2l+1}}{B^{2}} + \left(1 - A^{-(4l+1)} + \frac{4l+1}{4l} A^{-(2l+1)} \left[1 - R^{4} + \frac{4}{5} R^{2} \right] \right\} \right] - 2m \left(\frac{D}{u}\right)^{2} \right] \dots (27)$$

Region III :- For ohmic region $x_{\Omega} \leq x \leq L$

 $F_0 + 0.7 \ kT \ge F(x) \ge F$ $n_t(x) = 0 \quad \text{and} \quad n_0 \approx n(x).$

The value of three dimensionless variables at the imaginary transition plane x_{Ω} are derived as

The Poisson's equation for the ohmic region is obtained as

$$\frac{du}{duw} = 0$$

$$v = V(x_{\Omega}) + \frac{1}{u^2} \int u^2 duw$$

$$v = V(x_{\Omega}) + uw - u(x_{\Omega}) w(x_{\Omega}) \qquad \dots (30)$$

The current J and voltage V are evaluated in terms of three dimensionless variables from the equations (5) - (7) as

$$J = \frac{e^2 n_0^3 hL}{\epsilon} \frac{1}{u_a w_a}; \qquad V = \frac{e n_0 L^2}{\epsilon} \left(\frac{v_a}{w_a^2}\right) \qquad \dots (31)$$

where u_a , w_a , v_a are the values of u, w and v at x = L *i.e.* at the collecting electrode or anode.

COMPLETE CURRENT-VOLTAGE CHARACTERISTICS OF THE PROBLEM

The complete current – voltage characteristics is evaluated with the help of the above equations as described below :

True Ohm's Regime :

It occurs in the insulator at very low injection level of current. This current – voltage regime is found to be under thermal – equilibrium condition. The equations (5) - (7) and (31) give

$$\frac{v_a}{w_a^2} = \frac{1}{u_a w_a}$$

$$J = \frac{eh n_0^2}{L} V \implies J \propto V \qquad \dots (31a)$$

and

which is the pure Ohm's law for the planar current flow in insulator under carrier density dependent mobility regime.

Ohmic Regime :

At low currents, the dimensionless *J*-*V* relation is obtained by putting $v = v_a$ and $w = w_a$ in the equations (24) – (30) as

$$\frac{v_a}{w_a^2} = \frac{1}{u_a w_a} + \frac{u(x_{\Omega}) - w(x_{\Omega})}{u_a^2 w_a^2}$$

where the current flow is contributed by mainly the regions II 'd' and III. Substituting the values of dimensionless variables at the transition plane x_3 the above dimensionless characteristic becomes

$$\frac{v_a}{w_a^2} = \frac{1}{u_a w_a} - \frac{4Qm^2 D^{-\frac{1}{m}}}{(4m+1)(2m+1)} \left(\frac{1}{u_a w_a}\right)^2 \qquad \dots (32)$$

which is equivalent to Ohm's law with a small correction.

The critical current $J = J_{\Omega}$ at which the ohmic region leaves the insulator *i.e.* $x_{\Omega} = L$ then

$$J_{\Omega} = \frac{e^2 n_0^3 hL}{2 \epsilon Q} \left(\frac{2m+1}{m}\right) D^{\frac{1}{m}} \qquad ... (33)$$

$$V_{\Omega} = \frac{(2m+1)^2}{(4m+1)m} \frac{n_0 e L^2 D^{\frac{1}{m}}}{2Q\epsilon} \qquad \dots (34)$$

For Region II 'd', the dimensionless current-voltage characteristic is obtained from the equations (24)-(27) as

$$\frac{\mathbf{v}_{a}}{w_{a}^{2}} = \frac{QD^{4}}{(4m+1)} \left\{ \frac{1}{2} \left(\frac{1}{u_{a}w_{a}} \right)^{2} + 2m \left[\frac{2m+1}{2m} \frac{D^{\frac{1}{m}}}{QD} - \frac{D^{\frac{2}{m}}}{2m} \left(\frac{1}{u_{a}w_{a}} \right) \right]^{\frac{2m+1}{m+1}} \right\} \left[\frac{1}{u_{a}w_{a}} \right]^{\frac{1}{m+1}} \dots (35)$$

The approximation of equation (35) alongwith dimensionless variables (5)-(7) gives the current-voltage characteristic of ohmic regime as

$$J = \frac{(4m+1)^{m+1} (2mQ\epsilon)^m he^{1-m} n_0^{2-m}}{(2m+1)^{2m+1} (D^{\frac{1}{m}})^{2m+1} D^{2m-3}} \frac{V^{m+1}}{L^{2m+1}}$$

For critical current $J = J_{c_3}$ and $x_3 = L$, this transition regime is terminated from the insulator. The equations (24) yield

$$J_{c_3} = \frac{e^2 h L N_1 M^2}{\epsilon} \left[1 - \frac{A^l}{2l+1} \left(\frac{B}{D} \right) \right]^{-1} \qquad \dots (36)$$

$$V_{c_3} = \frac{eL^2 N_1}{2\epsilon} \left[1 - \frac{A^{4l}}{4l+1} \left(\frac{B}{D}\right)^4 \right] \left[1 - \frac{A^l}{2l+1} \left(\frac{B}{D}\right) \right]^{-2} \qquad \dots (37)$$

The expressions of J_{c_3} and V_{c_3} are different from the expressions for J_{Ω} and V_{Ω} because the ohmic region is absent in the first trap - filled – limit regime

For Region II 'c':

The dimensionless characteristic is derived from the equations (20)-(23) as

$$\frac{\mathbf{v}_a}{w_a^2} = \frac{1}{2Q} - \frac{B^2 A^{-2l} 2lR}{4l+1} \left(\frac{1}{u_a w_a}\right) - \frac{4l^2 Q B^4 A^{4l}}{2(4l+1)(2l+1)^2} \left(\frac{1}{u_a w_a}\right)^2 \quad \dots (38)$$

The values J_{c2} and V_{c2} are the critical current and critical voltage at $x_2 = L$ as

$$J_{c2} = \frac{e^2 h L (2l+1) N_2 N^2}{2 \epsilon l A^{l+2}} \left[1 + \frac{1}{2l} \left(\frac{1}{A} \right)^{l+1} \right]^{-1} \qquad \dots (39)$$

$$V_{c2} = \frac{ehL^2 (2l+1)^2 N_2 A^{2l-3}}{2 \epsilon l (4l+1)} \left[1 + \left(\frac{1}{A}\right)^{4l+1} \right] \left[1 + \frac{1}{2l} \left(\frac{1}{A}\right)^{l+1} \right]^{-2} \dots (40)$$

which are the constant quantities derived from the equations (5)-(7) and (20).

For Regions II 'b' :

The dimensionless characteristic is derived from the equations (16)-(18) as

$$\frac{v_a}{w_a^2} = \frac{PB^4}{(4l+1)} \left[\frac{1}{2} \left(\frac{1}{u_a w_a} \right)^2 + 2l \left\{ \left(\frac{2l+1}{2l} \right) \frac{1}{PB^2} - \frac{1}{2l} \left(\frac{1}{u_a w_a} \right) \right\}^{\frac{2l+1}{l+1}} \left[\frac{1}{u_a w_a} \right]^{\frac{1}{l+1}} \dots (41)$$

The equations (5)-(7) and (41) yield the current-voltage characteristic as

$$J = \left(\frac{4l+1}{2l+1}\right)^{l+1} \left(\frac{2l}{2l+1}\right)^{l} (en_0)^{1-l} h N \in^{l} \frac{(PB^2)^{2l+1}}{(PB^4)^{l+1}} \frac{V^{l+1}}{L^{2l+1}} \dots (42)$$

The perfect insulator region I and region II 'a' are present in the insulator for $J > V_{c_1}$. The critical current V_{c_1} and critical voltage V_{c_1} are obtained at $x_1 = L$. The equations (11) – (15) yield

$$J_{c_1} = \frac{e^2 n_0 h \, L \, N \, N_2}{\epsilon \left[B - \frac{P}{3} \right]} \qquad \dots (43)$$

$$V_{c1} = \frac{e^2 L^2 N_2}{\epsilon} \frac{\left[\frac{B^2}{2} - \frac{P^4}{2B^2} + \frac{2}{5}P^2\right]}{\left[B - \frac{P}{3}\right]^2} \dots (44)$$

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For Region II 'a': The dimensionless characteristic is derived from the equations (13)-(15) as

$$\frac{\mathbf{v}_a}{w_a^2} = \frac{1}{2Q} + \frac{2}{5}P^2 \left(\frac{1}{u_a w_a}\right) - \frac{P^5}{2} \left(\frac{1}{u_a w_a}\right)^2 \qquad \dots (45)$$

The perfect insulator region is present in insulator for $J > J_{TF}$.

At $x_{TF} = L$ $J = J_{TF}$ and $V = V_{TF}$ which are evaluated from the equations (12) as

$$J_{TF} = \frac{3}{2} \frac{e^2 h L N_2^2}{\epsilon} \qquad ... (46)$$

$$V_{TF} = \frac{9}{10} \frac{e \, hL^2 \, N_2^3}{\epsilon} \qquad \dots (47)$$

The equations (9) and (10) are applied to obtain the current-voltage characteristic of the insulator operating under perfect trap - free regime as

$$I = \frac{500}{243} \frac{\epsilon^2}{eL^5} \frac{h}{V^3} \qquad \dots (48)$$

which is the cube power law for the dependence of current on voltage $(J \propto V^3)$.

DISCUSSION AND CONCLUSIONS

In the current injection problem of insulator with distributed traps [5-7], the complete current-voltage characteristic of the solid is represented by $J \propto V^n$ where the exponent value n varies from 1 to 20. The influence of traps is observed mainly in the third and fourth trap-filled-limit regimes. This kind of trap distribution decides the locations and numbers of different transition current – voltage regimes present in the complete current-voltage characteristics.

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