ANALYSIS OF SUTHERLAND CONSTANT FROM COEFFICIENT OF VISCOSITY OF GASES USING ATTRACTIVE POTENTIAL MODEL

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One of the elementary model of potential energy is the attractive potential model (Sutherland potential model). This model is applied to the coefficient of viscosity of gases Ar, Ne and He and Sutherland constants have been analysed.

Introduction

We can calculate the Sutherland constant by applying attractive potential model (1, 4). to the coefficient of viscosity (1, 3). The formula of coefficient of viscosity (7, 10) has been obtained by semiempirical procedure in certain temperature zone. From slope and intercept of the curves, the values of Sutherland constant have been calculated in definite temperature zone.

Evaluation of Sutherland Constant

The coefficient of viscosity of gases (4, 10) can be written as

$$\eta = \frac{266.93\sqrt{MT} \times 10^{-7}}{\sigma^2 \Omega^{(2,2)}(T^*)} \qquad \ldots (1)$$

where η is the coefficient of viscosity of gas in poise; σ is the I collision diameter in A^0, M is the molecular weight in number.

T^* = Reduced temperature $= \dfrac{kT}{\phi_0}$

k = Boltzman Constant

$\Omega^{(2,\,2)}$ = Collision integral [4]

In Sutherland model the molecules are supposed to be hard spheres of diameter σ_0 surrounded by an attractive field by inverse power law. When attractive forces are acting among the molecules the cross-section for contact collision is given by

$$\sigma^2 = \sigma_0^2 \left[1 + \frac{\phi_0}{1.27\,kT}\right] \qquad \ldots (2)$$

It is clear from eq. (2) that the collision cross-section is increased by the attractive forces. Since the attractive intermolecular forces are acting among the molecules in Sutherland model, hence eq. (2) is applied to eq. (1) and it gives.

$$\eta = \frac{266.93\sqrt{M} \times 10^{-7} \times T^{1/2}}{\sigma_0^2 \left[1 + \frac{\phi_0}{1.27\,kT}\right]} \qquad \dots (3)$$

Putting $\frac{\phi_0}{1.27\,k} = S =$ Sutherland Constant in eq. (3), we have

$$\eta = \frac{266.93\sqrt{M} \times 10^{-7} \times T^{1/2}}{\sigma_0^2 \left[1 + \frac{S}{T}\right]} \qquad \dots (4)$$

Again putting $266.93\sqrt{M} \times 10^{-7} = Z$ in eq. (4), we get

$$\eta = \frac{Z\,T^{1/2}}{\sigma_0^2 \left[1 + \frac{S}{T}\right]} \qquad \dots (5)$$

From eq. (5),

$$\frac{T^{1/2}}{\eta} = \frac{\sigma_0^2}{Z} + \frac{\sigma_0^2 S}{ZT} \qquad \dots (6)$$

If we plot a graph between $\frac{T^{1/2}}{\eta}$ corresponding to observed value of η and $\frac{1}{T}$, we have straight lines having

$$\text{Slope} = \frac{\sigma_0^2 S}{Z}$$

$$\text{Intercept} = \frac{\sigma_0^2}{Z}$$

Now we can write

$$\frac{\text{Slope}}{\text{Intercept}} = \frac{\sigma_0^2 S / Z}{\sigma_0^2 / Z} = S = \frac{\phi_0}{1.27\,K} \qquad \dots (7)$$

The graph is plotted between $\frac{T^{1/2}}{\eta}$ corresponding to observed value of η [6] and $\frac{1}{T}$, we obtain the straight lines. The least square equations fit can be written as

Ar :

$$\eta = T^{1/2}\{5.1729204 \times 10^4 + 7.361422 \times 10^6\}^{-1} \; (100 \le T \le 2000\,K) \qquad \dots (8)$$

Ne :

$$\eta = T^{1/2}\{4.213432\times10^4 + 4.0215022\times10^6\}^{-1}\,(200 \le T \le 1200\,K) \qquad \dots (9)$$

He:

$$\eta = T^{1/2}\{6.2349098\times10^4 + 7.886096\times10^6\}^{-1}\,(280 \le T \le 1800\,K) \qquad \dots(10)$$

The correlation factors, intercepts and slopes of the straight lines have been given in table 1.

Table 1. Correlation factors, intercepts and slopes of the straight line curves [Eq. (8)-(10)] for Argon, Neon and Helium.

Gas	Correlation factor	Intercept	Slope	Temp Zone(K)
Argon	0.998	5.1728×10^4	7.361421×10^6	200-2000
Neon	0.995	4.2133×10^4	4.021502×10^5	200-1200
Helium	0.989	6.2348×10^4	7.888601×10^6	280-1800

From slopes and intercepts of the curves [c.f. eq. (8)–(10)], the values of Sutherland constant have been calculated and are given in table 2. Danilov and Obukhov [9] have reported the validity of Sutherland model in wide range of temperature for η of these gases. The present results are in conformity with that of Danilov and Obukhov.

Table 2. Calculated and Reported Values of Sutherland Constant

$$S = \frac{\sigma_0}{1.27\,k} \text{ [c.f. eq.(7)]}$$

Gas	Calculated Value of 'S'	Range of Temp. (K)	Reported Value of S	Range of Temp. (K)
Argon	141.49	100-2000	148.0 [Chapman (1970)	300-400
			153 $\pm$ 2 Danilov & Obukhov (1983)	— —
Neon	95.8	200-1200	64.1 [Chapman (1970)	300-400
			87 $\pm$ 1 Danilov & Obukhov (1983)	– –
Helium	126.7	280-1800	72.9 [Chapman (1970)	300-400
			118 $\pm$ 4 Danilov & Obukhov (1983)	

Conclusions

Since Sutherland constant, does not depend upon the temperature, is the function of ϕ_0, hence ϕ_0 is constant which suggest that σ_0 is constant and does not depend upon temperature. These results confirm the Sutherland assumption of hard molecules of diameter σ_0 .

References

1. Present, R.D., *Kinetic Theory of Gases* (McGraw Hill Book Co. Inc., New York (1958).
2. Seung-Kyo-Oh, *Journal of Thermodynamics*, **2013**, 29 (2013).
3. Chapman, S., *The Mathematical theory of non- uniform gases*, Cambridge (1970)
4. Hirschfelder, J.O., Curtiss, C.F. and Bird, R.B., *Molecular theory of Gases and Liquids*, John Wiley & Sons, Inc., New York (1954)
5. Sutherland, W., *Phil. Mag*., **36**, 507 (1893).
6. Touloukian, Y.S., Saxena, S.C. and Hestermans, P., *Thermophysical properties of Matter*, Vol. **II**, IFI/Plenum, New York-Washington (1974).
7. Jeans, J.H., *Dynamic theory of Gases*, Dover Pub. Inc., (1954).
8. Rio, F.D., *Mol. Phy*., **76**, 29 (1992).
9. Danilov, A.V and Obukhov, A.M., *Solv. Phys. Doki, (U.S.A.)*, **28**, 173 (1983).
10. Mandal, J.N and Thakur, M.L., *Indian Journal Pure & Appl. Phys*., **36**, 516 (1998).
11. Mandal, J.N., *Concepts of Solid State Physics*, Pragati Prakashan, Meerut (2011).

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