EFFECT OF THERMAL DIFFUSION ON MHD FREE CONVECTION AND MASS TRANSFER FLOW OF KUVSHINSHKI FLUID THROUGH POROUS MEDIUM WITH HEAT SOURCE ACROSS MOVING PLATE

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The aim of present investigation is to study the effect of thermal diffusion on unsteady two-dimensional free convection and mass transfer flow of a Kuvshinshki fluid through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat and mass flux across moving plate in the presence of a uniform magnetic field. The effects of G_r (Grashof Number), Gm (Modified Grashof Number) and A (Thermal diffusion parameter) on the velocity, temperature and skin friction are discussed with the help of tables and graphs. It is concluded that the velocity increases with the increase in G_r, G_m and A.

INTRODUCTION

The effect of variable permeability on combined free and forced convection in porous media was studied by Chandrasekhara and Namboodiri [4]. Later on mixed convection in porous media adjacent to a vertical uniform heat flux surface was studied by Joshi and Gebhart [6]. Heat and mass transfer in a porous medium was discussed by Bejan and Khair [3]. The above problem was studied in presence of Buoyancy effect by Trevisan and Bejan [11]. Lai and Kulacki [8] studied the effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. The study two dimensional flow through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in presence of free convection current was studied by Sharma [10]. Convection in a porous medium with inclined temperature gradient was investigated by Nield [9]. The problem of mixed convection along an isothermal vertical plate in porous medium with injection and suction was studied by Hooper et al [5]. Acharya, Das and Singh [1] have discussed magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Varshney and Kumar [12] have studied the unsteady effect on MHD free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Recently, Varshney and Dwivedi [13] have discussed unsteady effect on MHD free convection and mass transfer flow of Kuvshinshki fluid through porous medium with constant suction and constant heat and mass flux. Varshney and Sharma [14] have analysed effect of heat source on MHD free convection and mass transfer flow of Kuvshinshki fluid through porous medium with constant heat and mass flux across moving plate.

In present study, we consider the problem [14] with thermal diffusion.

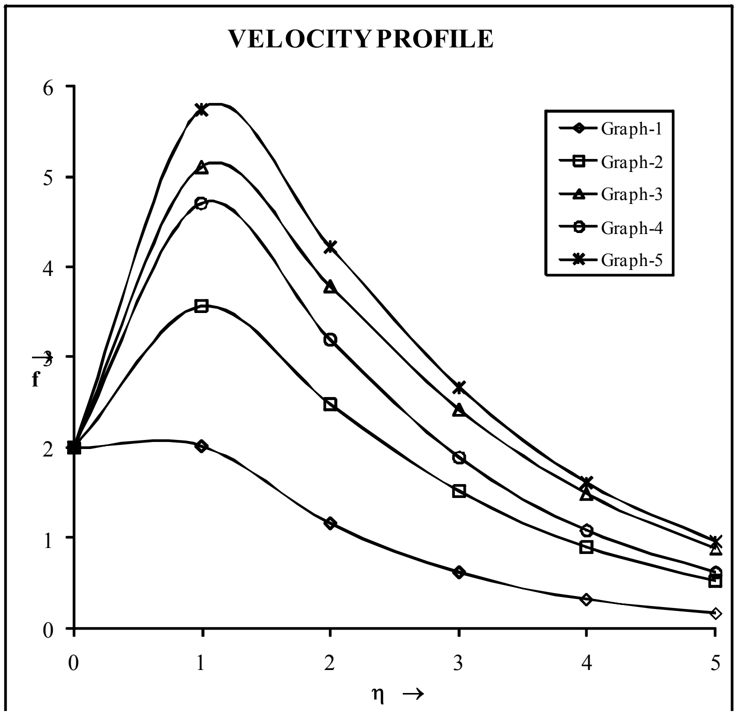

Fig. 1

FORMULATION OF THE PROBLEM

We consider unsteady two dimensional motion of Kuvshinshki fluid through a porous medium occupying semi-infinite region of space bounded by a vertical infinite surface under the action of uniform magnetic field applied normal to the direction of flow. The effect of induced magnetic field is neglected. The Reynolds number is assumed to be small. Further magnetic field is not strong enough to cause Joule heating. Hence, the term due to electrical dissipation is neglected in energy equation (3). The *X*-axis is taken along the surface in the upward direction and *Y*-axis is taken normal to it. The fluid properties are assumed constant except for the influence of density in the body force term. As the bounding surface is infinite in length, all the variables are functions of *Y*. Hence, by the usual boundary layer approximation the basic equations for unsteady flow through porous medium with heat source and moving plate are :

$$\frac{\partial F}{\partial Y} = 0 \qquad \dots (1)$$

$$(1+\lambda_0)\frac{\partial}{\partial t_0}.\frac{\partial U}{\partial t_0} + V\frac{\partial U}{\partial Y} = \nu\frac{\partial^2 U}{\partial Y^2} + g\,\beta\,(T - T_\infty) + g\beta'\,(C - C_\infty)$$

$$-\left(\sigma\frac{B_0^2}{\rho} + \frac{\nu}{K_0}\right)\left(1 + \lambda_0\frac{\partial}{\partial t_0}\right)U \qquad \dots (2)$$

$$\frac{\partial T}{\partial t_0}+V\frac{\partial T}{\partial Y}=\frac{\lambda}{\rho C_p}\cdot\frac{\partial^2 T}{\partial Y^2}+\frac{S^*}{\rho C_p}(T-T_\infty) \quad \ldots (3)$$

$$\frac{\partial C}{\partial t_0}+V\frac{\partial C}{\partial Y}=D\frac{\partial^2 C}{\partial Y^2} \quad \ldots (4)$$

where U and V are the corresponding velocity components along and perpendicular to the surface, ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, β' is the coefficient of concentration expansion, ν is the kinematic viscosity, T_∞ is the temperature of the fluid in the free stream, C_∞ is the concentration at infinite, σ is the electric conductivity, B_o is the magnetic induction, K_o is porosity parameter, λ is the thermal conductivity, D is the concentration diffusivity, C_p is the specific heat at constant pressure, λ_o is the coefficient of viscoelastic, S^* is the coefficient of heat source.

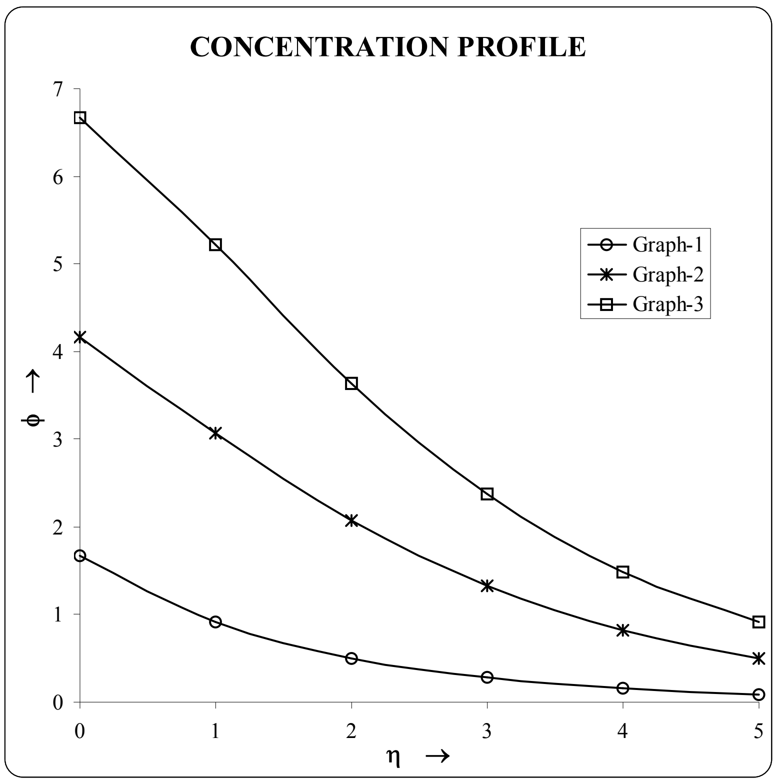

Fig. 2

METHOD OF SOLUTION

The equation of continuity (1) gives

$$V = \text{constant} = -V_o \quad \ldots (5)$$

where $V_o > 0$ corresponds to steady suction velocity at the surface.

In view of equation (5), Equations (2), (3) and (4) can be written as

$$\left(1+\lambda_0 \frac{\partial}{\partial t_0}\right)\frac{\partial U}{\partial t_0} - V_0 \frac{\partial U}{\partial Y} = \nu \frac{\partial^2 U}{\partial Y^2} + g\beta (T - T_\infty) + g\beta' (C - C_\infty)$$

$$- \left(\sigma \frac{B_0^2}{\rho} + \frac{\nu}{K_0}\right)\left(1+\lambda_0 \frac{\partial}{\partial t_0}\right) U \qquad \ldots (6)$$

$$\frac{\partial T}{\partial t_0} - V \frac{\partial T}{\partial Y} = \frac{\lambda}{\rho C_p} . \frac{\partial^2 T}{\partial Y^2} + \frac{S^*}{\rho C_p}(T - T_\infty) \qquad \ldots (7)$$

$$\frac{\partial C}{\partial t_0} - V_0 \frac{\partial C}{\partial Y} = D \frac{\partial^2 C}{\partial Y^2} + D_1 \frac{\partial^2 T}{\partial Y^2} \qquad \ldots (8)$$

The boundary conditions of the problem are

$$\begin{aligned} &U = U_w, \frac{dT}{dY} = -\frac{q}{\lambda}, \frac{dC}{dY} = -\frac{m}{D} \text{ at } Y = 0, t_0 = 0 \\ &U \to 0, \quad T = T_\infty \qquad \text{as } Y \to \infty, \ t_0 > 0 \end{aligned} \qquad \ldots(9)$$

On introducing the following non dimensional quantities into equations (6), (7) and (8).

$$f(\eta) = \frac{U}{V_0} \text{ (Velocity)}, \quad \eta = \frac{V_0 Y}{\nu} \text{ (Distance)}, \quad \theta = \frac{(T - T_\infty) V_0 \lambda}{q\nu}$$

$$Q = \frac{U_w}{V_0}, \quad P_r = \frac{\mu C_p}{\lambda} \text{ (Prandtl no.)}, \quad S_e = \frac{\nu}{D} \text{ (Schimdt no.)}$$

$$\phi = \frac{(C - C_\infty) V_0 D}{m\nu}, \quad t = \frac{t_0 V_0^2}{\nu}, \quad \alpha = \frac{V_0^2 K_0}{\nu^2} \text{ (Porosity Parameter)}$$

$$M = \frac{\sigma B_0^2 \nu}{\rho V_0^2} \text{ (Magnetic number)},$$

$$G_r = g\beta = \frac{q\nu^2}{V_0^4 \lambda} \text{ (Grashof number for heat transfer)},$$

$$G_m = g\beta' \frac{q\nu^2}{V_0^4 \lambda} \text{ (Grashof number for mass transfer)},$$

$$S = \frac{S^* \nu^2}{V_0^2 \lambda} \text{ (Heat source parameter)},$$

$$A = \frac{q D_1}{\lambda m} \text{ (Thermal diffusion parameter)}$$

where 'q' is the heat flux per unit area and 'm' is the mass flux per unit area.

$$-[1+\lambda_1(M+\alpha^{-1})]\frac{\partial f}{\partial t}+\frac{\partial^2 f}{\partial \eta^2}+\frac{\partial f}{\partial \eta}-\lambda_1\frac{\partial^2 f}{\partial t^2}-f(\alpha^{-1}+M)=-G_r\theta-G_m\phi \qquad \dots (10)$$

$$-P_r\frac{\partial \theta}{\partial t}+\frac{\partial^2\theta}{\partial \eta^2}+P_r\frac{\partial \theta}{\partial \eta}+SP_r\theta=0 \qquad \dots (11)$$

$$-S_c\frac{\partial \phi}{\partial t}+\frac{\partial^2\phi}{\partial \eta^2}+S_c\frac{\partial \phi}{\partial \eta}+AS_c\frac{\partial^2\theta}{\partial \eta^2}=0 \qquad \dots (12)$$

where viscoelastic parameter is $\lambda_1=\frac{\lambda_0 V_0^2}{\nu}$.

The corresponding boundary conditions become

$$\eta=0,\quad f=Q,\quad \theta'=-1,\ \phi'=-1$$
$$\eta\to\infty,\quad f\to 0,\quad \theta\to 0,\ \phi\to 0 \qquad \dots (13)$$

Following Mitra (1980), we assume the solution of

$$f(\eta,t)=f_0(\eta)\,e^{-nt}$$
$$\theta(\eta,1)=\theta_0(\eta)\,e^{-nt} \qquad \dots (14)$$
$$\phi(\eta,t)=\phi_0(\eta)\,e^{-nt}$$

Substituting equation (14) into equations (10), (11) and (12), we find

$$f_0''+f_0'-(\alpha^{-1}+M-n)(1-n\lambda_1)f_0=-G_r\theta_0-G_m\phi_0 \qquad \dots (15)$$

$$\theta_0''+P_r\theta_0'+P_r(n+S)\theta_0=0 \qquad \dots (16)$$

$$\theta_0''+S_c\phi_0'+nS_c\phi_0=-AS_c\theta_0'' \qquad \dots (17)$$

with corresponding boundary conditions

$$f_0=Q,\quad \theta_0'=-1,\quad \phi_0'=-1 \quad \text{at} \quad \eta=0$$
$$f_0\to 0,\quad \theta_0\to 0,\quad \phi_0\to 0,\quad \text{as} \quad \eta\to\infty \qquad \dots (18)$$

Solving equations (15)-(17) under boundary conditions (18), we get

$$f_0=(Q+A_4G_r+A_r(1+A_2A_6)G_m-A_4A_2A_6G_m)\,e^{-A_1\eta}-A_4G_re^{-A_2\eta}$$
$$-A_5(1+A_2A_6)G_me^{-A_3\eta}+A_4A_6A_2G_me^{-A_2\eta} \qquad \dots (19)$$

$$\theta_0=\frac{1}{A_2}e^{-A_2\eta} \qquad \dots (20)$$

$$\phi_0=\frac{(1+A_2A_6)}{A_3}e^{-A_3\eta}-A_6e^{-A_2\eta} \qquad \dots (21)$$

where $$A_1=\frac{1+[1+4(1-n\lambda_1)(\alpha^{-1}+M-n)]^{1/2}}{2}$$

$$A_2 = \frac{P_r + [P_r^2 - 4P_r(n+S)]^{1/2}}{2}$$

$$A_3 = \frac{S_c + [S_c^2 - 4nS_c]^{1/2}}{2}$$

$$A_4 = \frac{1}{A_2[A_2^2 - A_2 - (\alpha^{-1} + M - n)(1 - n\lambda_1)]}$$

$$A_5 = \frac{1}{A_3[A_3^2 - A_3 - (\alpha^{-1} + M - n)(1 - n\lambda_1)]}$$

$$A_6 = \frac{A\,S_c A_2}{(A_2^2 - A_2 S_c + nS_c)}$$

Hence, the equations for f, θ and ϕ will be as follows

$$f = [(Q + A_4 G_r + A_5(1 + A_2 A_6)G_m - A_4 A_2 A_6 G_m)e^{-A_1\eta} - A_4 G_r e^{-A_2\eta}$$

$$- A_5(1 + A_2 A_6)G_m e^{-A_3\eta} + A_4 A_6 A_2 G_m e^{-A_2\eta}].e^{-nt} \quad \ldots (22)$$

$$\theta = \frac{1}{A_2} e^{-A_2\eta}.e^{-nt} \quad \ldots (23)$$

$$\phi = \left[\frac{(1 + A_2 A_6}{A_3} e^{-A_1\eta} - A_6 e^{-A_2\eta}\right].e^{-nt} \quad \ldots (24)$$

Results and discussion

Fluid Velocity Profiles of boundary layer flow are plotted in Fig. I to III for $P_r = 0.71$, $M = 1.0$, $\alpha = 1.0$, $S_c = 0.6$, $S = 2.0$, $n = 0.1$, $\lambda_1 = 2$ and different values of G_r (Grashof No.), G_m (Modified Grashof No.) and A (Thermal diffusion parameter).

	G_r	G_m	A
For Graph-1	5	2	0
For Graph-2	5	2	1.5
For Graph-3	5	2	3.0
For Graph-4	10	2	1.5
For Graph-5	5	4	1.5

From all the Graphs of Fig. 1 it is found that the velocity increases sharply till $\eta = 1.2$ after it velocity decreases sharply till $\eta = 3.5$ then after it velocity decreases continuously with the increase in η. It is also concluded that the velocity increases with the increase in G_r and G_m and A.

Concentration Profile is tabulated in Table 2 and plotted in Fig. 2 for different values of A. From this figure it is concluded that concentration decreases with the increase in η. It is also concluded that concentration increases with the increase in A.

Table 1. Values of velocity f at M = 1.0, α = 1.0, P_r = 0.71, S_c = 0.6, S = 2.0 and different values of G_r, G_m and A.

η	Graph-1	Graph-2	Graph-3	Graph-4	Graph-5
0	2.00000	2.00000	2.00000	2.00000	2.00000
1	2.02338	3.56335	5.10266	4.70163	5.74229
2	1.17076	2.47921	3.78701	3.19263	4.21863
3	0.61874	1.52384	2.42848	1.89535	2.67530
4	0.32065	0.90380	1.48665	1.08923	1.62298
5	0.16569	0.52747	0.88907	0.61900	0.96377

Table 2. Values of concentration ϕ at P_r = 0.71, S_c = 0.6, and different values of A.

η	Graph-1	Graph-2	Graph-3
0	1.66667	4.16660	6.66672
1	0.91469	3.06621	5.21790
2	0.50199	2.06603	3.63018
3	0.27550	1.32228	2.36915
4	0.15120	0.81832	1.48550
5	0.08298	0.49465	0.90635

Table 3. Values of skin friction τ at M = 1.0, α = 1.0, P_r = 0.71, S_c = 0.6, S = 2.0 and different values of G_r, G_m and A.

G_r	G_m	A	τ
5	2	0	2.20110
5	2	1.5	6.44605
5	2	3.0	10.68692
10	2	1.5	10.56155
5	4	1.5	12.72777

The values of skin friction are tabulated in Table 3 at the different values of G_r, G_m and A as taken for velocity profile. From this table it is concluded that skin friction increases with the increase in G_r and G_m and A.

Particular case

When $A = 0$, this problem reduces to the problem of Varshney and Sharma [14].

Conclusion

1. Velocity increases with the increase in A (Thermal diffusion parameter).
2. Skin friction increases with the increase in A (Thermal diffusion parameter).

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