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SOME RESULTS ON TOTALLY na-FEEBLY CONTINUOUS FUNCTIONS

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The notion of totally na-feebly continuous, totally *na*-feebly regular continuous functions are introduced and some properties and characterizations of such functions are being discussed.

KEYWORDS : feebly regular door space, totally *na*-feebly continuous functions, totally na-feebly regular continuous functions, feebly regular irresolute.

\PRELIMINARIES

During the last few years the study of generalized closed and feebly closed mappings has found considerable interest among general topologists. Feebly closed and generalized mappings suggest some new concepts which have been to be very useful in study of a topology. In fact S.N. Maheswari and P.C. Jain introduced the concepts of feebly open and feebly closed sets in a topological space (X, τ). Dalal [2] proved that the map $f : X \to Y$ is feebly continuous if every singleton set in X is feebly open. In 1991, Mahide Kucuk and Idris Zorlutuna introduced feebly normality and feebly regularity in "*S*-separable spaces, feebly continuous functions and feebly separation axioms", Ganit 11, (1-2), 19-24 MR95a : 54036 ZbI 818.54022. We recall that in the subset A of X, the closure of A is the intersection of all closed sets containing A and the interior of A is the union of all open sets contained in A, denoted by cl (A) and int (A) respectively. A is said to be *semi-open* if $A \subseteq$ cl int (A) and semi-closed if int cl $(A) \subseteq A$.

Definition 1.1 [5] : A subset A of a topological space (X, τ) is said to be *feebly open* (resp. *feebly closed*) if $A \subset s$ cl int (A) (resp. s int cl $(A) \subset A$). The feebly closure of A is the intersection of all feebly closed set containing A and is denoted by f cl A.

Remark 1.2 : Let X and Y be the set of real numbers with usual topology, let the mapping $f : X \to Y$ be defined as follows $f(x) = x$ if $x \neq 0$ and $x \neq 1, f(0) = 1$ and $f(1) = 0$. Then f is one-one. Recall that a function $f : X \to Y$ is continuous if the inverse image of every open set in Y is open in X.

Definition 1.3 : A map $f : X \to Y$ is said to be

(i) *Feebly closed* (resp. *feebly open*) if the image of each closed set in X (resp. open set) is feebly closed set in Y (resp. feebly open in Y).

(ii) *Feebly continuous* if $f^{-1}(V)$ is feebly open in X for each open set V of Y.

Remark 1.4 [1]: In a topological space,

(i) Every open set is feebly open

(ii) Every closed set is feebly closed

Definition 1.5 [1]: A topological space (X, τ) is called

(i) *Feebly-*T_1 if there are feebly open sets U and V, x and y are the distinct points such that $x \in U, x \notin V$ and $y \in V, y \notin U$.

(ii) Feebly-T_2 (*Feebly Hausdorff*) if for any pair of distinct points x, y of X, there exists disjoint feebly open sets U and V such that $x \in U$ and $y \in V$.

(iii) Feebly-regular if for all $x \in X$ and for all closed set A containing x there exists feebly open set H such that $x \in H \subset f\,\mathrm{cl}\,H \subset A$.

(iv) Feebly-normal if for all disjoint closed sets H_1, H_2 in X, there exists feebly open sets U_1, U_2 in X such that $H_1 \subset U_1$, $H_2 \subset U_2$ and $U_1 \cap U_2 = \varphi$.

Definition 2.1: The topological space X is a F. reg. door space if and only if every subset of X is either F. reg. open or F. reg. closed.

Definition 2.2: A F. reg. door space (X, τ) is said to be feebly regular door symmetrical space (briefly F. reg. door symmetrical space) if for x and y in X,

$$x \in \text{F. reg. cl } \{y\} \Rightarrow y \in \text{F. reg. cl } \{x\}.$$

Theorem 2.3: Let (X, τ) be a F. reg. door symmetrical space and Y is a F. reg. door space of X. Then Y is F. reg. door space with respective topology τ in Y.

Proof : Let $S \subseteq Y$. Then $S \subseteq X$.

So S is either F. reg. open in X or F. reg. closed in X.

Hence $S \cap Y$ is either F. reg. open in Y or F. reg. closed in Y. But $S \cap Y = S$.

Then S is either F. reg. open in Y or F. reg. closed in Y.

Thus (Y, τ_Y) is a F. reg. door symmetrical space.

Theorem 2.4: The property of being a F. reg. door symmetrical space is a topological property.

Proof : Let X be a F. reg. door symmetrical space and let a function f from the topological space X to Y be an homeomorphism.

Let $S \subseteq Y$, consider $f^{-1}(S) \subseteq X$, since X is a F. reg. door symmetrical space.

Then $f^{-1}(S)$ is either F. reg. open or F. reg. closed in X.

Now $f(f^{-1}(S)) = S$.

Then S is either F. reg. open or F. reg. closed in Y. Thus Y is F. reg. door symmetrical space.

Theorem 2.5 : Let (X, τ) be a F. reg. door symmetrical space, let $Y \subseteq X$ be a F. reg. clopen subset of X then (Y, τ_Y) is also a F. reg. door symmetrical space.

Proof : Let $M \subseteq Y$ be a subset of Y. Now $M \subseteq X$. But X is a F. reg. door symmetrical space.

Then M is either F. reg. open or F. reg. closed in X.

Since Y is either F. reg. open and F. reg. closed, M is either F. reg. open or F. reg. closed in Y.

Then Y is F. reg. door symmetrical space.

Definition 2.6 : A function f from the F. reg. door symmetrical topological space X to Y is said to be totally *na*-F. reg. continuous if the inverse image of every F. reg. open set in Y is δ-clopen in X.

Theorem 2.7 : A function $f : X \to Y$ is a totally na-F.reg.continuous function if and only if the inverse image of every F. reg. closed subset of Y is δ-clopen in X.

Proof : Let F be any F. reg. closed in Y.

Then $Y - F$ is F. reg. open set in Y.

By definition 2.6 $f^{-1}(Y\text{-}F)$ is δ-clopen in X.

That is $X - f^{-1}(F)$ is δ-clopen in X, this implies $f^{-1}(F)$ is δ-clopen in X.

On the other hand, if V is F. reg. open in Y, then $Y–V$ is F. reg. closed in Y, by hypothesis, $f^{-1}(Y - V) = X - f^{-1}(V)$ is δ-clopen in X, which implies $f^{-1}(V)$ is δ-clopen in X.

Thus inverse image of every F. reg. open set in Y is δ-clopen in X.

Therefore f is totally *na*-F. reg. continuous function.

Remark 2.8 : (i) Every F. reg. open set is feebly open.

(ii) δ-clopen set is δ-open and δ-closed.

(iii) δ-clopen = regular clopen.

Theorem 2.9 : Every totally *na*-F. reg. continuous is a *na*-continuous.

Proof : Let X and Y be topological space.

Suppose $f : X \to Y$ is totally *na*-F. reg. continuous and

U is any F. reg. open subset of Y.

The function $f : X \to Y$ is totally *na*-F. reg. continuous, it follows $f^{-1}(U)$ is δ-clopen in X, by remark 2.8, hence $f^{-1}(U)$ is δ-open in X.

Thus inverse image of every F. reg. open set in Y is δ-open in X. Therefore the function f is *na*-continuous.

Definition 2.10 : A function f from X to Y is said to be strongly totally *na*-F. reg. continuous if the inverse image of every F. reg. open set in Y is regular-clopen in X.

Theorem 2.11 : Every strongly totally *na*-F. reg. continuous function is totally *na*-F. reg. continuous and vice versa.

Proof : Suppose a function f from X to Y is strongly totally *na*-F. reg. continuous and let S be any F. reg. open set in Y, by definition 2.10, $f^{-1}(S)$ is regular-clopen in X. Thus the inverse image of each F. reg. open set in Y is δ-clopen in X. Therefore f is totally *na*-F. reg. continuous.

Theorem 2.12 : A function f from X to Y is totally *na*-F. reg. continuous if and only if for each $p \in X$ and each F. reg. open set O in Y with $f(p) \in O$, there is a δ-clopen set E in X such that $p \in E$ and $f(E) \subset O$.

Proof : Suppose a function f from X to Y is a totally *na*-F. reg. continuous function where X and Y are topological spaces and F. reg. door symmetrical space. Let O be any F. reg. open set in Y containing $f(p)$ so that $p \in f^{-1}(O)$. Since f is totally *na*-F. reg. continuous, $f^{-1}(O)$ is δ-clopen in X.

Let $E = f^{-1}(O)$. Then E is δ-clopen set in X and $p \in E$.

Also $f(E) = f(f^{-1}(O)) \subset O$, this implies $f(E) \subset O$.

On the other hand let O be F. reg. open in Y, let $p \in f^{-1}(O)$ be arbitrary, this implies $f(p) \in O$, therefore by theorem 2.11, there is a δ-clopen set $f(Mp) \subset X$ containing p such that $f(Mp) \subset O$, which implies $Mp \subset f^{-1}(O)$.

We have $p \in Mp \subset f^{-1}(O)$, implies $f^{-1}(O)$ is δ-clopen neighbourhood of p, since p is arbitrary, it implies $f^{-1}(O)$ is δ-clopen neighbourhood of its points, hence it is δ-clopen set in X.

Therefore f is totally *na*-F. reg. continuous.

Remark 2.13 : (X, τ) is F. reg. T_1 space if and only if singleton sets are F. reg. closed sets.

Theorem 2.14 : very totally *na*-F. reg. continuous function into a F. reg. T_1 space is strongly totally *na*-F. reg. continuous function.

Proof : Suppose a function f from X to Y is a totally *na*-F. reg. continuous function in a F. reg. T_1 space. Singletons are F. reg. closed sets by remark 2.13. Hence $f^{-1}(B)$ is δ-clopen in X for every subset B of Y.

By remark 2.8, $f^{-1}(B)$ is regular clopen in X.

Therefore f is strongly totally *na*-F. reg. continuous function.

Theorem 2.15 : A function $f : X \to Y$ is totally *na*-F. reg. continuous and P is δ-clopen subset of X, then the restriction $f \setminus P : X \to Y$ is totally *na*-F. reg. continuous.

Proof : Consider the function $f \setminus P : P \to Y$ and O be any F. reg. open set in Y. Since f is totally *na*-F. reg. continuous, $f^{-1}(O)$ is δ-clopen subset of X.

Since P is δ-clopen subset of X and $(f \setminus P)^{-1}(O) = P \cap f^{-1}(O)$ is δ-clopen in P, it follows $(f \setminus P)^{-1}(O)$ is δ-clopen in P. Hence $f \setminus P$ is totally *na*-F. reg. continuous.

Definition 2.16 : A function f from X to Y is said to be F. reg. irresolute if the inverse image of every F. reg. open set in Y is F. reg. open in X.

Theorem 2.17 : If the functions f and g from X to Y and from Y to Z are totally *na*-F. reg. continuous and F. reg. irresolute respectively then the function $g \circ f$ from X to Z is totally *na*-F. reg. continuous.

Proof : Let the functions f and g from X to Y and from Y to Z are totally *na*- F. reg. continuous and F. reg. irresolute respectively.

Let O be F. reg. open in Z.

Since g is F. reg. irresolute, $g^{-1}(O)$ is F. reg. open in Y.

Now since f is totally *na*-F. reg. continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is δ-clopen in X. Hence $g \circ f : X \to Z$ is totally *na*-F. reg. continuous.

References

1. Ali, K., On semi separations axioms, *M.Sc. Thesis*, Al. Mastaxiriyah Univ. (2003).
2. Dalal Ibraheem, Ressan, Some results of feebly open and feebly closed mappings, *Baghdad Science Journal*, Vol. **6(4)**, (2009).
3. Devi, R., Maki, H. and Balachandran, K., Semi-generalized closed maps and generalized semi closed maps, *Mem. Fac. Sci. Kochi Univ. (Math.)*, **14**, 41–54 (1993).
4. Dorsett, C., S-separable spaces, feebly continuous functions and feebly separation axioms, *Ganit*, **11(1-2)**, 19–24 MR.95a:54036zb1818.54022 (1991).

5. Maheswari, S.N. and Jain, U., Note on some application of feebly open sets, *Madhya Bharati Jun. Saugar*, (1978-1979).
6. Dhana Balan, A.P. and Buvaneswari, R., On *m*-Irresolute and *m*-Totally clopen continuous Functions, *Innovative Journal, Asian Journal of Current Engineering and Maths*, **2(6)**, 357-358 (2013).
7. Dhana Balan, A.P. and Buvaneswari, R., Totally Feebly Continous Functions, International conference on Mathematical Science organized by School of Mathematics, Madurai Kamaraj University, *Madurai and Internation Multidisciplinary Research Foundation*, (2014).
8. Dhana Balan, A.P. and Buvaneswari, R., On Almost Feebly Totally Continuous Functions in Topological Spaces, *International Journal of Computer Science and Information Technology Research*, Vol. **3**, Issue **1**, 274-279 (2015).

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