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A STUDY ON NEUTROSOPHIC SEMI-GENERALIZED CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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The purpose of this paper is to introduce and study the concepts of neutrosophic semi-generalized closed sets and neutrosophic semi-generalized open sets in neutrosophic topological space.

KEYWORDS : Neutrosophic semi-generalized closed set and neutrosophic semi-generalized open set.

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Introduction

Fuzzy set was proposed by Zadeh [18] in 1965 which as a framework to encounter uncertainty, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. By adding the degree of non-membership to Fuzzy set, Atanassov was proposed intuitionistic fuzzy set (IFS) in 1983 [1] which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. Later on fuzzy topology was introduced by Chang in 1967. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. In last few years various concepts infuzzy sets were extended to intuitionistic fuzzy sets. In 1997, Coker introduced the concept of intuitionistic fuzzy topological space.

The neutrosophic set was introduced by Smarandache [6] and explained,neutrosophic set is a generalization of intuitionistic fuzzy set. In 2012, Salama, Alblowi [12], introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of Intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]0^-, 1^+[$ is nonstandard unitinterval.

The concepts of neutrosophic semi-open sets, neutrosophic semi-closed sets, neutrosophic semi-interior and neutrosophic semi-closure in neutrosophic topological spaces were introduced by Iswarya *et. al*. [8] in 2016.Also intuitionistic fuzzy semi-generalized closed sets and its applications were introduced by Santhi *et. al*. [16] in 2010 and semi $^\#$ generalized closed sets in topological spaces were introduced by Saranya *et. al*. [15] in 2016.In this paper, we are extending the above concepts to neutrosophic topological spaces. We study some of the basic properties of neutrosophic semi-generalized closed sets and neutrosophicsemi-generalized open sets in neutrosophic topological spaces with examples.

PRELIMINARIES

In this section, we give the basic definitions for neutrosophic sets and its operations.

Definition 1.1. [8] Let (X, τ) be a neutrosophic topological space. A neutrosophic subset A of the neutrosophictopological space X is said to be neutrosophic semi-open if $A \subseteq NCl\ (N\ Int\ (A))$ and neutrosophic semi-closed if $N\ Int\ (NCl\ (A)) \subseteq A$.

Definition 1.2. [14] Let (X, τ) be a neutrosophictopological space. A neutrosophic set A in (X, τ) is said to be neutrosophic closed (N-closed for short) if NCl $(A) \subseteq G$ whenever $A \subseteq G$ and G is neutrosophic open.

Definition 1.3. [17] Let (X, τ_N) be a neutrosophic topological space. Then a neutrosophic set A is called Neutrosophic ω closed set (N_ω-closed set for short) if $NCl\ (A) \subseteq G$ whenever $A \subseteq G$ and G is N_s-open set.

NEUTROSOPHIC SEMI-GENERALIZED CLOSED SETS

In this section, we introduce neutrosophic semi-generalized closed sets and studied some of their basic properties.

Definition 2.1. Aneutrosophicset A of aneutrosophic topological space (X, τ) is called aneutrosophicsemi-generalized closed set ($NSGC$ set for short)if $NSCl\ (A) \subseteq U$, whenever $A \subseteq U$ and U is a neutrosophic semi-open set.

Example 2.2. Let $X = \{a\}$ with $\tau = \{0_N, A, B, C, D, 1_N\}$ where $A = \langle(0.2, 0.5, 0.6)\rangle$, $B = \langle(0.5, 0.1, 0.4)\rangle$, $C = \langle(0.5, 0.5, 0.4)\rangle$ and $D = \langle(0.2, 0.1, 0.6)\rangle$. Let us take $S = \langle(0.3, 0.1, 0.6)\rangle$. Then $NCl\ (N\ Int\ (S)) = C\ (C)$. Therefore $S \subseteq C\ (C)$. Hence S is neutrosophic semi-open set. So we have $\tau_{NSO} = 0_N, A, B, C, D, S, C\ (A), C\ (C), 1_N$. Let $S \subseteq C$ and C is neutrosophic semi-open set. Then $NSCl\ (S) = C\ (C) \subseteq C$. Therefore $NSCl\ (S) \subseteq C$, $S \subseteq C$ and C is neutrosophic semi-open set. Hence S is neutrosophicsemi-generalized closed set.

Theorem 2.3. Every neutrosophic closed set in neutrosophic topological space (X, τ) is aneutrosophic semi-generalized closed set.

Proof : Let A be a neutrosophic closed set in neutrosophic topological space X. Then by Definition 1.16 (b) [8], $A = NCl\ (A)$. Again by Proposition 6.4 [8], $A \subseteq NS\ Cl\ (A) \subseteq NCl\ (A)$. Therefore $A = NSCl\ (A)$. Hence A is a neutrosophic semi-generalized closed set in X.

The converse of the above theorem is not true as shown by the following example.

Example 2.4. From Example 2.2, S is neutrosophic semi-generalized closed set but not neutrosophic closed set.

Theorem 2.5. Every neutrosophic semi-closed set in neutrosophic topological space (X, τ) is aneutrosophic semi-generalized closed set.

Proof : Obvious.

The converse of the above theorem is not true as shown by the following example.

Example 2.6. From Example 2.2, S is neutrosophic semi-generalized closed set but not neutrosophic semi-closed set.

Theorem 2.7. If A and B are neutrosophic semi-generalized closed sets, then $A \cap B$ is also a neutrosophicsemi-generalized closed set.

Proof : Let A and B are neutrosophic semi-generalized closed sets. If $A \cap B \subseteq U$ and U is neutrosophic semi-open set, then $A \subseteq U$ and $B \subseteq U$. Since A and B are neutrosophic semi-generalized closed sets, $NSCl\ (A) \subseteq U$ and $NSCl\ (B) \subseteq U$. Hence $NSCl\ (A) \cap NSCl\ (B) \subseteq U$. By Proposition 6.5 (ii) [8], $NSCl\ (A \cap B) \subseteq NSCl\ (A) \cap NSCl\ (B) \subseteq U$. This implies that $NSCl\ (A \cap B) \subseteq U$. Therefore $NSCl\ (A \cap B) \subseteq U$, $A \cap B \subseteq U$ and U is neutrosophic semi-open set. Thus $A \cap B$ is neutrosophic semi-generalized closed set.

Remark 2.8. Union of any two neutrosophic semi-generalized closed sets in (X, τ) need not be a neutrosophic semi-generalized closed set, as seen from the following example.

Example 2.9. Let $X = \{a\}$ with $\tau = \{0_N, A, B, C, D, E, F, G, H, 1_N\}$ where $A = \langle(0.6, 0.3, 0.7)\rangle$, $B = \langle(0.4, 0.9, 0.2)\rangle$, $C = \langle(0.5, 0.2, 0.8)\rangle$, $D = \langle(0.6, 0.9, 0.2)\rangle$, $E = \langle(0.5, 0.9, 0.2)\rangle$, $F = \langle(0.4, 0.3, 0.7)\rangle$, $G = \langle(0.4, 0.2, 0.8)\rangle$ and $H = \langle(0.5, 0.3, 0.7)\rangle$. Also $\tau_{NSO} =\ 0_N, A, B, C, D, E, F, G, H, C\ (A), 1_N$. Now we consider the two neutrosophic semi-generalized closed sets C and F. Their intersection G is neutrosophic semi-generalized closed set but their union H is not neutrosophicsemi-generalized closed set.

Theorem 2.10. Let A and B be neutrosophic semi-generalized closed sets in (X, τ) such that $NCl\ (A) = NSCl\ (A)$ and $NCl\ (B) = NSCl\ (B)$. Then $A \cup B$ is neutrosophic semi-generalized closed set in X.

Proof : Let $A \cup B \subseteq U$, where U is neutrosophic semi-open set. Then $A \subseteq U$, $B \subseteq U$. Since A and B are neutrosophic semi-generalized closed sets, $NSCl\ (A) \subseteq$ U and $NSCl\ (B) \subseteq U$. Now by Proposition 1.18 (h) [8], $NCl\ (A \cup B\ ) = NCl\ (A) \cup NCl\ (B) = NSCl\ (A) \cup NSCl\ (B) \subseteq U$. Again by Proposition 6.4 [8], $NSCl\ (A \cup B) \subseteq NCl\ (A \cup B\ )$. We have $NSCl\ (A \cup B) \subseteq NCl\ (A \cup B) \subseteq U$. Therefore $NSCl\ (A \cup B) \subseteq U$, $A \cup B \subseteq U$ and U is neutrosophic semi-open set. Hence $A \cup B$ is neutrosophic semi-generalized closed set.

Theorem 2.11. Suppose that $B \subseteq A \subseteq X$, B is a neutrosophic semi-generalized closed set relative to A and that A is neutrosophic semi-generalized closed set in X. Then B is neutrosophic semi-generalized closed set in X.

Proof : Let $B \subseteq G$, where G is neutrosophic semi-open set in X. Then $B \subseteq A \cap G$ and $A \cap G$ is neutrosophic semi-open set in A. But B is a neutrosophic semi-generalized closed setrelative to A. Hence $NSCl_A\ (B) \subseteq A \cap G$. Since $NSCl_A\ (B)) = A \cap NSCl\ (B)$, $A \cap NSCl\ (B)) \subseteq A \cap G$. This implies that $A \subseteq G \cup C\ (NSCl\ (B))$ and $G \cup C\ (NSCl\ (B))$ is a neutrosophic semi-open set in X. Since A is neutrosophic semi-generalized closed setin X, $NSCl\ (A) \subseteq G \cup C\ (NSCl\ (B))$. Hence $NSCl\ (B) \subseteq G \cup C\ (NSCl\ (B))$ and $NSCl\ (B) \subseteq G$. Therefore B is neutrosophic semi-generalized closed set relative to X.

Theorem 2.12. If A is aneutrosophic semi-generalized closed setin X and $A \subseteq B \subseteq NSCl\ (A)$, then B is a neutrosophic semi-generalized closed set in X.

Proof : Let U be a neutrosophic semi-generalized open set in X such that $B \subseteq U$. Since $A \subseteq B$, $A \subseteq U$. Again since A is aneutrosophic semi-generalized closed set, $NSCl\ (A) \subseteq U$. By hypothesis, $B \subseteq NSCl\ (A)$. By Proposition 6.3 (iii) [8], $NSCl\ (B) \subseteq NSCl\ (NSCl\ (A)) = NSCl\ (A)$. That is $NSCl\ (B) \subseteq NSCl\ (A)$. This implies that $NSCl\ (B) \subseteq U$. Hence B is a neutrosophic semi-generalized closed set in X.

Theorem 2.13. Let $A \subseteq Y \subseteq X$ and A beneutrosophic semi-generalized closed set in X. Then A isneutrosophic semi-generalized closed relative to Y.

Proof : Let $A \subseteq Y \cap G$ where G is neutrosophic semi-open set in X. Then $A \subseteq G$ and hence $NSCl\ (A) \subseteq G$. This implies that $Y \cap NSCl\ (A) \subseteq Y \cap G$. Thus A is neutrosophic semi-generalized closed relative to Y.

Theorem 2.14. If A is aneutrosophic semi-open set and neutrosophic semi-generalized closed set in (X, τ), then A is aneutrosophic semi-closed set in X.

Proof : Given that A is aneutrosophic semi-open set and neutrosophic semi-generalized closed set in X. Therefore $NSCl\ (A) \subseteq A$. By Proposition 6.3 (i) [8], $A \subseteq NSCl\ (A)$. This implies that $NSCl\ (A) = A$. Hence A is aneutrosophic semi-closed set in X.

Theorem 2.15. The concepts of neutrosophic semi-closed set and neutrosophic semi-open set coincide if and only if every neutrosophic subset of X is aneutrosophic semi-generalized closed set.

Proof : Let A be aneutrosophic subset of X such that $A \subseteq U$ where U is aneutrosophic semi-open set. Then U is a neutrosophic semi-closed set such that $NSCl\ (A) \subseteq NSCl\ (U) = U$. Hence $NSCl\ (A) \subseteq U$. Therefore A is aneutrosophic semi-generalized closed set. Conversely, assume that U is aneutrosophic semi-open set. By our hypothesis, every neutrosophic subset of X is aneutrosophic semi-generalized closed set. Therefore U is aneutrosophic semi-generalized closed set. Hence $NSCl\ (U) \subseteq U$. This implies that U is a neutrosophic closed set. Next let us assume that B is aneutrosophic semi-closed set. Then $C\ (B)$ is aneutrosophic semi-open set. Byour hypothesis, $C\ (B)$ is aneutrosophic semi-generalized closed set. Hence $NSCl\ (C\ (B)) \subseteq C\ (B)$. This implies that $C\ (B)$ is aneutrosophic semi-closed set. Therefore B is a neutrosophic semi-open set in X.

Theorem 2.16. Let A be aneutrosophic semi-generalized closed subset of (X, τ). Then $NSCl\ (A) - A$ does not contain any non-empty neutrosophic semi-closed set.

Proof : Assume that A is a neutrosophic semi-generalized closed set in X. Let B be a non-empty neutrosophic semi-closed set such that $B \subseteq NSCl\ (A) - A = NSCl\ (A) \cap \mathrm{C}\ (A)$. That is, $B \subseteq NSCl\ (A)$ and $B \subseteq C\ (A)$. Therefore $A \subseteq C\ (B)$. Since $C\ (B)$ is aneutrosophic semi-open set, $NSCl\ (A) \subseteq C\ (B) \Rightarrow B \subseteq C\ (NSCl\ (A))$. But $B \subseteq NSCl\ (A) - A$. Thus $B \subseteq (NSCl\ (A) - A) \cap C\ (NSCl\ (A)) \subseteq NSCl\ (A) \cap C\ (NSCl\ (A))$. That is, $B \subseteq \phi$. Therefore B is empty.

Theorem 2.17. A neutrosophicset A is aneutrosophic semi-generalized closed setin X if and only if $NSCl\ (A) - A$ contains no nonempty neutrosophic semi-closed set.

Proof : Suppose that A is neutrosophic semi-generalized closed set in X. Let S be a neutrosophic semi-closed subset of $NSCl\ (A) - A$. Then $A \subseteq C\ (S)$. Since A is neutrosophic semi-generalized closed set, we have $NSCl\ (A) \subseteq C\ (S)$. This implies that $S \subseteq C\ (NSCl\ (A))$. Hence $S \subseteq NSCl\ (A) \cap C\ (NSCl\ (A)) = \phi$. Therefore S is empty. Conversely, suppose that $NSCl\ (A) - A$ contains no nonempty neutrosophic semi-closed set. Let $A \subseteq G$ and that G be neutrosophic semi-open. If $NSCl\ (A) \nsubseteq G$, then $NSCl\ (A) \cap C\ (G)$ is a non-empty neutrosophic semi-closed subset of $NSCl\ (A) - A$. Hence A is neutrosophic semi-generalized closed set in X.

Corollary 2.18. Aneutrosophic semi-generalized closed set$A$ is aneutrosophic semi-closed set if and only if $NSCl\ (A) - A$ is a neutrosophic semi-closed set.

Proof : Let A be aneutrosophic semi-generalized closed set. If A is aneutrosophic semi-closed set, then by Theorem 2.16, $NSCl\ (A) - A = \phi$. Therefore $NSCl\ (A) - A$ is aneutrosophic semi-closed set. Conversely, assume that $NSCl\ (A) - A$is a neutrosophic semi-closed set. Then

A is aneutrosophic semi-generalized closed set. By Theorem 2.16, *NSCl* (*A*) – *A* = ϕ. Therefore *NSCl* (*A*) = *A*. Hence *A* is aneutrosophic semi-closed set.

NEUTROSOPHIC SEMI-GENERALIZED OPEN SETS

In this section, we introduce neutrosophic semi-generalized open sets and studied some of their basic properties.

Definition 3.1. A Neutrosophic set *A* in *X* is called neutrosophic semi-generalized open set (*NSGO* set for short) in *X* if *C* (*A*) is neutrosophic semi-generalized closed set in *X*.

That is, $U \subseteq NS\ Int\ (A)$, whenever $U \subseteq A$ and *U* is a neutrosophic semi-closed set.

The family of all *NSGC* set (resp, *NSGO* set) of a neutrosophic topological space (*X*, τ) will be denoted by *NSGC* (*X*) (resp. *NSGO* (*X*)).

Example 3.2. From Example 2.2, let us take *T* = ⟨(0.3, 0.5, 0.6)⟩. Then *T* and *C* (*T*) are neutrosophic semi-open sets. Also $\tau_{NSO} = 0_N$, *A*, *B*, *C*, *D*, *S*, *T*, *C* (*T*), *C* (*A*), *C* (*C*), 1_N. Let $T \subseteq C\ (D)$ and T is neutrosophic semi-closed set. Then *NS Int* (*C* (*D*)) = *C* (*A*). Also $T \subseteq C$ (*A*). Therefore $T \subseteq NS\ Int\ (C\ (D))$, $T \subseteq C\ (D)$ and *T* is neutrosophic semi-closed set. Hence *C* (*D*) is neutrosophic semi-generalized open set.

Theorem 3.3. Every neutrosophic open set in neutrosophic topological space (*X*, τ) is aneutrosophicsemi-generalized open set.

Proof : Let *A* be a neutrosophic open set in neutrosophic topological space *X*. Then by Definition 1.16 (a) [8], *A* = *N Int* (*A*). Again by Proposition 6.4 [8], $N\ Int\ (A) \subseteq NSInt\ (A) \subseteq A$. Therefore *A* = *NS Int* (*A*). Hence *A* is a neutrosophic semi-generalized open set in *X*.

The converse of the above theorem is not true as shown by the following example.

Example 3.4. From Example 3.2, *C* (*D*) is neutrosophic semi-generalized open set but not neutrosophic open set.

Theorem 3.5. Every neutrosophic semi-open set in neutrosophic topological space (*X*, τ) is aneutrosophic semi-generalized open set.

Proof : Obvious.

The converse of the above theorem is not true as shown by the following example.

Example 3.6. From Example 3.2, *C* (*D*) is neutrosophic semi-generalized open set but not neutrosophic semi-open set.

Theorem 3.7. If *A* and *B* are neutrosophic semi-generalized open sets, then $A \cup B$ is also a neutrosophic semi-generalized open set.

Proof : Let *A* and *B* are neutrosophic semi-generalized open sets. If $U \subseteq A \cup B$ and *U* be neutrosophic semi-closed set, then $U \subseteq A$ and $U \subseteq B$. Since *A* and B are neutrosophic semi-generalized open sets, $U \subseteq NS\ Int\ (A)$ and $U \subseteq NS\ Int\ (B)$. Hence $U \subseteq NS\ Int\ (A) \cup NS\ Int\ (B)$. By Theorem 5.3 (ii) [8], $NS\ Int\ (A \cup B) \supseteq NS\ Int\ (A) \cup NS\ Int\ (B) \supseteq U$. This implies that $U \subseteq NS\ Int\ (A \cup B)$. Therefore $U \subseteq NS\ Int\ (A \cup B)$, $U \subseteq A \cup B$ and *U* is neutrosophic semi-closed set. Thus $A \cup B$ is neutrosophic semi-generalized open set.

Remark 3.8. Intersection of any two neutrosophic semi-generalized open sets in (*X*, τ) need not be a neutrosophic semi-generalized open set, as seen from the following example.

Example 3. From Example 2.9, we consider the two neutrosophic semi-generalized open sets *C* (*C*) and *C* (*F*). Their union *C* (*G*) is neutrosophic semi-generalized open set but their intersection *C* (*H*) is not neutrosophic semi-generalized open set.

Theorem 3.10. Let *A* and *B* be neutrosophic semi-generalized open sets in (*X*, τ) such that *N Int* (*A*) = *NS Int* (*A*) and *N Int* (*B*) = *NS Int* (*B*). Then $A \cap B$ is neutrosophic semi-generalized open set in *X*.

Proof : Let $U \subseteq A \cap B$, where *U* is neutrosophic semi-closed set. Then $U \subseteq A$, $U \subseteq B$. Since *A* and *B* are neutrosophic semi-generalized open sets, U ⊆ *NS Int* (*A*) and *U* ⊆ *NS Int* (*B*). Now by Proposition 1.18 (g) [8], *N Int* ($A \cap B$) = *N Int* (*A*) ∩ *N Int* (*B*) = *NS Int* (*A*) ∩ *NS Int* (*B*) ⊇ *U*. Now by Proposition 6.4 [8], *N Int* ($A \cap B$) ⊆ *NS Int* ($A \cap B$). We have *U* ⊆ *N Int* ($A \cap B$) ⊆ *NS Int* ($A \cap B$). Therefore *U* ⊆ *NS Int* ($A \cap B$), $U \subseteq A \cap B$ and *U* is neutrosophic semi-closed set. Hence $A \cap B$ is neutrosophic semi-generalized open set.

Theorem 3.11. If *A* is a neutrosophic semi-generalized open set in *X* and if *NS Int* (*A*) ⊆ *B* ⊆ *A*, then *B* is a neutrosophic semi-generalized open set in *X*.

Proof : Let *A* be aneutrosophic semi-generalized open set in *X*. Since *NS Int* (*A*) ⊆ *B* ⊆ *A* and by Proposition 6.2 (i) [8], we have *C* (*A*) ⊆ *C* (*B*) ⊆ *C* (*NS Int* (*A*)) = *NSCl* (C (*A*)). Again since *C* (*A*) is a neutrosophic semi-generalized closed set and by Theorem 2.12, we have *C* (*B*) is a neutrosophic semi-generalized closed set in *X*. Hence *B* is a neutrosophic semi-generalized open set in *X*.

Theorem 3.12. Aneutrosophic set *A* of aneutrosophic topological space (*X*, τ) is aneutrosophic semi-generalized open set if and only if *B* ⊆ *NS Int* (*A*), where *B* is aneutrosophic semi-closed set and *B* ⊆ *A*.

Proof : Assume that *A* is aneutrosophic semi-generalized open set in *X*. Let *B* be aneutrosophic semi-closed setin *X* such that *B* ⊆ *A*. Then *C* (*B*) is aneutrosophic semi-open set in *X* such that *C* (*A*) ⊆ *C* (*B*). Since *C* (*A*) is a neutrosophic semi-generalized closed set, *NSCl* (*C* (*A*)) ⊆ *C* (*B*). By Proposition 6.2 (i) [8], *NSCl* (*C* (*A*)) = *C* (*NS Int* (*A*)). Therefore *C* (*NS Int* (*A*)) ⊆ *C* (*B*) implies that *B* ⊆ *NS Int* (*A*). Conversely, assume that *B* ⊆ *NS Int* (*A*), where *B* is aneutrosophic semi-closed set and *B* ⊆ *A*. Then *C* (*NS Int* (*A*)) ⊆ *C* (*B*), where *C* (*B*) is aneutrosophic semi-open set and *NSCl* (*C* (*A*)) ⊆ *C* (*B*). Therefore *C* (*A*) is aneutrosophic semi-generalized closed set. This implies that *A* is aneutrosophic semi-generalized open set.

Theorem 3.13. A neutrosophic set *A* is neutrosophic semi-generalized open set in *X* if and only if *G* = *X* whenever *G* is neutrosophic semi-open set and *NS Int* (*A*) ∪ *C* (*A*) ⊆ *G*.

Proof : Assume that *A* is a neutrosophic semi-generalized open set, *G* is a neutrosophic semi-open set and *NS Int* (*A*) ∪ *C* (*A*) ⊆ *G*. By Proposition 6.2 (i) [8], *C* (*G*) ⊆ *C* (*NS Int* (*A*)) ∩ *C* (*C* (*A*)) = *C* ((*NS Int* (*A*)) – *C* (*A*) = *NSCl* (*C* (*A*)) – *C* (*A*). Since *C* (*A*) is neutrosophic semi-generalized closed set and *C* (*G*) is neutrosophic semi-closed set and by Theorem 2.17, we have *C* (*G*) = ϕ. Therefore *G* = *X*. Conversely, suppose that *F* is neutrosophic semi-closed set and *F* ⊆ *A*. Then *NS Int* (*A*) ∪ *C* (*A*) ⊆ *NS Int* (*A*) ∪ *C* (*F*). This implies that *NS Int* (*A*) ∪ *C* (*F*) = *X* and hence *F* ⊆ *NS Int* (*A*). Therefore *A* is neutrosophic semi-generalized open set.

Theorem 3.14. A neutrosophic set *A* is neutrosophic semi-generalized closed set if and only if *NSCl* (*A*) – *A* is neutrosophic semi-generalized open set.

Proof : Suppose that *A* is neutrosophic semi-generalized closed set. Let *F* ⊆ *NSCl* (*A*) – *A* where *F* is neutrosophic semi-closed set. By Theorem 2.17, *F* = ϕ. Therefore *F* ⊆ *NS Int* (*NSCl* (*A*) –*A*). By Theorem 3.12, *NSCl* (*A*) – *A* is neutrosophic semi-generalized open set. Conversely, let $A \subseteq G$ where G is a neutrosophic semi-open set. Then $NSCl\ (A) \cap C\ (G) \subseteq NSCl\ (A) \cap C\ (A) = NSCl\ (A) - A$. Since $NSCl\ (A) \subseteq C\ (G)$ is neutrosophic semi-closed set and $NSCl\ (A) - A$ is neutrosophic semi-generalized open set. By Theorem 3.12, $NSCl\ (A) \cap C\ (G) \subseteq NS\ Int\ (NSCl\ (A) - A) = \phi$. Hence A is neutrosophic semi-generalized closed set.

Conclusion

In this paper, we studied the concepts of neutrosophic semi-generalized closed sets, neutrosophic semi-generalized open sets and their properties in neutrosophic topological spaces. In future, we extended this neutrosophic topology concepts by neutrosophic continuous, neutrosophic semi-continuous, neutrosophicpre-continuous, neutrosophic almost continuous and neutrosophic weakly continuous in neutrosophic topological spaces. Also we extended this neutrosophic concepts by nets, filters and borders.

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