Acta Ciencia Indica, Vol. XLIII M, No. 4, 247 (2017)

A GAME WITH NON ZERO TRIANGULAR NUMBERS

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RECEIVED : 29 December, 2017

The paper is focused on a row and column both dominance prime game with rown's Algorithm. A 6 × 6 game with non zero triangular numbers is constructed on dominance strategy. The Interactions between the Lower bounds and/or Upper bounds of this game at different Stages are computed. Few findings are established.

KEYWORDS : Game Theory, players, strategy, Pay-off matrix, optimal solution, Lower bound, Upper bound

AMS Classification : 91A05,91A18,91A43, 91A90.

INTRODUCTION

K.V.L.N. Acharyulu [2-6] applied Brown's algorithm on various types of game problems. Billy E.Gillett [1], Levin and Desjardins [7] invented new ways of approaching game theory problems. Mathematicians like Rapoport [11], Dresher [8], Raiffa [10], McKinsey [9] etc discussed and attempted various situations of OR with useful applications of game theory. Brown's algorithm is one of the useful methods to analize any type of game in a scientific manner.

BASIC FORMATION OF 6 × 6 GAME

The triangular game is framed with 6 rows and 6 columns with increasing triangular numbers according to player A and Player B. It involves six possible actions of A *i.e.* $A1$, $A2$, $A3$, $A4$, $A5$, $A6$ which will impact on the other six possible actions of player B *i.e.* $B1$, $B2$, $B3$, $B4$, $B5$, $B6$. It is assumed that the influence of Player B will be on each component of Player A.

The pay-off matrix of the game having the size 6 × 6 is given below.

$$\begin{bmatrix} 1 & 3 & 6 & 10 & 15 & 21 \\ 28 & 36 & 45 & 55 & 66 & 78 \\ 91 & 105 & 120 & 136 & 153 & 171 \\ 190 & 210 & 231 & 253 & 276 & 300 \\ 325 & 351 & 378 & 406 & 435 & 465 \\ 496 & 528 & 561 & 595 & 630 & 666 \end{bmatrix}$$

Material and methods

Brown's Algorithm:

Step 1: Player A chooses one of the possible actions (Ai_1) from $A1$-$A6$ to play, and Player B then plays with the possible action Bj_1 corresponding to the smallest element in the selected action Ai_1.

Step 2 : Player A then picks out the possible action (Ai_2) from $A1 - A6$ to play corresponding to the largest element in the possible action (Bj_1) selected by Player B in step 1.

Step 3 : Player B sums the actions of Player A who has played thus far, and plays with the possible action of Bj_2 corresponding to a smallest sum element.

Step 4 : Player A sums the actions of Player B who has played thus far, and plays the possible action (Ai_3) corresponding to a largest sum element. After the required iterations are computed,then go to step 5; otherwise, come back to step 3.

Step 5: Compute an Lower and Upper bound $\underline{\gamma}$ and $\overline{\gamma}$ respectively.

$$\overline{\gamma} = \frac{\text{Largest sum element from step 4}}{\text{Number of plays of the game thus far}} \quad \text{and} \quad \underline{\gamma} = \frac{\text{Smallest sum element from step 3}}{\text{Number of plays of the game thus far}}$$

Step 6 : Let Xi be the portion of the time Player A played row i with $i = 1, 2, ..., m$ and let Yi be the proportion of the time Player B played column j with $j = 1, 2, ..., n$. These strategies approximate the optimal mini max strategies. Upper and Lower bounds of the value of the game where $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$ are calculated in step 5. The Process completes.

Results

Brown's Algorithm is used on triangular game 500 iterations to obtain better accuracy in the results with the help of Java program. The influences of player B on each component of Player A are determined and listed from Table (1) to Table (10) at each iteration.

Table 1. Player *A* vs Player *B* at 50th Iteration

Player *A*			**Player *B***			
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**
50	24305	24332	24395	24494	24629	24800
1400	25875	25908	25977	26082	26223	26400
4550	27495	27534	27609	27720	27867	28050

9500	29165	29210	29291	29408	29561	29750
16250	30885	30936	31023	31146	31305	31500
24800	32655	32712	32805	32934	33099	33300

Table 2. Player *A* vs Player *B* at 100th Iteration

Player *A*			**Player *B***			
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**
100	100	100	100	100	100	100
2800	2800	2800	2800	2800	2800	2800
9100	9100	9100	9100	9100	9100	9100
19000	19000	19000	19000	19000	19000	19000
32500	32500	32500	32500	32500	32500	32500
49600	49600	49600	49600	49600	49600	49600

Table 3. Player *A* vs Player *B* at 150th Iteration

Player *A*			**Player *B***			
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**
150	73905	73932	73995	74094	74229	74400
4200	78675	78708	78777	78882	79023	79200
13650	83595	83634	83709	83820	83967	84150
28500	88665	88710	88791	88908	89061	89250
48750	93885	93936	94023	94146	94305	94500
74400	99255	99312	99405	99534	99699	99900

Table 4. Player *A* vs Player *B* at 200th Iteration

Player *A*			**Player *B***			
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**
200	98705	98732	98795	98894	99029	99200
5600	105075	105108	105177	105282	105423	105600
18200	111645	111684	111759	111870	112017	112200
38000	118415	118460	118541	118658	118811	119000
65000	125385	125436	125523	125646	125805	126000
99200	132555	132612	132705	132834	132999	133200

Table 5. Player *A* vs Player *B* at 250th Iteration

Player *A*			**Player *B***			
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**
250	123505	123532	123595	123694	123829	124000
7000	131475	131508	131577	131682	131823	132000
22750	139695	139734	139809	139920	140067	140250
47500	148165	148210	148291	148408	148561	148750
81250	156885	156936	157023	157146	157305	157500
124000	165855	165912	166005	166134	166299	166500

Table 6. Player *A* vs Player *B* at 300th Iteration

Player *A*			**Player *B***			
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**
300	148305	148332	148395	148494	148629	148800
8400	157875	157908	157977	158082	158223	158400
27300	167745	167784	167859	167970	168117	168300
57000	177915	177960	178041	178158	178311	178500
97500	188385	188436	188523	188646	188805	189000
148800	199155	199212	199305	199434	199599	199800

Table 7. Player *A* vs Player *B* at 350th Iteration

Player *A*			**Player *B***			
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**
350	173105	173132	173195	173294	173429	173600
9800	184275	184308	184377	184482	184623	184800
31850	195795	195834	195909	196020	196167	196350
66500	207665	207710	207791	207908	208061	208250
113750	219885	219936	220023	220146	220305	220500
173600	232455	232512	232605	232734	232899	233100

Table 8. Player *A* vs Player *B* at 400th Iteration

Player *A*			**Player *B***			
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**
400	197905	197932	197995	198094	198229	198400
11200	210675	210708	210777	210882	211023	211200
36400	223845	223884	223959	224070	224217	224400
76000	237415	237460	237541	237658	237811	238000
130000	251385	251436	251523	251646	251805	252000
198400	265755	265812	265905	266034	266199	266400

Table 9. Player *A* vs Player *B* at 450th Iteration

Player *A*			**Player *B***			
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**
450	222705	222732	222795	222894	223029	223200
12600	237075	237108	237177	237282	237423	237600
40950	251895	251934	252009	252120	252267	252450
85500	267165	267210	267291	267408	267561	267750
146250	282885	282936	283023	283146	283305	283500
223200	299055	299112	299205	299334	299499	299700

Table 10. Player *A* vs Player *B* at 500th Iteration

Player *A*			**Player *B***			
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**
500	247505	247532	247595	247694	247829	248000
14000	263475	263508	263577	263682	263823	264000
45500	279945	279984	280059	280170	280317	280500
95000	296915	296960	297041	297158	297311	297500
162500	314385	314436	314523	314646	314805	315000
248000	332355	332412	332505	332634	332799	333000

4.1. Observations from the Iterations:

(i) The correlations between all iterations are good.

(ii) The variations are very normal at each iteration.

(iii) Constant differences between the values of possible actions of player *A* and Player *B* at any two consequent iterations are obtained.

OPTIMUM MIXED STRATEGIES OF PLAYER A AND PLAYER B:

The optimum mixed strategies of Player *A* and Player *B* are shown in **Table-11.**

Table 11. Optimum Mixted strategies of Player *A* and Player *B* (Iteration wise)

50		100		150		200		250		300		350		400		450		500	
A	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

LOWER BOUNDS AND UPPER BOUNDS AT ALL ITERATIONS

At each play of the game the minimum sum element selected by player B divided by the number of place of the game is known as lower bound.

Similarly at each play of the game the maximum sum element selected by player *A* divided by the number of place of the game is called as upper bound.

The Values of U.Bs and L.Bs in 6 × 6 game are tabulated in Table-12.

Table-12

U.B	Lower Bounds									
Iterations / 50-500	50	100	150	200	250	300	350	400	450	500
496	486.10	491.05	492.70	493.52	494.02	494.35	494.58	494.76	494.90	495.01
496	486.64	491.32	492.88	493.66	494.12	494.44	494.66	494.83	494.96	495.06
496	487.91	491.95	493.30	493.97	494.38	494.65	494.84	494.98	495.10	495.19
496	489.88	492.94	493.96	494.47	494.77	494.98	495.12	495.23	495.32	495.38
496	492.58	494.29	494.89	495.14	495.31	495.43	495.511	495.57	495.62	495.65
496	496.00	496.00	496.00	496.00	496.00	496.00	496.00	496.00	496.00	496.00

CONCLUSIONS

(i) The value of the game is 496.

(ii) The optimum mixed strategies of player *A* and player *B* in all iterations are same.

(iii) The upper bound value for any iterartion is same.

(iv) The error is 9.9 and it will be reduced gradually.

(v) The obtained strategies for Player *A* & Player *B* are best.

(vi) The Game is identified as strictly determinable game.

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