Acta Ciencia Indica, Vol. XLIII M, No. 4, 221 (2017)

A SPECIAL SYMMETRIC GAME PROBLEM WITH TRIANGULAR NUMBERS – BROWN'S ALGORITHM

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RECEIVED : 29 December, 2017

The paper aims to discuss a row and column both dominance triangular symmetric game problem with Brown's Algorithm. A 8 × 8 symmetric game problem is based on dominance strategy. The relations among the Lower bounds and/or Upper bounds of the game are obtained at different stages. The derived Conclusions are drawn at the end.

KEYWORDS : Game Theory, players, strategy, Pay-off matrix, optimal solution, Lower bound, Upper bound.

AMS Classification : 91A05, 91A18, 91A43, 91A90.

INTRODUCTION

Brown's algorithm is one of the accurate procedures to classify the nature of any game in an effective manner. Many Influences of the players on one and other may have different policies to get success. The Optimum mixed strategies may help us to evaluate any game effectively. Even Some games are complicated to evaluate the complexity and nature, Brown's algorithm will help us to know the nature of the game.

Billy E. Gillett [1] introduced few algorithms to solve many games. Many games in operations research (OR) are discussed by Levin and Desjardins [2]. Few mathematicians like Rapoport [3], Dresher [4], Raiffa [5], McKinsey [6] etc concentrated on various cases of OR and useful applications of game theory.

BASIC FORMATION OF 8X8 GAME

The symmetric game problem is constructed with 8 rows and 8 columns with non zero triangular numbers according to player A and Player B. One player selects only one single action from his/her set possible actions. The actions of A are considered as $A1$, $A2$, $A3$, $A4$, $A5$, $A6$, $A7$, $A8$ which will be influenced by the other actions of player B *i.e.* $B1$, $B2$, $B3$, $B4$, $B5$, $B6$, $B7$, $B8$. It is better to consider the influence of Player B on each component of Player A.

The pay-off matrix of the game having the size 8 × 8 is given below.

$$\begin{bmatrix} 1 & 3 & 6 & 10 & 15 & 21 & 28 & 36 \\ 3 & 45 & 55 & 66 & 78 & 91 & 105 & 120 \\ 6 & 55 & 136 & 153 & 171 & 190 & 210 & 231 \\ 10 & 66 & 153 & 253 & 276 & 300 & 325 & 351 \\ 15 & 78 & 171 & 276 & 378 & 406 & 435 & 465 \\ 21 & 91 & 190 & 300 & 406 & 496 & 528 & 561 \\ 28 & 105 & 210 & 325 & 435 & 528 & 595 & 630 \\ 36 & 120 & 231 & 351 & 465 & 561 & 630 & 666 \end{bmatrix}$$

MATERIAL AND METHODS

The author applied Brown's algorithm to solve this special case of 8 × 8 game in which row and columns both dominated. Brown's Algorithm:

Step 1: Player A chooses one of the possible actions (Ai_1) from $A1$-$A8$ to play, and Player B then plays with the possible action Bj_1 corresponding to the smallest element in the selected action Ai_1.

Step 2 : Player A then picks out the possible action (Ai_2) from $A1 - A8$ to play corresponding to the largest element in the possible action (Bj_1) selected by Player B in step 1.

Step 3 : Player B sums the actions of Player A who has played thus far, and plays with the possible action of Bj_2 corresponding to a smallest sum element.

Step 4 : Player A sums the actions of Player B who has played thus far, and plays the possible action (Ai_3) corresponding to a largest sum element. After the required iterations are computed,then go to step 5; otherwise, come back to step 3.

Step 5 : Compute an Lower and Upper bound $\underline{\gamma}$ and $\overline{\gamma}$ respectively.

$$\overline{\gamma} = \frac{\text{Largest sum element from step 4}}{\text{Number of plays of the game thus far}} \quad \text{and} \quad \underline{\gamma} = \frac{\text{Smallest sum element from step 3}}{\text{Number of plays of the game thus far}}$$

Step 6 : Let Xi be the portion of the time Player A played row i with $i = 1, 2, ..., m$ and let Yi be the proportion of the time Player B played column j with $j = 1, 2, ..., n$. These strategies approximate the optimal mini max strategies. Upper and Lower bounds of the value of the game where $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$ are calculated in step 5. The Process completes.

RESULTS

Brown's Algorithm is applied on prime problem up to 500 iterations to get good accuracy in the results with the help of Java program. The influences of player B on each component of Player A are tabulated from Table (1) to Table (10) at each iteration.

Table 1. Player *A* vs Player *B* at 50th Iteration

Player *A*				**Player *B***				
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**	***A*7**	***A*8**
50	1765	1767	1770	1774	1779	1785	1792	1800
150	5883	5925	5935	5946	5958	5971	5985	6000
300	11325	11374	11455	11472	11490	11509	11529	11550
500	17209	17265	17352	17452	17475	17499	17524	17550
750	22800	22863	22956	23061	23163	23191	23220	23250
1050	27510	27580	27679	27789	27895	27985	28017	28050
1400	30898	30975	31080	31195	31305	31398	31465	31500
1800	32670	32754	32865	32985	33099	33195	33264	33300

Table 2. Player *A* vs Player *B* at 100th Iteration

Player *A*				**Player *B***				
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**	***A*7**	***A*8**
100	3565	3567	3570	3574	3579	3585	3592	3600
300	11883	11925	11935	11946	11958	11971	11985	12000
600	22875	22924	23005	23022	23040	23059	23079	23100
1000	34759	34815	34902	35002	35025	35049	35074	35100
1500	46050	46113	46206	46311	46413	46441	46470	46500
2100	55560	55630	55729	55839	55945	56035	56067	56100
2800	62398	62475	62580	62695	62805	62898	62965	63000
3600	65970	66054	66165	66285	66399	66495	66564	66600

Table 3. Player *A* vs Player *B* at 150th Iteration

Player *A*				**Player *B***				
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**	***A*7**	***A*8**
150	5365	5367	5370	5374	5379	5385	5392	5400
450	17883	17925	17935	17946	17958	17971	17985	18000
900	34425	34474	34555	34572	34590	34609	34629	34650
1500	52309	52365	52452	52552	52575	52599	52624	52650
2250	69300	69363	69456	69561	69663	69691	69720	69750
3150	83610	83680	83779	83889	83995	84085	84117	84150
4200	93898	93975	94080	94195	94305	94398	94465	94500
5400	99270	99354	99465	99585	99699	99795	99864	99900

Table 4. Player *A* vs Player *B* at 200th Iteration

Player *A*				Player *B*				
	*A*1	*A*2	*A*3	*A*4	*A*5	*A*6	*A*7	*A*8
200	7165	7167	7170	7174	7179	7185	7192	7200
600	23883	23925	23935	23946	23958	23971	23985	24000
1200	45975	46024	46105	46122	46140	46159	46179	46200
2000	69859	69915	70002	70102	70125	70149	70174	70200
3000	92550	92613	92706	92811	92913	92941	92970	93000
4200	111660	111730	111829	111939	112045	112135	112167	112200
5600	125398	125475	125580	125695	125805	125898	125965	126000
7200	132570	132654	132765	132885	132999	133095	133164	133200

Table 5. Player *A* vs Player *B* at 250th Iteration

Player *A*				Player *B*				
	*A*1	*A*2	*A*3	*A*4	*A*5	*A*6	*A*7	*A*8
250	8965	8967	8970	8974	8979	8985	8992	9000
750	29883	29925	29935	29946	29958	29971	29985	30000
1500	57525	57574	57655	57672	57690	57709	57729	57750
2500	87409	87465	87552	87652	87675	87699	87724	87750
3750	115800	115863	115956	116061	116163	116191	116220	116250
5250	139710	139780	139879	139989	140095	140185	140217	140250
7000	156898	156975	157080	157195	157305	157398	157465	157500
9000	165870	165954	166065	166185	166299	166395	166464	166500

Table 6. Player *A* vs Player *B* at 300th Iteration

Player *A*				Player *B*				
	*A*1	*A*2	*A*3	*A*4	*A*5	*A*6	*A*7	*A*8
300	10765	10767	10770	10774	10779	10785	10792	10800
900	35883	35925	35935	35946	35958	35971	35985	36000
1800	69075	69124	69205	69222	69240	69259	69279	69300
3000	104959	105015	105102	105202	105225	105249	105274	105300
4500	139050	139113	139206	139311	139413	139441	139470	139500
6300	167760	167830	167929	168039	168145	168235	168267	168300
8400	188398	188475	188580	188695	188805	188898	188965	189000
10800	199170	199254	199365	199485	199599	199695	199764	199800

Table 7. Player *A* vs Player *B* at 350th Iteration

Player *A*				**Player *B***				
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**	***A*7**	***A*8**
350	12565	12567	12570	12574	12579	12585	12592	12600
1050	41883	41925	41935	41946	41958	41971	41985	42000
2100	80625	80674	80755	80772	80790	80809	80829	80850
3500	122509	122565	122652	122752	122775	122799	122824	122850
5250	162300	162363	162456	162561	162663	162691	162720	162750
7350	195810	195880	195979	196089	196195	196285	196317	196350
9800	219898	219975	220080	220195	220305	220398	220465	220500
12600	232470	232554	232665	232785	232899	232995	233064	233100

Table 8. Player *A* vs Player *B* at 400th Iteration

Player *A*				**Player *B***				
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**	***A*7**	***A*8**
400	14365	14367	14370	14374	14379	14385	14392	14400
1200	47883	47925	47935	47946	47958	47971	47985	48000
2400	92175	92224	92305	92322	92340	92359	92379	92400
4000	140059	140115	140202	140302	140325	140349	140374	140400
6000	185550	185613	185706	185811	185913	185941	185970	186000
8400	223860	223930	224029	224139	224245	224335	224367	224400
11200	251398	251475	251580	251695	251805	251898	251965	252000
14400	265770	265854	265965	266085	266199	266295	266364	266400

Table 9. Player *A* vs Player *B* at 450th Iteration

Player *A*				**Player *B***				
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**	***A*7**	***A*8**
450	16165	16167	16170	16174	16179	16185	16192	16200
1350	53883	53925	53935	53946	53958	53971	53985	54000
2700	103725	103774	103855	103872	103890	103909	103929	103950
4500	157609	157665	157752	157852	157875	157899	157924	157950
6750	208800	208863	208956	209061	209163	209191	209220	209250
9450	251910	251980	252079	252189	252295	252385	252417	252450
12600	282898	282975	283080	283195	283305	283398	283465	283500
16200	299070	299154	299265	299385	299499	299595	299664	299700

Table-10: Player *A* vs Player *B* at 500th Iteration

Player *A*				**Player *B***				
	***A*1**	***A*2**	***A*3**	***A*4**	***A*5**	***A*6**	***A*7**	***A*8**
500	17965	17967	17970	17974	17979	17985	17992	18000

1500	59883	59925	59935	59946	59958	59971	59985	60000
3000	115275	115324	115405	115422	115440	115459	115479	115500
5000	175159	175215	175302	175402	175425	175449	175474	175500
7500	232050	232113	232206	232311	232413	232441	232470	232500
10500	279960	280030	280129	280239	280345	280435	280467	280500
14000	314398	314475	314580	314695	314805	314898	314965	315000
18000	332370	332454	332565	332685	332799	332895	332964	333000

4.1. Observations from the Iterations:

(i) The influence of Player *B* occurs on the possible action of Player *A* in each iteration.

(ii) Too much variations are not observed at any iteration.

(iii) Influenced Fluctuations are not traced.

(iv) Constant differences between the values of possible actions of player *A* at any two consequent iterations have been determined

(v) Constant differences between the values of possible actions of player *B* at any two consequent iterations have been ascertained.

OPTIMUM MIXED STRATEGIES OF PLAYER A AND PLAYER B

The optimum mixed strategies of Player *A* and Player *B* are shown in **Table-11.**

Table-11. Optimum Mixted strategies of Player *A* and Player *B* (Iteration wise)

50		100		150		200		250		300		350		400		450		500	
A	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

LOWER BOUNDS AND UPPER BOUNDS AT ALL ITERATIONS

At each play of the game the minimum sum element selected by player *B* divided by the number of place of the game is known as lower bound.

Similarly, at each play of the game the maximum sum element selected by player *A* divided by the number of place of the game is called as upper bound.

The Values of U.Bs and L.Bs in 8 × 8 game are tabulated in Table-12.

Table-12

U.B	**Lower Bounds**									
Iterations / 50-500	**50**	**100**	**150**	**200**	**250**	**300**	**350**	**400**	**450**	**500**
36.00	35.30	35.65	35.76	35.82	35.86	35.88	35.90	35.91	35.92	35.93
36.00	35.34	35.67	35.78	35.83	35.87	35.89	35.90	35.91	35.92	35.93
36.00	35.40	35.70	35.80	35.85	35.88	35.90	35.91	35.92	35.93	35.94
36.00	35.48	35.74	35.82	35.87	35.89	35.91	35.92	35.93	35.94	35.94
36.00	35.58	35.79	35.86	35.89	35.91	35.93	35.94	35.94	35.95	35.95
36.00	35.70	35.86	35.90	35.92	35.94	35.95	35.95	35.96	35.96	35.97
36.00	35.84	35.92	35.94	35.96	35.96	35.97	35.97	35.98	35.98	35.98
36.00	36.00	36.00	36.00	36.00	36.00	36.00	36.00	36.00	36.00	36.00

Conclusions

(i) The value of the symmetric triangular game is 36.

(ii) The optimum mixed strategies of two players *A* and *B* are identical at any iteration.

(iii) Unique upper bound is traced at each iteration.

(iv) The Initial error is 1.3. It reduces systematically iteration by iteration.

(v) The best strategies are obtained for Player *A* & Player *B*.

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