HOMOTOPY ANALYSIS ON HARVESTED ENEMY SPECIES WITH VARIABLE RATE AND COVER PROTECTED AMMENSAL SPECIES

DR. K.V.L.N. ACHARYULU

Associate Professor, Department of Mathematics, Bapatla Engineering College, Bapatla-522101, India

RECEIVED : 30 November, 2017

The paper intends to find a series solution by homotopy analysis on harvested enemy species with variable rate and Cover Protected Ammensal species. The Ammensalism species is cover protected with limited resources. The model equations are constituted by a pair of non linear first order differential equations. The series solution for the considered model is derived with perturbation method.

KEYWORDS : Ammensalism, Homotopy Analysis, Stability, Dominance Reversal time.

INTRODUCTION

Abbasbandy, S [1] applied perturbation technique and derived many results in the area of asymptotic techniques .After wards Liao [5-8] developed and simplified Homotopy Perturbation Method (HPM) in 1992. Few other methods with independent physical parameters were also established by eminent Mathematicians [2, 4]. In the present years, the HPM methodology has been effectively applied in the field of Modern Sciences [3, 9-12].

BASIC IDEA OF HOMOTOPY PERTURBATION METHOD

Step (1): Let us consider nonlinear differential equation:

$$A(u) - f(r) = 0, \quad r \in \Omega \qquad \ldots \text{(I)}$$

With the boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma$$

where A is a general differential operator, B a boundary operator, $f(r)$ is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal drawn outwards from Ω.

Step (2): In general the operator A, is divided into two parts : a linear part L and a nonlinear part N. Therefore above differential equation (I) is expressed in the form of

$$L(u) - N(u) - f(r) = 0 \qquad \text{(II)}$$

Step (3): With the help of Homotopy Perturbation Method (HPM), one can constitute a homotopy $v(r,\ p):\Omega\times[0,\ 1]\rightarrow R$ which satisfies

$$H(v,p)=(1-p)\left[L(v)-L(u_0)\right]+p\left[A(v)-f(r)\right]=0,\ p\in[0,1],r\in\Omega \quad \text{… (III)}$$

It is nothing but

$$H(v,p)=L(v)-L(u_0)+pL(u_0)+p\left[A(v)-f(r)\right]=0 \quad \text{… (IV)}$$

where $p\in[0,1]$ is named as an embedding parameter, and u_0 is an initial approximation of equation (1), which satisfies the boundary conditions.

Step (4): Then equations (III), (IV) follow that

$$H(v,0)=L(v)-L(u_0)=0 \quad \text{and} \quad H(v,1)=A(v)-f(r)=0$$

Thus the changing process of P from zero to unity is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$.

Step (5): According to the HPM, we can first use the imbedding parameter p as a 'small parameter' and assume that the solutions of the equations (III) and (IV) can be written as a power series in p :

$$v=v_0+pv_1+p^2v_2+p^3v_3+p^4v_4+----------$$

The approximate solution of equation (I) can be obtained as

$$u=\underset{p\to 1}{Lt}\ v=v_0+v_1+v_2+v_3+v_4+-----------$$

Notations adopted

$N_1\ (t)$: The population rate of the species S_1 at time t

$N_2\ (t)$: The population rate of the species S_2 at time t

a_i : The natural growth rate of S_i, $i = 1, 2$.

a_{ii} : The rate of decrease of S_i; due to its own insufficient resources, $i = 1, 2$.

a_{12} : The inhibition coefficient of S_1 due to S_2 *i.e.* The Commensal coefficient.

m : Decrease of Enemy Species due to Harvesting.

b : Cover protection for Ammensal Species ($0 < b < 1$)

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, α, h_1, h_2 are assumed to be non-negative constants.

Basic equations

$$\frac{dN_1}{dt}=a_1N_1-a_{11}N_1^2-(1-b)a_{12}N_1N_2 \quad \text{... (1)}$$

$$\frac{dn_2}{dt}=(1-m)\,a_2N_2-a_{22}N_2^2 \text{ with initial conditions } N_1(0)=c_1 \text{ and } N_2(0)=c_2 \quad \text{… (2)}$$

The following system can be constructed by the concept of homotopy as follows

$$v_1' - N_{10}' + p(N_{10}' - a_1 v_1 + a_{11} v_1^2 - (1-b) a_{12} v_1 v_2) = 0 \quad \ldots (3)$$

$$v_2' - N_{20}' + p(N_{20}' - (1-m) a_2 v_2 + a_{22} v_2^2) = 0 \quad \ldots (4)$$

The initial approximations are considered as

$$v_{1,0}(t) = N_{10}(t) = v_1(0) = c_1 \quad \ldots (5)$$

$$v_{2,0}(t) = N_{20}(t) = v_2(0) = c_2 \quad \ldots (6)$$

and $v_1(t) = v_{1,0}(t) + p v_{1,1}(t) + p^2 v_{1,2}(t) + p^3 v_{1,3}(t) + p^4 v_{1,4}(t) + p^5 v_{1,5}(t) + \cdots$ ... (7)

$$v_2(t) = v_{2,0}(t) + p v_{2,1}(t) + p^2 v_{2,2}(t) + p^3 v_{2,3}(t) + p^4 v_{2,4}(t) + p^5 v_{2,5}(t) + \cdots \quad \ldots (8)$$

where $v_{i,J}(i = 1,2, J = 1,2,3 \ldots)$ are to be computed by substituting (5), (6), (7), (8) in (3), (4)

We get

$$\begin{aligned} & v_{1,0}'(t) + p v_{1,1}'(t) + p^2 v_{1,2}'(t) + p^3 v_{1,3}'(t) + p^4 v_{1,4}'(\mathrm{t}) + p^5 v_{1,5}'(\mathrm{t}) + \cdots - N_{10}' \\ & + p[N_{10}' - a_1(v_{1,0}(t) + p v_{1,1}(t) + p^2 v_{1,2}(t) + p^3 v_{1,3}(t) + p^4 v_{1,4}(t) + p^5 v_{1,5}(t) + \cdots) \\ & + a_{11}\big(v_{1,0}(t) + p v_{1,1}(t) + p^2 v_{1,2}(t) + p^3 v_{1,3}(t) + p^4 v_{1,4}(t) + p^5 v_{1,5}(t) + \cdots\big)(v_{1,0}(t) \\ & + p v_{1,1}(t) + p^2 v_{1,2}(t) + p^3 v_{1,3}(t) + p^4 v_{1,4}(t) + p^5 v_{1,5}(t) + \cdots + (1-b) a_{12}(v_{1,0}(t) \\ & + p v_{1,1}(t) + p^2 v_{1,2}(t) + p^3 v_{1,3}\big(t) + p^4 v_{1,4}(t) + p^5 v_{1,5}(t) + \cdots\big)\big(v_{2,0}(t) + p v_{2,1}(t) + \\ & p^2 v_{2,2}(t) + p^3 v_{2,3}(t) + p^4 v_{2,4}(t) + p^5 v_{2,5}(t) + \cdots)] = 0 \quad \ldots (9) \end{aligned}$$

From equation (4)

$$\begin{aligned} & v_{2,0}'(t) + p v_{2,1}'(t) + p^2 v_{2,2}'(t) + p^3 v_{2,3}'(t) + p^4 v_{2,4}'(t) + p^5 v_{2,5}'(t) + \cdots - N_{20}' \\ & + p[N_{20}' - (1-m) a_2 (v_{2,0}(t) + p v_{2,1}(t) + p^2 v_{2,2}(t) + p^3 v_{2,3}(t) + p^4 v_{2,4}(t) + p^5 v_{2,5}(t) \\ & + \cdots + a_{22}\big(v_{2,0}(t) + p v_{2,1}(t) + p^2 v_{2,2}(t) + p^3 v_{2,3}(t) + p^4 v_{2,4}(t) + p^5 v_{2,5}(t) + \cdots\big) \\ & \big(v_{2,0}(t) + p v_{2,1}(t) + p^2 v_{2,2}(t) + p^3 v_{2,3}(t) + p^4 v_{2,4}(t) + p^5 v_{2,5}(t) + \cdots\big)] = 0 \quad \ldots (10) \end{aligned}$$

From (9),

$$\begin{aligned} & 0 + p v_{1,1}'(t) + p^2 v_{1,2}'(t) + p^3 v_{1,3}'(t) + p^4 v_{1,4}'(t) + p^5 v_{1,5}'(t) + \cdots - 0 \\ & + p[0 - a_1 v_{1,0}(t) - a_1 p v_{1,1}(t) - a_1 p^2 v_{1,2}(t) - a_1 p^3 v_{1,3}(t) - a_1 p^4 v_{1,4}(t) \\ & - a_1 p^5 v_{1,5}(t) - \ldots + a_{11} v_{1,0}^2(t) + a_{11} p v_{1,0}(t) v_{1,1}(t) + a_{11} p^2 v_{1,0}(t) v_{1,2}(t) \\ & + a_{11} p^3 v_{1,0}(t) v_{1,3}(t) + a_{11} p^4 v_{1,0}(t) v_{1,4}(t) + \cdots + a_{11} p v_{1,0}(t) v_{1,1}(t) + a_{11} p^2 v_{1,1}^2(t) \\ & + a_{11} p^3 v_{1,1}(t) v_{1,2}(t) + a_{11} p^4 v_{1,1}(t) v_{1,3}(t) + a_{11} p^5 v_{1,1}(t) v_{1,4}(t) + \cdots \\ & + a_{11} p^2 v_{1,0}(t) v_{1,2}(t) + a_{11} p^3 v_{1,1}(t) v_{1,2}(t) + a_{11} p^4 v_{1,2}^2(t) + a_{11} p^5 v_{1,2}(t) v_{1,3}(t) + \cdots \\ & + a_{11} p^3 v_{1,0}(t) v_{1,3}(t) + a_{11} p^4 v_{1,1}(t) v_{1,3}(t) + a_{11} p^5 v_{1,2}(t) v_{1,3}(t) + \cdots \\ & + a_{11} p^4 v_{1,0}(t) v_{1,4}(t) + a_{11} p^5 v_{1,1}(t) v_{1,4}(t) + \cdots + a_{11} p^5 v_{1,0}(t) v_{1,5}(t) + \cdots \\ & + (1-b) a_{12}\, v_{1,0}(t) v_{2,0}(t) + (1-b) a_{12} p v_{1,0}(t) v_{2,1}(t) + (1-b) a_{12} p^2 v_{1,0}(t) v_{2,2}(t) \\ & + (1-b) a_{12} p^3 v_{1,0}(t) v_{2,3}(t) + (1-b) a_{12} p^4 v_{10}(t) v_{2,4}(t) \ldots\ldots (1-b) a_{12}\, p v_{1,1}(t) v_{2,0}(t) \\ & + (1-b) a_{12} p^2 v_{1,1}(t) v_{2,1}(t) + (1-b) a_{12} p^3 v_{1,1}(t) v_{2,2}(t) \\ & + (1-b) a_{12} p^4 v_{1,1}(t) v_{2,3}(t). \ldots + (1-b) a_{12} p^2 v_{2,0}(t) v_{1,2}(t) \\ & + (1-b) a_{12} p^3 v_{1,2}(t) v_{2,1}(t) + (1-b) a_{12} p^4 v_{1,2}(t) v_{2,2}(t) \ldots + (1-b) a_{12}\; p^3 v_{1,3}(t) v_{2,0}(t) \\ & + (1-b) a_{12} p^4 v_{1,3}(t) v_{2,1}(t). \ldots + (1-b) a_{12} p^4 v_{1,4}(t) v_{2,0}(t) \ldots.] = 0 \quad \ldots (11) \end{aligned}$$

From (10),

$$0 + pv'_{2,1}(t) + p^2v'_{2,2}(t) + p^3v'_{2,3}(t) + p^4v'_{2,4}(t) + p^5v'_{2,5}(t) + \cdots - 0$$
$$+p[0 - (1-m)a_2p^4v_{2,4}(t) - (1-m)a_2p^5v_{2,5}(t) - \cdots + a_{22}\ v_{2,0}^2(t)v_{2,0}(t)$$
$$-(1-m)a_2pv_{2,1}(t) - (1-m)a_2p^2v_{2,2}(t) - (1-m)a_2p^3v_{2,3}(t)$$
$$-(1-m)a_2p^4v_{2,4}(t) - (1-m)a_2p^5v_{2,5}(t) - \cdots + a_{22}\ v_{2,0}^2(t)$$
$$+a_{22}pv_{2,0}(t)v_{2,1}(t) + a_{22}p^2v_{2,0}(t)v_{2,2}(t) + a_{22}p^3v_{2,0}(t)v_{2,3}(t) + a_{22}p^4v_{2,0}(t)v_{2,4}(t) + \cdots$$
$$+a_{22}\ pv_{2,1}(t)v_{2,0}(t) + a_{22}p^2v_{2,1}^2(t) + a_{22}p^3v_{2,1}(t)v_{2,2}(t) + a_{22}p^4v_{2,1}(t)v_{2,3}(t)$$
$$+a_{22}p^5v_{2,1}(t)v_{2,4}(t) + \cdots + a_{22}p^2v_{2,0}(t)v_{2,2}(t) + a_{22}p^3v_{2,2}(t)v_{2,1}(t) + a_{22}p^4v_{2,2}^2(t)$$
$$+a_{22}p^5v_{2,2}(t)v_{2,3}(t) + \cdots + a_{22}p^3v_{2,0}(t)v_{2,3}(t) + a_{22}p^4v_{2,1}(t)v_{2,3}(t) + a_{22}$$
$$p^5v_{2,2}(t)v_{2,3}(t) + \cdots + a_{22}p^4v_{2,0}(t)v_{2,4}(t) + a_{22}p^5v_{2,1}(t)v_{2,4}(t) + \cdots$$
$$+a_{22}p^5v_{2,0}(t)v_{2,5}(t) + \cdots] = 0 \qquad \ldots (12)$$

Now comparing the coefficient of various powers of p in (11) & (12), we obtain

The coefficient of P^1:

$$v'_{1,1}(t) - a_1v_{1,0}(t) + a_{11}v_{1,0}^2(t) + (1-b)a_{12}v_{1,0}(t)v_{2,0}(t) = 0$$
$$v'_{2,1}(t) - (1-m)a_2v_{2,0}(t) + a_{22}v_{2,0}^2(t) = 0$$

The coefficient of P^2:

$$v'_{1,2}(t) - a_1v_{1,1}(t) + a_{11}v_{1,0}(t)v_{1,1}(t) + a_{11}v_{1,0}(t)v_{1,1}(t) + (1-b)a_{12}v_{1,0}(t)v_{2,1}(t)$$
$$+(1-b)a_{12}v_{1,1}(t)v_{2,0}(t) = 0$$
$$v'_{2,2}(t) - (1-m)a_2v_{2,1}(t) + a_{22}v_{2,0}(t)v_{2,1}(t) + a_{22}v_{2,0}(t)v_{2,1}(t) = 0$$

The coefficient of P^3:

$$v'_{1,3}(t) - a_1v_{1,2}(t) + a_{11}v_{1,0}(t)v_{1,2}(t) + a_{11}v_{1,1}^2(t) + a_{11}v_{1,0}(t)v_{1,2}(t)$$
$$+(1-b)a_{12}v_{1,0}(t)v_{2,2}(t) - (1-b)a_{12}v_{1,1}(t)v_{2,1}(t) + (1-b)a_{12}v_{2,0}(t)v_{1,2}(t) = 0$$
$$v'_{2,3}(t) - (1-m)a_2v_{2,2}(t) + a_{22}v_{2,0}(t)v_{2,2}(t) + a_{22}v_{2,1}^2(t) + a_{22}v_{2,0}(t)v_{2,2}(t) = 0$$

The coefficient of P^4:

$$v'_{1,4}(t) - a_1v_{1,3}(t) + a_{11}v_{1,0}(t)v_{1,3}(t) + a_{11}v_{1,1}(t)v_{1,2}(t) + a_{11}v_{1,1}(t)v_{1,2}(t)$$
$$+\ a_{11}v_{1,0}(t)v_{1,3}(t) + (1-m)a_{12}v_{1,0}(t)v_{2,3}(t) + (1-m)a_{12}v_{1,1}(t)v_{2,2}(t)$$
$$+(1-m)a_{12}v_{2,1}(t)v_{1,2}(t) + (1-m)\ a_{12}v_{2,0}(t)v_{1,3}(t) = 0$$
$$v'_{2,4}(t) - (1-m)a_2v_{2,3}(t) + a_{22}v_{2,0}(t)v_{2,3}(t) + a_{22}v_{2,1}(t)v_{2,2}(t)$$
$$+a_{22}v_{2,1}(t)v_{2,2}(t) + a_{22}v_{2,0}(t)v_{2,3}(t) = 0$$

Now $v_1(0) = c_{1,} v_2(0) = c_2$

$$v_{1,1}(t) = a_1\int_0^t v_{1,0}(t)dt - a_{11}\int_0^t v_{1,0}^2(t)dt - (1-b)a_{12}\int_0^t v_{1,0}(t)v_{2,0}(t)dt$$
$$= c_1a_1t - a_{11}c_1^2t + (1-b)a_{12}c_1c_2t$$

$$\therefore\ v_{1,1}(t) = (a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1t$$

$$v_{2,1}(t) = (1-m)a_2 \int_0^t v_{2,0}(t)dt - a_{22}\int_0^t v_{2,0}^2(t)dt \; = (1-m)a_2c_2t - a_{22}c_2^2t$$

$$\therefore \;\; v_{2,1}(t) = ((1-m)a_2 - a_{22}c_2)c_2t$$

$$v_{1,2}(t) = a_1\int_0^t v_{1,1}(t)dt - 2a_{11}\int_0^t v_{1,0}(t)v_{1,1}(t)dt - (1-b)a_{12}\int_0^t v_{1,0}(t)v_{2,1}(t)dt$$

$$-(1-b)a_{12}\int_0^t v_{1,1}(t)v_{2,0}(t)dt$$

$$= a_1(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1\frac{t^2}{2} - 2a_{11}c_1(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1\frac{t^2}{2}$$

$$-(1-b)a_{12}c_1((1-m)a_2 - a_{22}c_2)c_2\frac{t^2}{2} - (1-b)a_{12}c_2(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1\frac{t^2}{2}$$

$$\therefore \;\; v_{1,2}(t) = [(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1$$

$$-(1-b)a_{12}c_1((1-m)a_2 \; - a_{22}c_2)c_2]\frac{t^2}{2}$$

$$v_{2,2}(t) = (1-m)a_2\int_0^t v_{2,1}(t)dt - 2a_{22}\int_0^t v_{2,0}(t)v_{2,1}(t)dt$$

$$= [(1-m)a_2((1-m)a_2 - a_{22}c_2)c_2 - 2a_{22}c_2((1-m)a_2 - a_{22}c_2)c_2]\frac{t^2}{2}$$

$$\therefore \qquad v_{2,2}(t) = [(a_2 - a_{22}c_2)(a_2 - 2a_{22}c_2)c_2]\frac{t^2}{2}$$

$$v_{1,3}(t) = a_1\int_0^t v_{1,2}(t)dt - 2a_{11}c_1\int_0^t v_{1,2}(t)dt - a_{11}\int_0^t v_{1,1}^2(t)dt - (1-b)a_{12}c_1\int_0^t v_{2,2}(t)dt$$

$$-(1-b)a_{12}c_2\int_0^t v_{1,2}(t)dt - (1-b)a_{12}\int_0^t v_{1,1}(t)v_{2,1}(t)dt$$

$$= (a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)\{(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)$$

$$(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1 - (1-b)a_{12}c_1c_2((1-m)a_2 - a_{22}c_2)\}\frac{t^3}{6}$$

$$-a_{11}(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1^2\frac{t^3}{3}$$

$$-(1-b)a_{12}c_1\{((1-m)a_2 - a_{22}c_2)((1-m)a_2 - 2a_{22}c_2)c_2\}\frac{t^3}{6}$$

$$-(1-b)a_{12}c_2((1-m)a_2 - a_{22}c_2)(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1\frac{t^3}{3}$$

$$\therefore \;\; v_{1,3}(t) = [(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)\{(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)(a_1 - a_{11}c_1$$

$$-(1-b)a_{12}c_2)c_1 - (1-b)a_{12}c_1c_2((1-m)a_2 - a_{22}c_2)\}$$

$$+(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1\{2(1-b)a_{12}c_2((1-m)a_2 - a_{22}c_2)$$
$$-2a_{11}(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1\}$$
$$-(1-b)a_{12}c_1c_2\{((1-m)a_2 - a_{22}c_2)((1-m)a_2 - 2a_{22}c_2)\}]\frac{t^3}{6}$$

$$v_{2,3}(t) = (1-m)a_2\int_0^t v_{2,2}(t)dt - 2a_{22}\int_0^t v_{2,0}(t)v_{2,2}(t)dt - a_{22}\int_0^t v_{2,1}^2(t)dt$$
$$= ((1-m)a_2 - 2a_{22}c_2)\{((1-m)a_2 - a_{22}c_2)((1-m)a_2 - 2a_{22}c_2)c_2\}\frac{t^3}{6}$$
$$-a_{22}((1-m)a_2 - a_{22}c_2)^2c_2^2\frac{t^3}{3}$$

$$\therefore v_{2,3}(t) = [((1-m)a_2 - a_{22}c_2)c_2\{((1-m)a_2 - 2a_{22}c_2)((1-m)a_2 - 2a_{22}c_2)$$
$$-2a_{22}((1-m)a_2 - a_{22}c_2)c_2\}\frac{t^3}{6}$$

$$v_{1,4}(t) = (a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)\int_0^t v_{1,3}(t)dt - 2a_{11}\int_0^t v_{1,1}(t)v_{1,2}(t)dt$$
$$-(1-b)a_{12}\int_0^t v_{1,1}(t)v_{2,2}(t)dt - (1-b)a_{12}\int_0^t v_{1,2}(t)v_{2,1}(t)dt$$
$$-(1-b)a_{12}c_1\int_0^t v_{2,3}(t)dt$$

$$= [(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)\{(a_1 - 2a_{11}c_1 - c_2)\{(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)$$
$$(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1 - (1-b)a_{12}c_1c_2((1-m)a_2 - a_{22}c_2)\}$$
$$+(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1\{2(1-b)a_{12}c_2((1-m)a_2 - a_{22}c_2)$$
$$-2a_{11}(a_1 - a_{11}c_1 - (1-m)a_{12}c_2)c_1\}$$
$$-(1-m)a_{12}c_1c_2\{((1-m)a_2 - a_{22}c_2)((1-m)a_2 - 2a_{22}c_2)\}]\frac{t^4}{24}$$
$$-2a_{11}(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1\{(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)(a_1 - a_{11}c_1$$
$$-(1-b)a_{12}c_2)c_1 - (1-m)a_{12}c_1((1-m)a_2 - a_{22}c_2)c_2\}\frac{t^4}{8}$$
$$-(1-b)a_{12}c_1\{((1-m)a_2 - 2a_{22}c_2)[((1-m)a_2 - a_{22}c_2)c_2((1-m)a_2 - 2a_{22}c_2)]$$
$$2((1-m)a_2 - a_{22}c_2)c_2a_{22}((1-m)a_2 - a_{22}c_2)c_2\}\frac{t^4}{24}$$
$$-(1-b)a_{12}c_1(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)$$
$$[((1-m)a_2 - a_{22}c_2)c_2((1-m)a_2 - 2a_{22}c_2)]\frac{t^4}{8}$$
$$+a_{12}((1-m)a_2 - a_{22}c_2)c_2[(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)$$

$$(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1 - (1-b)a_{12}c_1c_2((1-m)a_2 - 2a_{22}c_2)]\ \frac{t^4}{8}$$

$$\therefore \quad v_{1,4}(t) = \{c_1 + (1-b)a_{12}c_1c_2((1-m)a_2 - a_{22}c_2))$$
$$[(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)^2 - c_1 6a_{11}\ (a_1 - a_{11}c_1 - (1-b)a_{12}c_2)]$$
$$+(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)\{2(a_1 - a_{11}c_1 - (1-b)a_{12}c_1)c_1((1\text{-b})$$
$$c_2(a_{22}c_2 - (1-m)a_2) - a_{11}(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1]$$
$$-(1-b)a_{12}c_1c_2[((1-m)a_2 - 2a_{22}c_2)((1-m)a_2 - a_{22}c_2)]$$
$$+[((1-m)a_2 - a_{22}c_2)c_2((1-m)a_2 - 2a_{22}c_2)]$$
$$[(1-b)a_{12}c_1(a_2 - 2a_{22}c_2) - 3(1-b)a_{12}c_1(a_1 - a_{11}c_1 - (1-b)1)a_{12}c_2)]$$
$$+((1-m)a_2 - a_{22}c_2)c_2\{[(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1[3(1\text{-b})$$
$$a_{12}(2a_{11}c_1 - a_1 - (1-b)a_{12}c_2)]$$
$$+((1-m)a_2 - a_{22}c_2)c_2[3(1-b)^2a_{12}^2c_1 + 2a_{22}(1-b)a_{12}c_1]\}\frac{t^4}{24}$$

$$v_{2,4}(t) = (1-m)a_2\int_0^t v_{2,3}(t)dt - 2a_{22}c_2\int_0^t v_{2,3}(t)dt - 2a_{22}\int_0^t v_{2,1}(t)v_{2,2}(t)dt$$
$$= ((1-m)a_2 - 2a_{22}c_2)\ ((1-m)a_2 - a_{22}c_2)c_2\{((1-m)a_2 - 2a_{22}c_2)^2$$
$$-2a_{22}c_2((1-m)a_2 - a_{22}c_2)\}\ \frac{t^4}{24}$$
$$-2a_{22}((1-m)a_2 - a_{22}c_2)c_2\{((1-m)a_2 - a_{22}c_2)c_2((1-m)a_2 - 2a_{22}c_2)\}\frac{t^4}{8}$$
$$\therefore \quad v_{2,4}(t) = ((1-m)a_2 - a_{22}c_2)c_2((1-m)a_2 - 2a_{22}c_2)\{((1-m)a_2 - 2a_{22}c_2)^2$$
$$-8a_{22}c_2((1-m)a_2 - a_{22}c_2)\}\frac{t^4}{24}$$

Up to the terms which contain maximum the power of four, we obtain

$$N_1(t) = \lim_{p\to 1} v_1(t) = \sum_{x=0}^{4} v_{1,x}(t) = v_{1,0}(t) + v_{1,1}(t) + v_{1,2}(t) + v_{1,3}(t) + v_{1,4}(t)$$

$$N_1(t) = \lim_{p\to 1} v_2(t) = \sum_{x=0}^{4} v_{2,x}(t) = v_{2,0}(t) + v_{2,1}(t) + v_{2,2}(t) + v_{2,3}(t) + v_{2,4}(t)$$

The solutions by Homotopy Perturbation Method are derived as

$$N_1(t) = c_1 + [(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1]t$$
$$+[(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1$$
$$-(1-b)a_{12}c_1((1-m)a_2 - a_{22}c_2)c_2]\frac{t^2}{2} + \{(a_1 - 2a_{11}c_1$$
$$-(1-b)a_{12}c_2)[(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1$$
$$-(1-b)a_{12}c_1c_2((1-m)a_2 - a_{22}c_2)] + (a_1 - a_{11}c_1 - (1-b)a_{12}c_2)$$

$$c_1[2(1-b)a_{12}c_2(a_{22}c_2 - (1-m)a_2) - 2a_{11}(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1]$$

$$-(1-b)a_{12}c_1c_2[((1-m)a_2 - a_{22}c_2)((1-m)a_2 - 2a_{22}c_2)]\frac{t^3}{6}$$

$$+\{[(a_1 - 2a_{11}c_1 - (1-m)a_{12}c_2)(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1$$

$$-(1-b)a_{12}c_1c_2((1-m)a_2 - a_{22}c_2)]$$

$$[(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)^2 - 6a_{11}c_1(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)]$$

$$+(a_1 - 2a_{11}c_1 - (1-b)a_{12}c_2)\{2(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1(1-b)a_{12}$$

$$[(a_{22}c_2 - (1-m)a_2) - a_{11}(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1]$$

$$-(1-b)a_{12}c_1c_2[((1-m)a_2 - a_{22}c_2)((1-m)a_2 - 2a_{22}c_2)]\}$$

$$+[((1-m)a_2 - a_{22}c_2)c_2((1-m)a_2 - 2a_{22}c_2)]$$

$$[(1-b)a_{12}c_1(2a_{22}c_2 - (1-m)a_2) - 3(1-b)a_{12}c_1(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)]$$

$$+((1-m)a_2 - a_{22}c_2)c_2\{(a_1 - a_{11}c_1 - (1-b)a_{12}c_2)c_1$$

$$[3(a_1 - 2a_{11}c_1 - (1-m)a_{12}c_2)(1-m)a_{12}]$$

$$+\big((1-m)a_2 - a_{22}c_2\big)c_2(3(1-b)^2a_{12}^2c_1 + 2a_{22}(1-b)a_{12}c_1)\}\ \frac{t^4}{24}$$

$$\therefore\ N_2(t) = c_2 + \big[\big((1-m)a_2 - a_{22}c_2\big)c_2\big]t$$

$$+\big[\big((1-m)a_2 - a_{22}c_2\big)((1-m)a_2 - 2a_{22}c_2)c_2\big]\frac{t^2}{2}$$

$$+[((1-m)a_2 - a_{22}c_2)c_2\{((1-m)a_2 - 2a_{22}c_2)((1-m)a_2 - 2a_{22}c_2)$$

$$-2(a_{22}c_2)\big((1-m)a_2 - a_{22}c_2\big)\}]\frac{t^3}{6}$$

$$+\{((1-m)a_2 - 2a_{22}c_2)((1-m)a_2 - 2a_{22}c_2) - 8a_{22}c_2((1-m)a_2 - a_{22}c_2)\}]\frac{t^4}{24}$$

Conclusions

A mathematical model of harvested enemy species with variable rate and Cover Protected Ammensal species is formed by a couple of first order nonlinear differential equations. Harvesting at variable rate for Enemy Species and a Cover protection for Ammensal Species are also considered. A series solution of this Special model of Ammensalsim is successfully derived by Homotopy Perturbation Method.

Referenes

1. Abbasbandy, S., The application of the Homotopy analysis method to nonlinear equations arising in heat transfer, *Phys. Lett.*, **A.360**, pp. 109-113 (2006).
2. Hilton, P.J., An introduction to Homotopy Theory, Cambridge University Press, Cambridge (1953).
3. Acharyulu, K.V.L.N., Kumar, N. Phani, Bhargavi, G. and Nagamani, K., Ecological Harvested Ammensal Model - A Homotopy Analysis, *International Journal of Scientific and Innovative Mathematical Research (IJSIMR)*, Volume **3**, Special Issue **4**, pp 10-18 November (2015).
4. Liao, Shijun, Homotopy analysis method in nonlinear differential equations, *Springer*, pp. 1-562 (2012).

5. Liao, S.J., The proposed Homotopy analysis technique for the solution of nonlinear problems, *Ph.D dissertation*, Shanghai Jiao Tong university (1992).
6. Liao, S.J, On the Homotopy analysis method for nonlinear problems, *Appl. Math. Comput.*, **147**, pp. 499-513 (2004).
7. Liao, S.J., On the relationship between the homotopy analysis method and Euler transform, *Commun. Nonlinear Sci. Numer. Simulat.*, **15**, (2003-2016).
8. Liao, S.J., Tan, Y., A general approach to obtain series solutions of nonlinear differential equations, *Stud. Appl. Math.*, **119**, pp. 297-355 (2007).
9. Acharyulu, K.V.L.N., Kumar, N. Phani, Vasavi, S.V. and Jahan, S.K. Khamar, A Series Solution of Ecological Harvested Commensal Model by Homotopy Perturbation Method, *International Journal of Scientific and Innovative Mathematical Research (IJSIMR)*, Volume **3**, Special Issue **4**, pp 1-9 November (2015).
10. Acharyulu, K.V.L.N. and Kumar, Dr. Phani, Ammensalism with Mortal Enemy Species Series Solution, *International Journal of Advance research in Science & Engineering (IJARSE)*, Volume **5**, Issue **9**, pp 470-477 (2016).
11. Acharyulu, K.V.L.N., Kumar, N. Phani, Bhargavi, G. and Nagamani, K., Ecological Harvested Ammensal Model-A Homotopy Analysis, *International Journal of Scientific and Innovative Mathematical Research (IJSIMR)*, Volume **3**, Issue **12**, pp 27-35 (2015).
12. Acharyulu, K.V.L.N., Nagamani, K. and Bhargavi, G., "A Mathematical Model Of Mutualism With Harvested First Species-A Series Solution", *Acta Ciencia Indica*, Volume **42**, No. **4**, 271-281 (2016).

❑