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THE INSTABILITY ANALYSIS OF HYDRODYNAMIC SOUND WAVES PROPAGATION IN POROUS MEDIA

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In this paper the sound wave propagation in a stationary or flowing fluid in porous medium is studied. The problem is analyzed with varitional principal. The effect of Mach number, Darcy number and porosity has been analyzed analytically & numerically. Modulus of wave attenuation per unit distance and phase velocity far varying shear wave number and Mach number. The mach number increased the attenuation for the hydrodynamic sound waves is decreased and consequently the phase velocities are increased. Therefore, mach number has destabilizing effect. The effect of porosity on both attenuation and phase velocity. It is found that as the porosity is increased the attenuation is decreased and the phase velocity are increased for the hydrodynamic sound waves. Thus, porosity has destabilizing effect. The effect of Darcy number on both attenuation and phase velocities. The Darcy number is increased the attenuation is increased and phase velocity are increased for the hydrodynamic sound waves. Therefore the Darcy number has stabilizing effect.

KEYWORDS: Porous media, Sound waves, Mach number, Porosity, Darcy number.

INTRODUCTION

The study of acoustic wave propagation problem has importance in engineering & sciences such as geophysical exploration, seismology, earthquake engineering, and rock dynamics, etc. The fluid flowing through porous media has received considerable attention during recent years, because of its several important applications in many areas of applied science and engineering. In porous materials such as fibrous and granular, the absorption process of the acoustic wave takes place through viscosity and thermal losses of the acoustic energy. These materials are widely used in room acoustics, in order to control reverberation time, to avoid undesired reflections, and to fill double wall cavities, floors and ceilings etc. If the acoustic improvements are restricted to interior spaces, (buildings halls, theatres, dwellings, factories, vehicle cabins, etc.), usually mineral wools or open pore foams can be used. On the other hand, for outdoor problems for instant acoustic noise barriers against traffic noise, the absorption provided by granular materials such as porous concrete should be employed.

Thornhill (1993) found that the theory of characteristics of linear partial differential equations of the first order shows that the equations for the wave fronts in a general electromagnetic field are mathematically identical with those for wave fonts in general motion.

The problem of a propagation of sound waves in fluids contained in a plain medium is widely studied by Helmholtz (1863), Kirchhoff (1868) and Rayleigh (1896). Numerical solutions of the problem were published by Tsao (1968), Gerlasch and Parker (1976). Scarton and Rouleau (1973) and by Shields *et al.* (1965). It is shown that the two-main parameters governing the propagation of sound waves in gases contained in rigid cylindrical tubes are the shear wave number and the reduced frequency. It is demonstrated that most of the analytical solution are dependent only on the shear wave number and that they are covered completely by the solution for the first time by Zwikker and Kosten (1949). The complete numerical solution of the problem has been obtained by Tijdeman (1975). A first approximation to the effects of mean flow on sound propagation through cylindrical capillary tubes has been achieved by Peat (1993).

In this paper the sound waves propagation in a stationary or flowing fluid in a porous medium is studied. The basic equations governing acoustic wave propagation in porous media has been analyzed through small acoustic disturbance of frequency. The problem is analyzed with varitional principal. The effect of Mach number, Darcy number and porosity has been analyzed analytically & numerically.

PROBLEM FORMULATION

The thermo-viscous effects in the fluid filling interstices among the fibers or the particles are responsible for the energy loss of the propagation acoustic wave. Generally, thermal losses are much lower than viscous losses in this kind of material. For mean flow through a capillary porous duct, it is reasonable to apply the conventional boundary layer approximation that the axial velocity is much greater than the radial velocity $u' >> v'$, and the changes in the radial direction occur much rapidly than those in the axial direction. The equation of continuity, equation of motion and equation of energy are the basic equations governing acoustic wave propagation in porous media.

$$\varepsilon \frac{\partial \rho'}{\partial t'} + u' \frac{\partial \rho'}{\partial x'} + \rho' \frac{\partial u'}{\partial x'} = 0 \qquad ...(1)$$

$$\rho' \left[\frac{1}{\varepsilon} \frac{\partial u'}{\partial t'} + \frac{1}{\varepsilon} u' \frac{\partial u'}{\partial x'} \right] = -\frac{\partial p'}{\partial x'} - \frac{\mu}{K} u' + \frac{\mu}{\varepsilon} \left[\left(\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r} \frac{\partial u'}{\partial r'} \right) + \frac{1}{3} \frac{\partial}{\partial x'} \frac{\partial u'}{\partial x'} \right] \qquad ...(2)$$

$$p' c_p \left[\frac{\partial T'}{\partial t'} + u' \frac{\partial T'}{\partial x'} \right] = k_{eff} \left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'} \frac{\partial T'}{\partial r'} \right) + \frac{\partial p'}{\partial t'} + u' \frac{\partial p'}{\partial x'} + \mu \left(\frac{\partial u'}{\partial r'} \right)^2 \qquad ....(3)$$

where u' is the velocity component in the axial direction. ρ', p' and T' are the fluid density, pressure and temperature, μ is the absolute viscosity and K is the permeability and ε is the porosity of the porous medium, c_p and k_{eff} are the specific heat and effective thermal conductivity of the porous media. The fluid is assumed to be perfect gas governed by the equation of state:

$$p' = \rho' R_0 T' \qquad ...(4)$$

where R_0 is the gas constant. It is also assumed that the flow through the circular duct is a superposition of a fully developed laminar, incompressible, axial steady flow and a small harmonic acoustic disturbance of frequency ω. The steady flow is taken to have constant density $\bar{\rho}$ and a mean speed of sound of steady flow $\bar{a}$ such that the fluid variables can be expanded in the form

$$\rho' = \bar{\rho}(1 + \alpha\rho(\eta)\, e^{Z\xi}\, e^{i\omega t'}) \qquad \text{...(5)}$$

$$u' = \bar{a}(M_0(\eta) + \alpha u(\eta) e^{Z\xi}\, e^{i\omega t'}) \qquad \text{...(6)}$$

$$p' = \left(\frac{\overline{\rho a}^2}{\gamma}\right)(p_0(\xi) + \alpha p(\eta) e^{Z\xi} e^{i\omega t'}) \qquad \text{...(7)}$$

$$T' = \left(\frac{\bar{a}^{\;2}}{\gamma R_0}\right)(1 + \alpha T\ (\eta) e^{Z\xi} e^{i\omega t'}) \qquad \text{...(8)}$$

where $\alpha << 1$ (perturbation parameter), γ is the ratio of specific heat. It is seen that the steady flow variables p_0 and Mach number M_0 together with acoustic variables ρ, u, p and T are dimensionless. Introduce the following variables in the transformations.

$$\xi = \frac{\omega x'}{\bar{a}} \quad n = \frac{r'}{R} \qquad \text{...(9)}$$

where R is the radius of the capillary duct. The axial acoustic wave motion has been assumed to have complex propagations constant Z which can be expanded as

$$Z = Z(R) + iZ(I) \qquad \text{...(10)}$$

where $Z(R)$ represents the wave attenuation per unit distance and $Z(I)$ represents the phase shift over the same distance. Substituting values of ρ', u', p' and T' from equation (5)-(8) in to (1)-(3) and simplifying, we get

$$\frac{s^2}{\gamma}\frac{dp_0}{d\xi} = \frac{1}{\varphi\eta}\frac{d}{d\eta}\left(\eta\frac{dM_0}{d\eta}\right) - DaM_0 \qquad \text{...(11)}$$

Here $s = R\sqrt{\frac{\bar{\rho}\omega}{\mu}}$ is the shear wave number, $Da = \frac{R^2}{K}$ is Darcy number. This equation is similar to the classical equation of Hagenpoiseuilli flow, the solution of which, with no-slip boundary conditions, gives a parabolic velocity profile:

$$M_0 = \frac{s^2}{\gamma}\frac{dp_0}{d\xi}\left(\frac{1-\eta^2}{4}\right) = 2\bar{M}\,(1-\eta^2) \qquad \text{...(12)}$$

The $\bar{M}$ is the mean Mach number of the steady flow. The linearized acoustic equations obtained by equating the first order terms in α from the governing equations are

$$k\left[\frac{i\rho}{\varepsilon} + Zu + 2\bar{M}Z(1-\eta^2)\rho\right] = 0 \qquad \text{...(13)}$$

$$\frac{iu}{\varepsilon}+\frac{2\bar{M}Z}{\varepsilon^2}(1-\eta^2)u=\left(-\frac{Z}{\gamma}\right)p+\frac{1}{s^2\varepsilon}\left[\frac{d^2u}{d\eta^2}+\frac{1}{\eta}\frac{du}{d\eta}\right]-\left(\frac{Da}{s^2}\right)u \qquad ...(14)$$

$$iT+2\bar{M}z(1-\eta^2)T=\frac{1}{\sigma^2 s^2}\left[\frac{d^2T}{d\eta}+\frac{1}{\eta}\frac{dT}{d\eta}\right]+\left(\frac{\gamma-1}{\gamma}\right)$$

$$\left[i+2\bar{M}Z(1-\eta^2)p\right]-8\left(\frac{\gamma-1}{s^2}\right)\bar{M}\frac{d}{d\eta}(u\eta) \qquad ...(15)$$

$$p=\rho+T \qquad ...(16)$$

where p is constant, $\sigma=\sqrt{\frac{\mu c_p}{k_{eff}}}$ is the square root of the Prandtl number and $k=\frac{\omega R}{\bar{a}}$ is the reduced frequency. The ε and Da reflect the effect of the porous matrix size on the acoustic problem under consideration. The case of $\varepsilon=1$ or $Da=0$ corresponds to the plain medium without the presence of the solid matrix and any values of $0<\varepsilon<1$ or $Da>0$ represent a porous medium with different pore spaces. The no-slip boundary conditions of the fluid velocity at wall:

$$u=0,\ T=0,\ \text{at}\ \eta=1 \qquad ...(17)$$

Since the steady flow profile is also parabolic, variational solutions with the following form of axial acoustic velocity variation is sought.

$$u=C(1-\eta^2)\ \text{where } C \text{ is constant} \qquad ...(18)$$

VARIATIONAL SOLUTIONS

A variational solution of the momentum equation results in expression for constant C is

$$\frac{\partial f}{\partial u}=\frac{d}{d\eta}\left(\frac{\partial u}{\partial(du/d\eta)}\right) \qquad ...(19)$$

$$f=\left[\left(\frac{1}{s^2\varepsilon}\right)\eta\left(\frac{du}{d\eta}\right)^2+\frac{iu^2\eta}{\varepsilon}+\left(\frac{2Z}{\gamma}\right)pu\eta+\frac{2\bar{M}Z}{\varepsilon^2}(1-\eta)^2\eta u^2+\frac{2Da}{s^2}u^2\eta\right]d\eta \qquad ...(20)$$

The best approximation to the equation (2) corresponds to the minimum of the functional is

$$F=\int_0^1\left[\left(\frac{1}{s^2\varepsilon}\right)\eta\left(\frac{du}{d\eta}\right)^2+\frac{iu^2\eta}{\varepsilon}+\left(\frac{2Z}{\gamma}\right)pu\eta+\frac{2\bar{M}Z}{\varepsilon^2}(1-\eta)^2\eta u^2+\frac{Da}{s^2}u^2\eta\right]d\eta \qquad ...(21)$$

The trial solution for u, equation (3.18), is substituted into this expression and the minimum is found by setting:

$$\frac{\partial F}{\partial C}=0 \qquad ...(22)$$

We find

$$C = -\frac{\dfrac{pZ}{2\gamma}}{\left(\dfrac{2}{s^2\varepsilon} + \dfrac{i}{3} + \dfrac{\bar{M}Z}{2\varepsilon^2} + \dfrac{Da}{3s^2}\right)} \qquad ...(23)$$

It is necessary here to consider the equations of energy and state to relate the pressure to the density, eliminating the temperature, the energy equation corresponds to the minimum of the functional is

$$H = \int_0^1 \left[\left(\frac{1}{s^2\sigma^2}\right)\eta\left(\frac{dT}{d\eta}\right)^2 - 2\left(\frac{\gamma-1}{\gamma}\right)\left[i + 2\bar{M}Z(1-\eta)\right]\eta pT + 2\bar{M}Z\eta(1-\eta^2)\eta T^2 + iu^2\eta + \frac{16}{s^2}(\gamma-1)\bar{M}\,\eta\frac{d}{d\eta}(\eta u)\right] \qquad ...(24)$$

The trial solution of the temperature has been assumed as

$$T = D(1-\eta^2),\ D \text{ is constant} \qquad ...(25)$$

Which identically satisfies the boundary conditions on the temperature, then the minimum of the functional is found by setting:

$$\frac{\partial H}{\partial D} = 0 \qquad ...(26)$$

We find

$$D = \frac{\left(\dfrac{\gamma-1}{\gamma}\right)\left(\dfrac{i}{2} + \dfrac{2}{3}\bar{M}Z\right)p}{\left(\dfrac{2}{s^2\sigma^2} + \dfrac{i}{3} + \dfrac{\bar{M}Z}{2}\right)} \qquad ...(27)$$

Now from equation (16) and (9), we get

$$\rho = p - D(1-\eta^2) \qquad ...(28)$$

And hence the integral form of the continuity equation (23) becomes

$$\int_0^1 \eta\left[\frac{i}{\varphi} + 2\bar{M}Z(1-\eta^2)\,[p - D(1-\eta^2)] + ZC(1-\eta^2)\right]d\eta = 0 \qquad ...(29)$$

Evaluating the integral and substituting from equation (23) and equation (27) for *C* and *D* leads to an equation for the propagation constant:

$$\frac{i}{\varepsilon} + \bar{M}Z - \frac{\dfrac{Z^2}{4\gamma}}{\left(\dfrac{2}{s^2\varepsilon} + \dfrac{i}{3\varepsilon} + \dfrac{\bar{M}Z}{2\varepsilon^2} + \dfrac{Da}{3s^2}\right)} - \frac{\left(\dfrac{\gamma-1}{\gamma}\right)\left(\dfrac{i}{2} + \dfrac{2}{3}\bar{M}Z\right)\left(\dfrac{i}{2\varepsilon} + \dfrac{2\bar{M}Z}{3}\right)}{\left(\dfrac{2}{\sigma^2\varepsilon^2} + \dfrac{i}{3} + \dfrac{\bar{M}Z}{2}\right)} = 0 \qquad ...(30)$$

This is a cubic equation in Z for which no single analytical solution can be found, in the two limiting cases of either zero steady flow $\overline{M} = 0,$ or fluid of unit Prandtl number, $\sigma = 1,$ the equation becomes quadratic in Z and an analytical solution results.

Result and Discussion

Here, we give attention to the imaginary part of propagation constant $Z(I),$ the greatest physical insight follows from a study of phase velocity W, which when written in non dimensional form is simply the inverse of $Z(I),$ or

$$w = \left| \frac{W}{\overline{a}} \right| = \left| \frac{1}{Z(I)} \right| \qquad ....(31)$$

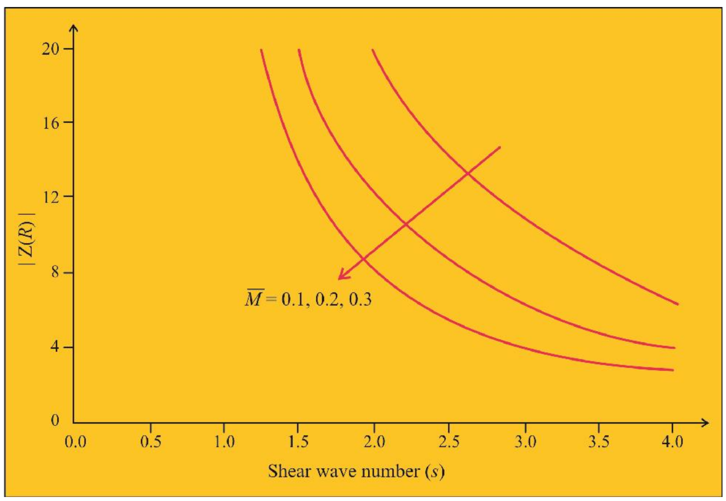

Fig. 1. Attenuation for different values of Mach Numbers taking $Da = 1.0$ **and** $\varepsilon = 0.8$

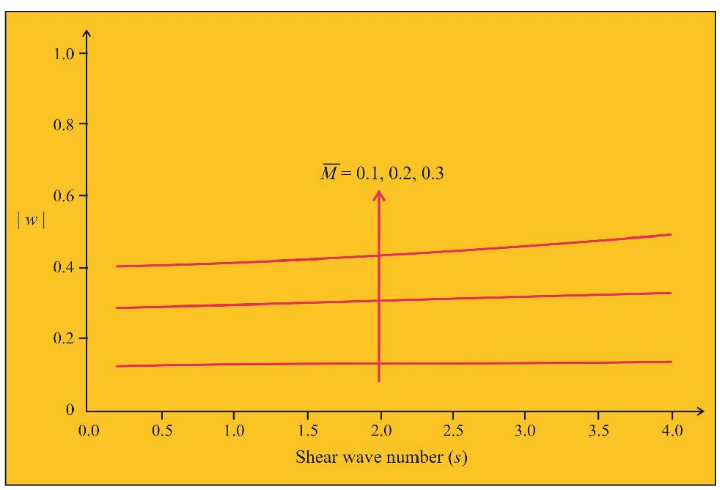

Fig. 2. Phase velocities for different values of Mach Numbers taking $Da = 1.0$ **and** $\varepsilon = 0.8$

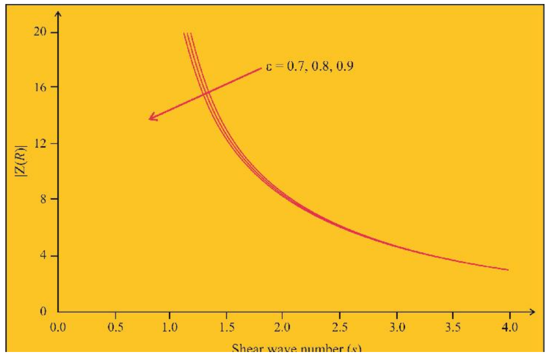

Fig. 3. Attenuation for different values of porosity taking $Da = 1.0$ **and** $\bar{M} = 0.3$ **except for** $Da = 0,\ \varepsilon = 1.0$

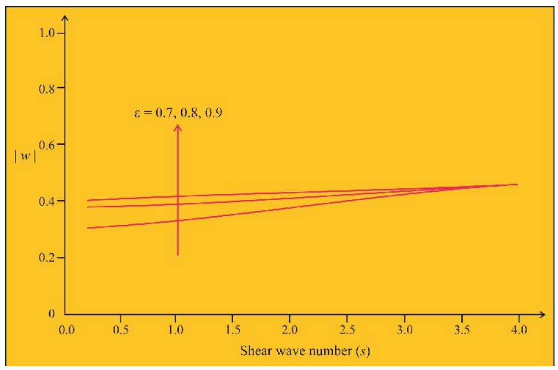

Fig. 4. Phase velocities for different values of Porosity taking $Da = 1.0$ **and** $\bar{M} = 0.3$ **except for** $Da = 0,\ \varepsilon = 1.0$

Figure 1 and 2 is a plot of modulus of wave attenuation per unit distance, $Z(R)$ and phase velocity $\left|\frac{1}{Z(I)}\right|$ for varying shear wave number and Mach numbers. It is clear that as the Mach number is increased the attenuation for the hydrodynamic sound waves is decreased and consequently the phase velocities are increased; this is due to unfavourable collision effect at higher velocities of the hydrodynamic sound waves. Therefore, Mach number has destabilizing effect on the system.

Figure 4 and 4 is a plot of modulus of wave attenuation per unit distance, $Z(R)$ and phase velocity $\left|\frac{1}{Z(I)}\right|$ for varying shear wave number and porosity, it is found that as the porosity is increased the attenuation is decreased and the phase velocities are increased for the hydrodynamic sound waves; this is due to neglecting porous medium effects in damping sound waves, as we move toward the plain media limit. Thus, porosity has destabilizing effect.

Figure (5) and (6) shows the effect of Darcy number on both attenuation and phase velocities. It is clear that as the Darcy number is increased the attenuation is increased for the hydrodynamic sound waves; this is due to favourable damping effect of the solid matrix of the three types of sound waves. It is also found that as the Darcy number is increased the phase velocities are increased for the hydrodynamic sound waves; this is due to favourable effect of the porous matrix in increasing phase shift of the three sound waves. Therefore, Darcy number has stabilizing effect on the system.

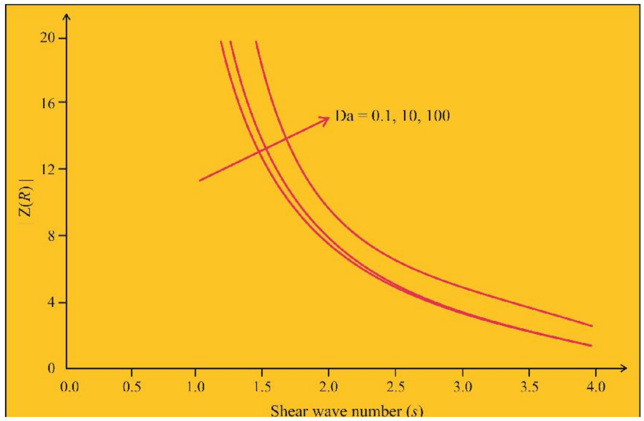

Fig. 5. Attenuation for different values of Darcy number taking $\varepsilon = 0.8$ **and** $\bar{M} = 0.3$ **except for** $Da = 0,\ \varepsilon = 1.0$

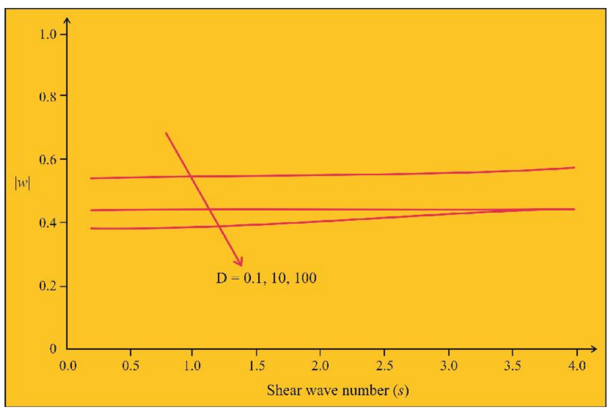

Fig. 6. Phase velocities for different values of Darcy number taking $\varepsilon = 0.8$ **and** $\bar{M} = 0.3$ **except for** $Da = 0,\ \varepsilon = 1.0$

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