Acta Ciencia Indica, Vol. XLIII M, No. 2, 141 (2017)

A SPECIAL CASE STUDY ON A 10 × 10 SYMMETRIC PROBLEM IN GAME THEORY – BROWN'S ALGORITHM

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RECEIVED : 6 October, 2017

The paper mainly intends to study a row and column both dominance game with the help of Brown's Algorithm. A 10 × 10 game is constructed on dominance strategy. Few results are obtained by considering maximum number of possible iterations. The relations among the Lower bounds and/or Upper bounds of this game at various levels are identified.

KEYWORDS: Game Theory, players, strategy, Pay-off matrix, optimal solution, Lower bound, Upper bound.

AMS Classification : 91A05, 91A18, 91A43, 91A90.

Introduction

In 1979, Billy E. Gillett [1] established many algorithms to solve many complicated problem in game theory. The theory of games and it's applications were thoroughly discussed by Levin and Desjardins [2] in 1970. Many other famous mathematicians like Rapoport [3], Dresher [4], Raiffa [5], McKinsey [6] etc studied on different strategies and useful applications of game theory.

Basic formation of 10X10 game

The game is constructed with 10 rows and 10 columns according to player *A* and Player *B*. One player selects only one single action from his/her set possible actions. It consists of ten possible actions of *A i.e* *A*1, *A*2, *A*3, *A*4, *A*5, *A*6, *A*7, *A*8, *A*9, *A*10 which will effect on the other ten possible actions of player *B i.e.* *B*1, *B*2, *B*3, *B*4, *B*5, *B*6, *B*7, *B*8, *B*9, *B*10. It is very much convenient to us to consider the influence of Player *B* on the components of Player *A* with maximum possible extent ten units. With this strategy, the game is framed by increasing one unit in Actions of Player *B* and systematic increase with quantity ten units in actions of player *A* to meet the required criteria of row and column dominance in the game.

The pay off matrix of constituted game having the size 10 × 10 is given below.

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 \\ 3 & 12 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 \\ 4 & 13 & 21 & 28 & 29 & 30 & 31 & 32 & 33 & 34 \\ 5 & 14 & 22 & 29 & 35 & 36 & 37 & 38 & 38 & 40 \\ 6 & 15 & 23 & 30 & 36 & 41 & 42 & 43 & 44 & 45 \\ 7 & 16 & 24 & 31 & 37 & 42 & 46 & 47 & 48 & 49 \\ 8 & 17 & 25 & 32 & 38 & 43 & 47 & 50 & 51 & 52 \\ 9 & 18 & 26 & 33 & 39 & 44 & 48 & 51 & 53 & 54 \\ 10 & 19 & 27 & 34 & 40 & 45 & 49 & 52 & 54 & 55 \end{bmatrix}$$

MATERIAL AND METHODS

The author applied Brown's algorithm to solve this special case of 10 × 10 game in which row and columns both dominated. Brown's Algorithm:

Step 1: Player A chooses one of the possible actions (Ai_1) from $A1$-$A10$ to play, and Player B then plays with the possible action Bj_1 corresponding to the smallest element in the selected action Ai_1.

Step 2: Player A then picks out the possible action (Ai_2) from $A1$-$A10$ to play corresponding to the largest element in the possible action (Bj_1) selected by Player B in step 1.

Step 3: Player B sums the actions of Player A who has played thus far, and plays with the possible action of Bj_2 corresponding to a smallest sum element.

Step 4: Player A sums the actions of Player B who has played thus far, and plays the possible action (Ai_3) corresponding to a largest sum element. After the required iterations are computed,then go to step 5; otherwise, come back to step 3.

Step 5: Compute an upper and lower bound $\underline{\gamma}$ and $\overline{\gamma}$ respectively.

$$\overline{\gamma} = \frac{\text{Largest sum element from step 4}}{\text{Number of plays of the game thus far}}$$

and
$$\underline{\gamma} = \frac{\text{Smallest sum element from step 3}}{\text{Number of plays of the game thus far}}$$

Step 6: Let Xi be the portion of the time Player A played row i with $i = 1, 2, ..., m$ and let Yi be the proportion of the time Player B played column j with $j = 1, 2, ..., n$. These strategies approximate the optimal mini max strategies. Upper and Lower bounds of the value of the game are where $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$ are calculated in step 5. The Process completes.

RESULTS

This case is studied with Brown's Algorithm to obtain best strategies by considering 50th iteration to 500th iterations by Java program. The influences of player B on Player A are presented in the tabular forms from Table (1) to Table (10) at each iteration.

Table 1. Player A vs. Player B at 50^{th} Iteration

Player A					**Player B**					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**
50	491	493	494	494	495	496	497	498	499	500
100	933	943	944	944	945	946	947	948	949	950
150	1326	1343	1344	1344	1345	1346	1347	1348	1349	1350
200	1670	1687	1694	1694	1695	1696	1697	1698	1699	1700
250	1965	1982	1989	1989	1995	1996	1997	1998	1999	2000
300	2211	2228	2235	2235	2241	2246	2247	2248	2249	2250
350	2408	2425	2432	2432	2438	2443	2447	2448	2449	2450
400	2556	2573	2580	2580	2586	2591	2595	2598	2599	2600
450	2655	2672	2679	2679	2684	2690	2694	2697	2699	2700
500	2705	2722	2729	2729	2735	2740	2744	2747	2749	2750

Table 2. Player *A* vs Player *B* at 100th Iteration

Player A					**Player B**					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**
100	991	992	993	994	995	996	997	998	999	1000
200	1883	1892	1893	1894	1895	1896	1897	1898	1899	1900
300	2676	2685	2693	2694	2695	2696	2697	2698	2699	2700
400	3370	3379	3387	3394	3395	3396	3397	3398	3399	3400
500	3965	3974	3982	3989	3995	3996	3997	3998	3999	4000
600	4461	4470	4478	4485	4491	4496	4497	4498	4499	4500
700	4858	4867	4875	4882	4888	4893	4897	4898	4899	4900
800	5156	5165	5173	5180	5186	5191	5195	5198	5199	5200
900	5355	5364	5372	5379	5384	5390	5394	5397	5399	5400
1000	5455	5464	5472	5479	5485	5490	5494	5497	5499	5500

Table 3. Player *A* vs Player *B* at 150th Iteration

Player A	**Player B**									
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**
150	1491	2833	4026	5070	5965	6711	7308	7756	8055	8205
300	1492	2842	4035	5079	5974	6720	7317	7765	8064	8214
450	1493	2843	4043	5087	5982	6728	7325	7773	8072	8222
600	1494	2844	4044	5094	5989	6735	7332	7780	8079	8229
750	1495	2845	4045	5095	5995	6741	7338	7786	8084	8235
900	1496	2846	4046	5096	5996	6746	7343	7791	8090	8240
1050	1497	2847	4047	5097	5997	6747	7347	7795	8094	8244

1200	1498	2848	4048	5098	5998	6748	7348	7798	8097	8247
1350	1499	2849	4049	5099	5999	6749	7349	7799	8099	8249
1500	1500	2850	4050	5100	6000	6750	7350	7800	8100	8250

Table 4. Player *A* vs Player *B* at 200th Iteration

Player A					**Player B**					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**
200	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
400	3783	3792	3793	3794	3795	3796	3797	3798	3799	3800
600	5376	5385	5393	5394	5395	5396	5397	5398	5399	5400
800	6770	6779	6787	6794	6795	6796	6797	6798	6799	6800
1000	7965	7974	7982	7989	7995	7996	7997	7998	7999	8000
1200	8961	8970	8978	8985	8991	8996	8997	8998	8999	9000
1400	9758	9767	9775	9782	9788	9793	9797	9798	9799	9800
1600	10356	10365	10373	10380	10386	10391	10395	10398	10399	10400
1800	10755	10764	10772	10779	10784	10790	10794	10797	10799	10800
2000	10955	10964	10972	10979	10985	10990	10994	10997	10999	11000

Table 5. Player *A* vs Player *B* at 250th Iteration

Player A					**Player B**					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**
250	2491	2492	2493	2494	2495	2496	2497	2498	2499	2500
500	4733	4742	4743	4744	4745	4746	4747	4748	4749	4750
750	6726	6735	6743	6744	6745	6746	6747	6748	6749	6750
1000	8470	8479	8487	8494	8495	8496	8497	8498	8499	8500
1250	9965	9974	9982	9989	9995	9996	9997	9998	9999	10000
1500	11211	11220	11228	11235	11241	11246	11247	11248	11249	11250
1750	12208	12217	12225	12232	12238	12243	12247	12248	12249	12250
2000	12956	12965	12973	12980	12986	12991	12995	12998	12999	13000
2250	13455	13464	13472	13479	13484	13490	13494	13497	13499	13500
2500	13705	13714	13722	13729	13735	13740	13744	13747	13749	13750

Table 6. Player *A* vs Player *B* at 300th Iteration

Player A					**Player B**					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**
300	2991	2992	2993	2994	2995	2996	2997	2998	2999	3000
600	5683	5692	5693	5694	5695	5696	5697	5698	5699	5700
900	8076	8085	8093	8094	8095	8096	8097	8098	8099	8100
1200	10170	10179	10187	10194	10195	10196	10197	10198	10199	10200
1500	11965	11974	11982	11989	11995	11996	11997	11998	11999	12000

1800	13461	13470	13478	13485	13491	13496	13497	13498	13499	13500
2100	14658	14667	14675	14682	14688	14693	14697	14698	14699	14700
2400	15556	15565	15573	15580	15586	15591	15595	15598	15599	15600
2700	16155	16164	16172	16179	16184	16190	16194	16197	16199	16200
3000	16455	16464	16472	16479	16485	16490	16494	16497	16499	16500

Table 7. Player *A* vs Player *B* at 350th Iteration

Player A					**Player B**					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**
350	3491	3492	3493	3494	3495	3496	3497	3498	3499	3500
700	6633	6642	6643	6644	6645	6646	6647	6648	6649	6650
1050	9426	9435	9443	9444	9445	9446	9447	9448	9449	9450
1400	11870	11879	11887	11894	11895	11896	11897	11898	11899	11900
1750	13965	13974	13982	13989	13995	13996	13997	13998	13999	14000
2100	15711	15720	15728	15735	15741	15746	15747	15748	15749	15750
2450	17108	17117	17125	17132	17138	17143	17147	17148	17149	17150
2800	18156	18165	18173	18180	18186	18191	18195	18198	18199	18200
3150	18855	18864	18872	18879	18884	18890	18894	18897	18899	18900
3500	19205	19214	19222	19229	19235	19240	19244	19247	19249	19250

Table 8. Player *A* vs Player *B* at 400th Iteration

Player A					**Player B**					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**
400	3991	3992	3993	3994	3995	3996	3997	3998	3999	4000
800	7583	7592	7593	7594	7595	7596	7597	7598	7599	7600
1200	10776	10785	10793	10794	10795	10796	10797	10798	10799	10800
1600	13570	13579	13587	13594	13595	13596	13597	13598	13599	13600
2000	15965	15974	15982	15989	15995	15996	15997	15998	15999	16000
2400	17961	17970	17978	17985	17991	17996	17997	17998	17999	18000
2800	19558	19567	19575	19582	19588	19593	19597	19598	19599	19600
3200	20756	20765	20773	20780	20786	20791	20795	20798	20799	20800
3600	21555	21564	21572	21579	21584	21590	21594	21597	21599	21600
4000	21955	21964	21972	21979	21985	21990	21994	21997	21999	22000

Table 9. Player *A* vs Player *B* at 450th Iteration

Player A					**Player B**					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**
450	4491	4492	4493	4494	4495	4496	4497	4498	4499	4500
900	8533	8542	8543	8544	8545	8546	8547	8548	8549	8550
1350	12126	12135	12143	12144	12145	12146	12147	12148	12149	12150

1800	15270	15279	15287	15294	15295	15296	15297	15298	15299	15300
2250	17965	17974	17982	17989	17995	17996	17997	17998	17999	18000
2700	20211	20220	20228	20235	20241	20246	20247	20248	20249	20250
3150	22008	22017	22025	22032	22038	22043	22047	22048	22049	22050
3600	23356	23365	23373	23380	23386	23391	23395	23398	23399	23400
4050	24255	24264	24272	24279	24284	24290	24294	24297	24299	24300
4500	24705	24714	24722	24729	24735	24740	24744	24747	24749	24750

Table 10. Player *A* vs Player *B* at 500th Iteration

Player A					**Player B**					
	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**
500	4991	4992	4993	4994	4995	4996	4997	4998	4999	5000
1000	9483	9492	9493	9494	9495	9496	9497	9498	9499	9500
1500	13476	13485	13493	13494	13495	13496	13497	13498	13499	13500
2000	16970	16979	16987	16994	16995	16996	16997	16998	16999	17000
2500	19965	19974	19982	19989	19995	19996	19997	19998	19999	20000
3000	22461	22470	22478	22485	22491	22496	22497	22498	22499	22500
3500	24458	24467	24475	24482	24488	24493	24497	24498	24499	24500
4000	25956	25965	25973	25980	25986	25991	25995	25998	25999	26000
4500	26955	26964	26972	26979	26984	26990	26994	26997	26999	27000
5000	27455	27464	27472	27479	27485	27490	27494	27497	27499	27500

4.1. Observation from the Iterations:

(i) Maximum possible correlation is maintained at a possible action of Player *B* and possible actions of Player of *A*.

(ii) The influence of Player *B* uniformly effects on the possible action of Player *A* in each iteration.

(iii) The uniform variations among the iterations are observed

(iv) Damaging fluctuations in the game are not traced.

(v) Periodic developments have been established.

(vi) Constant differences between the values of possible actions of player *A* at any two consequent iterations have been determined

(vii) Constant differences between the values of possible actions of player *B* at any two consequent iterations have been ascertained.

OPTIMUM MIXED STRATEGIES OF PLAYER *A* AND PLAYER *B*

The optimum mixed strategies of Player *A* and Player *B* are illustrated and shown in Table-11.

Table 11.

Optimum Mixted strategies of Player A and Player B (Iteration wise)																			
50		**100**		**150**		**200**		**250**		**300**		**350**		**400**		**450**		**500**	
A	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

UPPER BOUNDS AND LOWER BOUNDS AT ALL ITERATIONS

At each play of the game the minimum sum element selected by player *B* divided by the number of place of the game is known as lower bound.

Similarly at each play of the game the maximum sum element selected by player *A* divided by the number of place of the game is called as upper bound.

The Values of U.Bs and L.Bs in 10 × 10 game are tabulated in Table-12.

Table-12

U.B	**Lower Bounds**									
Iterations / 50-500	**50**	**100**	**150**	**200**	**250**	**300**	**350**	**400**	**450**	**500**
10	9.82	9.91	9.940	9.955	9.964	9.970	9.974	9.974	9.980	9.982
10	9.86	9.92	9.946	9.960	9.968	9.973	9.977	9.977	9.982	9.984
10	9.88	9.93	9.953	9.965	9.972	9.976	9.980	9.981	9.984	9.986
10	9.88	9.94	9.960	9.970	9.976	9.980	9.982	9.982	9.986	9.988
10	9.90	9.95	9.967	9.975	9.980	9.983	9.985	9.985	9.988	9.990
10	9.92	9.96	9.973	9.980	9.840	9.986	9.988	9.988	9.991	9.992
10	9.94	9.97	9.980	9.985	9.988	9.990	9.991	9.991	9.993	9.994
10	9.96	9.98	9.986	9.990	9.992	9.993	9.994	9.994	9.995	9.996
10	9.98	9.99	9.993	9.995	9.996	9.996	9.997	9.997	9.997	9.998
10	10	10	10	10	10	10	10	10	10	10

Conclusions

(i) The value of the game is 10.

(ii) The optimum mixed strategies of player A and player B are identical in any iteration.

(iii) The value of lower bound does not equal to the value of the game but it approached gradually towards the value of the game at final stage in any iteration.

(iv) The value of upper bound is unique throughout the game at every level of iteration.

(v) The Initial error is 0.18.It will be reduced from iteration by iteration.

Over all conclusions

(i) The best strategies are obtained for the both players.

(ii) Variations in the iterations are very nominal.

(iii) The considered game is classified as a strictly determinable game because of the same values of lower bound and upper bound at final stage.

(iv) The errors are minimized step by step.

(v) The revolution of the conflict is obtained in this game.

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