

## **ARITHMETIC PROGRESSION ON MOST LIKELY TIME ESTIMATE-A CASE STUDY**

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In this Paper, The impact of Arithmetic Progression (A.P.) on a network is observed in one case. A network is constructed in a symmetric way with 28 activities and 22 nodes. A.P is applied on a most likely time estimate among the three time estimates namely optimistic, most likely and pessimistic. Project analysis and Periodical analysis both are employed on the constructed network. Some special results are established.

**KEYWORDS** : Network, Time estimates, Float, Critical path, Normal distribution.

**AMS Classification** : 90-08, 90B10, 90C90.

### **INTRODUCTION**

The main objective Target of any Project is to achieve specified set of goals in an efficient manner. The constraints of a project are influenced by available resources cost management and duration of the time. PERT/CPM are the good techniques in the network analysis which reach our necessary requirements to accomplish the given project. Sometimes the implementations will create unexpected problems based on size and nature of the project. In such case the errors are to be fixed first and then move further. The results and observations are to be identified and the necessary corrections are to be carried out simultaneously from time to time. The successful completion of any project manages perfectly budgeted cost, quality, quantity and specified time.

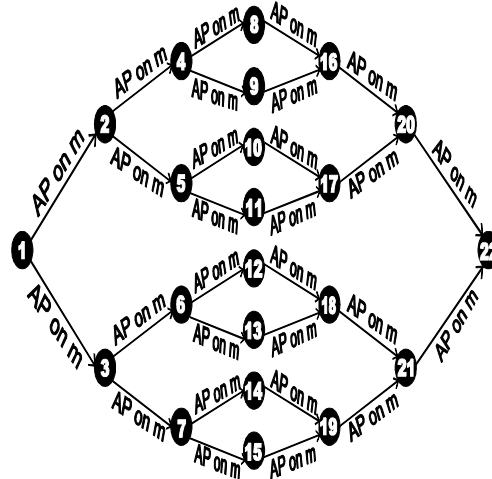
In 1966 Levin, Kirk Patrick [6] discussed about planning and explained us how to control the constraints with the help of PERT and CPM. Wiest and Levy [7] invented many things in network analysis which help us as management guide for the beginners of research community in operations research. Various models in network analysis were properly explained by Billy E. Gillett [8] in 1979. Later S.D. Sarma [9] continued the same spirit to attempt many difficult models in operations research and established the uses and drawbacks of the existing computational techniques. K.V.L.N. Acharyulu *et. al* [1-5] introduced many Network projects which are influenced by various progressions on different time estimates.

In this Paper the Arithmetic Progression is applied on most likely time estimate among the three time estimates. It is mainly observed whether Arithmetic Progression will strengthen a network or not. A network is drawn in a symmetric and systematic way with 22 nodes and 28 activities. Computational study has been applied on the constructed network and some fruitful results are established. Float values are also calculated and critical path is traced.

Project analysis and periodical analysis both are used to illustrate the project with standard normal distributive curves.

## BASIC CONSTRUCTION OF NETWORK

A network is considered with 28 activities and 22 nodes in a symmetric way for analyzing the impact of **Arithmetic Progression**. A.P is applied on most likely time estimate (m) in this case among the three estimates. No Dummy activity has involved.



Network with 22 nodes and 28 activities

## PRELIMINARIES AND NOTATIONS:

**(i)** **TE** = Earliest expected completion time of event (TE)

**Def:** For the fixed value of  $j=TE(j)=\text{Max}[TE(i)+ET(i,j)]$  which ranges over all activities from  $i-j$ .

(ii) **TL** = Latest allowable event completion time (TL)

**Def:** For the fixed value of  $i=TL(i)=\text{Min}[TL(j)+ET(i,j)]$  which ranges over all activities from  $i-j$ .

(iii) **ET** = Expected completion time of activity (I,J)

(iv) **a** = Optimistic time estimate

(v) **m** = Most likely time estimate

(vi) **b** = Pessimistic time estimate

(vii) **ES** = Earliest start of an activity

(viii) **EF** = Earliest finish of an activity

(ix) **LS** = Latest start of an activity

(x) **LF** = Latest finish of an activity

(xi) **TF** = Total Float

**Def:** TF of activity i-j =  $LF_{i-j} - EF_{i-j}$  (or)  $LS_{i-j} - ES_{i-j}$

(xii) **FF** = Free Float

**Def:** FF of activity i-j = TF - (TL-TE) of node j

(xiii) **IF** = Independent Float

**Def:** IF of activity i-j = FF - (TL-TE) of node i

(xiv) **SE**=Slack event time

(xv) **CPI**=Critical Path Indicator

(xvi) **SCT**= Scheduled Time

(xvii)  $\sigma$  = Standard deviation of project length

## MATERIAL AND METHODS

**Step 1:** Draw the project network completion time

**Step 2:** Compute the expected duration of each activity by using the formula

$$ET = \frac{a + 4m + b}{6}$$

From the time estimates  $a$ ,  $m$  and  $p$ . Also calculate the expected variance.  $\sigma^2$  of each activity

**Step 3:** Calculate  $TE$ ,  $TL$

**Step 4:** Find Total Float, Free Float and Independent Float

**Step 5:** Find the critical path and identify the critical activities

**Step 6:** Compute project length which is a square root of sum of variance of all the critical activities.

**Step 7:** Estimate the probability of completing project within specified time,

Using the standard normal variable  $z = \frac{SCT - ET}{\sigma}$ , where  $SCT$  is scheduled Completion time of event,  $\sigma$  = standard deviation of project length.

## RESULTS

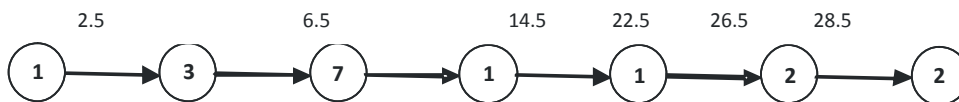
**B**y using CPM and PERT algorithm on the Network, the critical path is traced from Table-1. Time estimates, ET, Variance. ES, EF, LS, LF and all Float values are calculated and tabulated in the Table-1. The Critical path indicator marked the critical Actives with \* in the Table-1.

**Table-1**

Activity	Time Estimates				$\sigma$	Earliest		Latest		TF	FF	IF	CPI
	a	m	b	ET		ES	EF	LS	LF				
	1--2	1	1.5	2		1.5	0.027	0	1.5				

1--3	2	2.5	3	2.5	0.027	0	2.5	0	2.5	0	0	0	*
2--4	3	3.5	4	3.5	0.027	1.5	5	21.5	25	20	0	-14	
2--5	4	4.5	5	4.5	0.027	1.5	6	15.5	20	14	0	-14	
3--6	5	5.5	6	5.5	0.027	2.5	8	8.5	14	6	0	0	
3--7	6	6.5	7	6.5	0.027	2.5	9	2.5	9	0	0	0	*
4--8	7	7.5	8	7.5	0.027	5	12.5	27	34.5	22	0	-20	
4--9	8	8.5	9	8.5	0.027	5	13.5	25	33.5	20	0	-20	
5--10	9	9.5	10	9.5	0.027	6	15.5	22	31.5	16	0	-14	
5--11	10	10.5	11	10.5	0.027	6	16.5	20	30.5	14	0	-14	
6--12	11	11.5	12	11.5	0.027	8	19.5	16	27.5	8	0	-6	
6--13	12	12.5	13	12.5	0.027	8	20.5	14	26.5	6	0	-6	
7--14	13	13.5	14	13.5	0.027	9	22.5	11	24.5	2	0	0	
7--15	14	14.5	15	14.5	0.027	9	23.5	9	23.5	0	0	0	*
8--16	15	15.5	16	15.5	0.027	12.5	28	34.5	50	22	2	-20	
9--16	16	16.5	17	16.5	0.027	13.5	30	33.5	50	20	0	-20	
10--17	17	17.5	18	17.5	0.027	15.5	33	31.5	49	16	2	-12	
11--17	18	18.5	19	18.5	0.027	16.5	35	30.5	49	14	0	-14	
12--18	19	19.5	20	19.5	0.027	19.5	39	27.5	47	8	2	-6	
13--18	20	20.5	21	20.5	0.027	20.5	41	26.5	47	6	0	-6	
14--19	21	21.5	22	21.5	0.027	22.5	44	24.5	46	2	2	0	
15--19	22	22.5	23	22.5	0.027	23.5	46	23.5	46	0	0	0	*
16--20	23	23.5	24	23.5	0.027	30	53.5	50	73.5	20	6	-14	
17--20	24	24.5	25	24.5	0.027	35	59.5	49	73.5	14	0	-20	
18-21	25	25.5	26	25.5	0.027	41	66.5	47	72.5	6	6	0	
19-21	26	26.5	27	26.5	0.027	46	72.5	46	72.5	0	0	0	*
20-22	27	27.5	28	27.5	0.027	59.5	87	73.5	101	14	14	0	
21-22	28	28.5	29	28.5	0.027	72.5	101	72.5	101	0	0	0	*

**Critical Path:**



$$\begin{aligned} \text{Project Length} &= \sqrt{\text{Sum of Variances of each Critical activity}} \\ &= \sqrt{0.027+0.027+0.027+0.027+0.027+0.027} = 0.4024 \end{aligned}$$

The values of *TE*, *TL* and *SE* corresponding to every node are given in Table (2).

The slack event time may be positive, negative or zero.

It is also observed that the values of slack event time vanish at each critical activity.

Slack event time is defined as the amount of time in which the event can be retarded without involving the scheduled completion time for the project. Any activity on the critical path necessitates time in excess of its expected completion time and delays the project completion consequently.

**Table-2**

Nodes	TE	TL	SE	Nodes	TE	TL	SE
1	0	0	0	12	19.5	27.5	8
2	1.5	15.5	14	13	20.5	26.5	6
3	2.5	2.5	0	14	22.5	24.5	2
4	5	25	20	15	23.5	23.5	0
5	6	20	14	16	30	50	20
6	8	14	6	17	35	49	14
7	9	9	0	18	41	47	6
8	12.5	34.5	22	19	46	46	0
9	13.5	33.5	20	20	59.5	73.5	14
10	15.5	31.5	16	21	72.5	72.5	0
11	16.5	30.5	14	22	101	101	0

## PERIODICAL ANALYSIS

The percentage of possibilities of completion of the Project are computed and given in the following Table-3.

**Table-3**

ST	TE	Z	PROBABILITY	PERCENT OF POSSIBILITY (%)
25	28.5	- 8.69781	0	0
26	28.5	- 6.21272	0	0
27	28.5	- 3.72763	0.0001	0.01
28	28.5	- 1.24254	0.1075	10.75
29	28.5	1.242545	0.8925	89.25
30	28.5	3.727634	0.9999	99.99
31	28.5	6.212724	1	100

The Obtained Standard Normal Curves are given from Fig.1-Fig.6

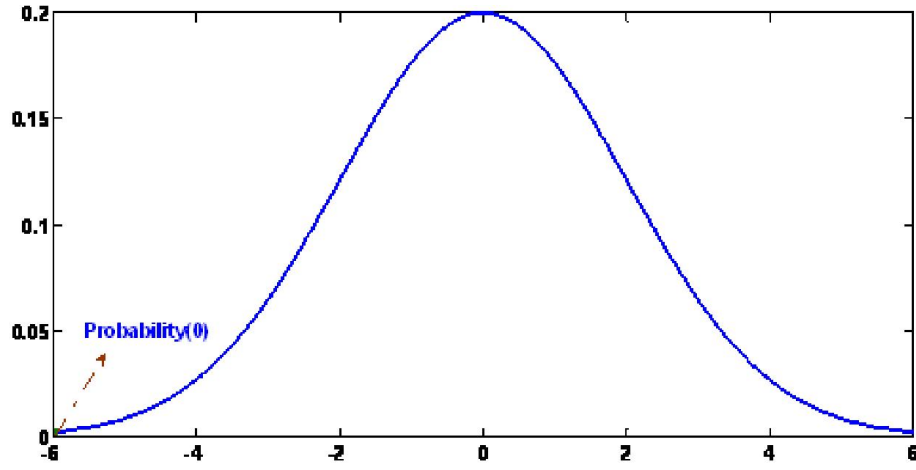


Fig. 1. Target of completion with null probability.

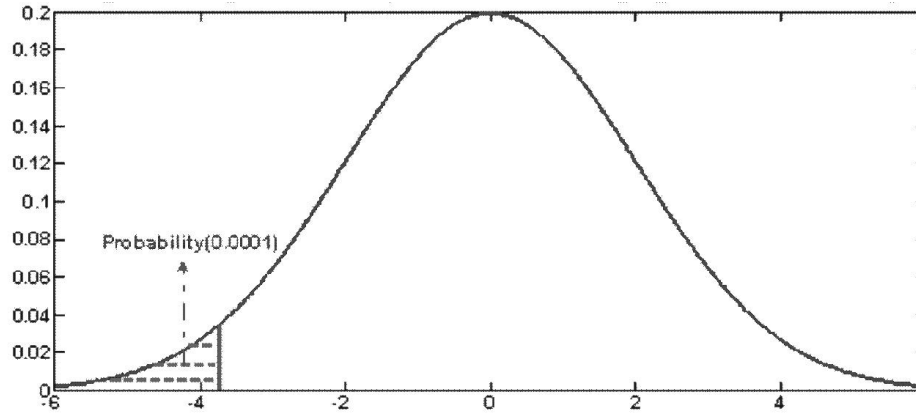


Fig. 2. Target of completion with negligible probability

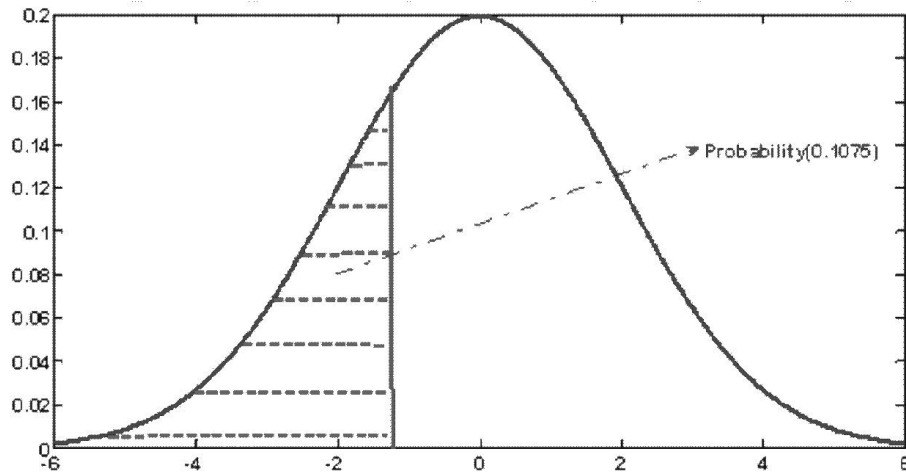


Fig. 3. Target of completion with partial probability.

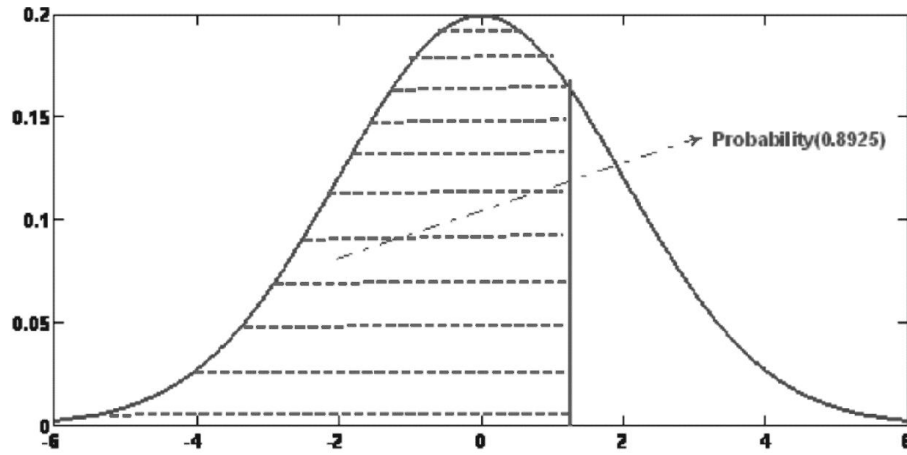


Fig. 4. Target of completion with considerable probability.

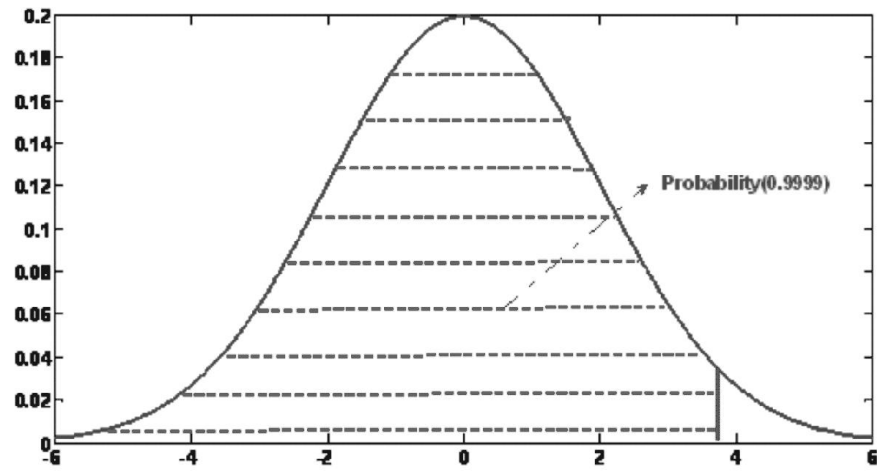


Fig. 5. Target of completion with acceptable probability.

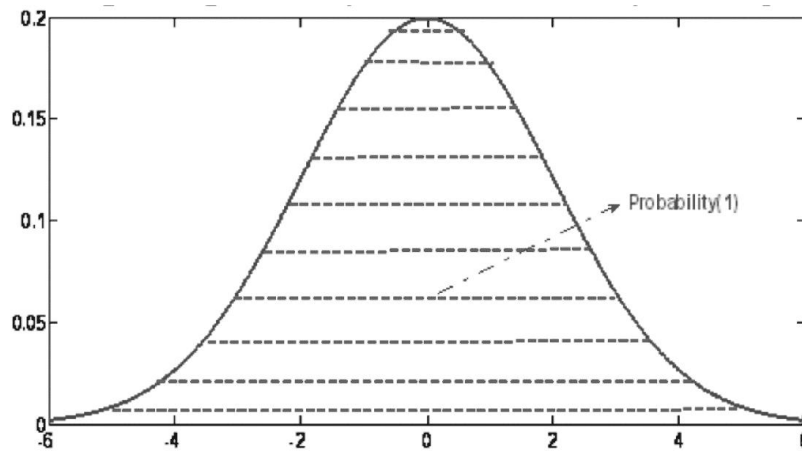


Fig. 6. Target of completion with accurate probability.

## CONCLUSIONS

The following conclusions are obtained from the computational study of PERT

- (i) Arithmetic Progression upholds exactly only when SCT is greater than ET.
- (ii) Arithmetic Progression does not hold up efficiently when SCT is less than or equal to ET.
- (iii) The percentages of completion of project at various probabilities are shown in standard Normal distributive curves.
- (iv) No considerable variances are observed in any activity of the network.
- (v) In the critical path, the total float values are zeroes, slack event of each node is vanished and TE & TL are identical at each node.
- (vi) Finally it is observed that Arithmetic Progression on most likely time estimate will enhance the successfulness of completion of the project gradually.

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