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ON g^* $b^\#$-CLOSED SETS

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In this paper the authors introduce a new class of sets called generalized g^* $b^\#$-closed sets in topological spaces (briefly g^* $b^\#$-closed set). Also we study some of its basic properties and investigate the relations between the associated topology.

KEYWORDS : $gb^\#$-closed sets, g^* $b^\#$-closed sets.

INTRODUCTION

N. Levine [7] introduced the notion of generalized closed (briefly g–closed) sets in topological space (X, τ) in 1970. J. Dontchev [4], H. Maki, R. Devi and K.Balachandran [9], N. Nagaveni [14], M.K.R.S. Veerakumar [21], A.A. Noorani *et. al* [16] introduced and investigated the concept of generalized semi pre closed sets, generalized α-closed sets, regular weakly generalized closed sets, g^*-closed sets and generalized b-closed sets respectively. R. Usha Parameswari *et. al* [20] introduced a class of generalized open sets in a topological space called $b^\#$-open sets. N.Vithya *et. al* [26] introduced and studied the concept of generalized $b^\#$-closed sets (briefly $gb^\#$-closed) in topological spaces. In this paper we introduce a new class of sets called g^* $b^\#$-closed set which is between the class of $b^\#$-closed sets and the class of g $b^\#$-closed sets.

PRELIMINARIES

Throughout this paper, (X, τ), (Y, τ) and (Z, τ) (or simply, X, Y and Z) denote the non empty topological spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of X, then Cl (A) and Int (A) denote the closure and the interior of A respectively**.** The following known definitions and results are used in this paper.

Definition 2.1: A subset A of a topological space (X, τ) is called

(i) **Semi-open [7]** if $A \subseteq \text{cl}(\text{int}(A))$ and **semi-closed** if $\text{int}(\text{cl}(A)) \subseteq A$,

(ii) **Pre-open [13]** if $A \subseteq \text{int}(\text{cl}(A))$ and **pre-closed** if $\text{cl}(\text{int}(A)) \subseteq A$,

(iii) **α-open [15]** if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and **α-closed** if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,

(iv) Semi pre open or β-open [1] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and semi pre closed or β-closed if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$,

(v) **Regular open [19]** if $\text{int}(\text{cl}(A)) = A$ and **regular closed** if $\text{cl}(\text{int}(A)) = A$.

(vi) ***b*-open**[2] if $A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$ and **bclosed** if $cl(\text{int}(A)) \cap \text{int}(cl(A)) \subseteq A$,

(vii) ***b*#-open** [22] if A = cl (int (A)) $\cup$ int (cl (A)) and ***b*#-closed** if A = cl (int (A)) $\cap$ int (cl (A)).

(viii) ***p*-set** [20] if cl (int (A)) $\subseteq$ int (cl (A)).

(ix) ***q*-set** [21] if int (cl (A)) $\subseteq$ cl (int (A)).

The semi closure (respectively, pre closure, α closure, semi pre closure, b closure and $b^{\#}$-closure) of a subset A of a space (X, τ) is the intersection of all semi-closed sets (respectively, pre-closed, α-closed, semi pre closed, bclosed and $b^{\#}$-closed) sets containing A and is denoted by scl (A) (respectively, pcl (A), αcl (A), spcl (A), bcl (A) and $b^{\#}$-cl (A)).

Definition 2.2 [8]: A subset A of a topological space (X, τ) is called ***g*-closed** if cl (A) $\subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.3 [17]: A subset A of a topological space (X, τ) is called ***rg*-closed** if cl (A) $\subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).

Definition 2.4 [3]: A subset A of a topological space (X, τ) is called ***sg*-closed** if scl (A) $\subseteq U$ whenever $A \subseteq U$ and U is semi open in (X, τ).

Definition 2.5 [11]: A subset A of a topological space (X, τ) is called ***gp*-closed** if pcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.6 [5]: A subset A of a topological space (X, τ) is called ***gpr*-closed** if pcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).

Definition 2.7 [24]: A subset A of a topological space (X, τ) is called ***g**p*-closed** if pcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).

Definition 2.8 [9]: A subset A of a topological space (X, τ) is called ***g*α-closed** if αcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).

Definition 2.9 [10]: A subset A of a topological space (X, τ) is called **α*g*-closed** if αcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.10 [6]: A subset A of a topological space (X, τ) is called ***gspr*-closed** if spcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is pre-open in (X, τ).

Definition 2.11 [4]: A subset A of a topological space (X, τ) is called ***gsp*-closed** if spcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).

Definition 2.12 [25]: A subset A of a topological space (X,τ) is called **pre semi closed** if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X,τ).

Definition 2.13 [16]: A subset A of a topological space (X, τ) is called ***gb*-closed** if bcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.14 [12]: A subset A of a topological space (X, τ) is called ***rgb*-closed** if bcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).

Definition 2.15 [27]: A subset A of a topological space (X, τ) is called ***g*α*b*-closed** if bcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).

Definition 2.16 [26]: A subset A of a topological space (X, τ) is called ***g**b*-closed** if bcl (A) $\subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).

Definition 2.17 [23]: A subset A of a topological space (X, τ) is called ***gb*#-closed** if $b^{\#}$cl (A) $\subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.18 [14]: A subset A of a topological space (X, τ) is called ***wg*-closed** if cl int $(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.19 [18]: A subset A of a topological space (X, τ) is called mildly ***g*-closed** if cl int $(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).

Definition 2.20 [14]: A subset A of a topological space (X, τ) is called ***rwg*-closed** if cl int $(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).

Lemma 2.21: Let A be a sub set of a topological space X. Then cl (int (A)) $\cap$ int (cl (A)) $\subseteq$ bcl $(A) \subseteq b^{\#}$ – cl (A).

Theorem 2.22:

(i) If A is a p-set then cl (int (A)) $\subseteq b^{\#}$ – cl (A).

(ii) If A is a q-set then int (cl (A)) $\subseteq b^{\#}$ – cl(A).

$g^*b^{\#}$-CLOSED SETS

In this section we introduce g^* $b^{\#}$-closed set and discuss their properties.

Definition 3.1: A subset A of a space X is called *a* g^* $b^{\#}$-closed set if $b^{\#}$-cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

Theorem 3.2:

(i) Every $b^{\#}$-closed set is g^* $b^{\#}$-closed set.

(ii) Every g^* $b^{\#}$-closed set is g $b^{\#}$-closed set.

(iii) Every g^* $b^{\#}$-closed set is g bclosed set.

(iv) Every $g^*b^{\#}$-closed set is g^* bclosed set.

(v) Every $g^*b^{\#}$-closed set is *rgb*-closed set.

(vi) Every $g^*b^{\#}$-closed set is *gsp*-closed set.

(vii) Every $g^*b^{\#}$-closed set is *gspr*-closed set.

(viii) Every $g^*b^{\#}$-closed set is pre semi closed set.

Proof:

(i) Let A be a $b^{\#}$-closed set in X such that $A \subseteq U$ where U is g-open in X. Since A is $b^{\#}$-closed, $b^{\#}$-cl $(A) = A$. Therefore $b^{\#}$-cl $(A) \subseteq U$. Hence A is a g^* $b^{\#}$-closed set in X.

(ii) Let A be a g^* $b^{\#}$-closed set in X such that $A \subseteq U$ where U is open in X. Since every open set is g-open, $b^{\#}$-cl $(A) \subseteq U$. Hence A is a g $b^{\#}$-closed set in X.

(iii) Let A be a g^* $b^{\#}$-closed set in X such that $A \subseteq U$ where U is open in X. Since every open set is g-open and since bcl $(A) \subseteq b^{\#}$-cl (A), bcl $(A) \subseteq U$. Hence A is a g b closed set in X.

(iv) Let A be a $g^*b^{\#}$-closed set in X such that $A \subseteq U$ where U is g-open in X. Since A is $g^*b^{\#}$-closed, $b^{\#}$-cl $(A) \subseteq U$. Since bcl $(A) \subseteq b^{\#}$-cl (A), bcl $(A) \subseteq U$. Hence A is a g^* b closed set in X.

(v) Let A be a $g^*b^{\#}$-closed set in X such that $A \subseteq U$ where U is regular open in X. Since every regular open is g-open and since bcl $(A) \subseteq b^{\#}$-cl (A), bcl $(A) \subseteq U$. Hence A is a *rgb* closed set in X.

(vi) Let A be a $g^*b^\#$-closed set in X such that $A \subseteq U$ where U is open in X. Since every open set is g-open and since spcl (A) $\subseteq b^\#$-cl (A), spcl (A) $\subseteq U$. Hence A is a *gsp*-closed set in X.

(vii) Let A be a $g^*b^\#$-closed set in X such that $A \subseteq U$ where U is regular open in X. Since every regular open set is g-open and since spcl (A) $\subseteq b^\#$-cl (A), spcl (A) $\subseteq U$. Hence A is a *gspr*-closed set in X.

(viii) Let A be a $g^*b^\#$-closed set in X such that $A \subseteq U$ where U is g-open in X. Since A is $g^*\ b^\#$-closed and since spcl (A) $\subseteq b^\#$-cl (A), spcl (A) $\subseteq U$. Hence A is a pre semi closed set in X.

The converse of the above Theorem need not be true as shown from the following Example.

Example 3.3: Let $X = \{a, b, c, d\}$ with the topology $\tau_1 = \{\Phi, X, \{a\}, \{b\}, \{a, b\}\}$. Let $A_1 = \{a, c, d\}$. Here A_1 is a $g^*\ b^\#$-closed set but not a $b^\#$-closed set.

Consider the topology $\tau_2 = \{\Phi\ , X, \{b\}\}$. Let $A_2 = \{b, c, d\}$. Here A_2 is a $gb^\#$-closed set but not a $g^*\ b^\#$-closed set.

Consider the topology $\tau_3 = \{\Phi, X, \{c\}\}$. Let $A_3 = \{a, c, b\}$. Here A_3 is gbclosed and *gsp*-closed but not a $g^*b^\#$-closed set.

Consider the topology $\tau_4 = \{\Phi, X, \{a\}\}$. Let $A_4 = \{b, c\}$. Here A_4 is g^*b closed but not a $g^*b^\#$-closed set. Let $A_5 = \{a, c, d\}$. Here A_5 is *rgb* closed but not a $g^*b^\#$-closed set. Let $A_6 = \{a, c\}$. Here A_6 is a *gspr*-closed set but not a $g^*\ b^\#$-closed set.

Consider the topology $\tau_5 = \{\Phi, X, \{a\}, \{b\}, \{a, b\}\}$. Let $A_7 = \{b, c\}$. Here A_7 is pre semi closed set but not a $g^*\ b^\#$-closed set.

The following Example gives the independency of $g^*\ b^\#$-closed set with other closed sets in topological spaces.

Example 3.4: Let $X = \{a, b, c, d\}$ with the topology $\tau_1 = \{\Phi, X, \{a\}, \{b\}, \{a, b\}\}$. Here $\{c\}$ is pre closed, b-closed, g-closed, *rg*-closed, α-closed, αg-closed, mildly g-closed, *rwg* closed and *wg*-closed but not a $g^*\ b^\#$-closed set. Also $\{a\}$ is $g^*\ b^\#$-closed but not a pre closed set, g-closed set, *gp*-closed set, αg-closed set, *wg* closed set, mildly g-closed set, *rwg* closed set and α-closed set. Also $\{b\}$ is a $g^*\ b^\#$-closed set but not a *rg*-closed set. Here $\{d\}$ is *gp*-closed and gα-closed but not a $g^*\ b^\#$-closed set.

Consider the topology $\tau_2 = \{\Phi, X, \{a\}\}$. Here $\{b, d\}$ is a semi closed set but not a $g^*\ b^\#$-closed set. Also $\{b, c\}$ is $g\alpha b$-closed but not a $g^*b^\#$-closed set.

Consider $\tau_3 = \{\Phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$. Here $\{a, b, d\}$ is a $g^*\ b^\#$-closed set but not a semi closed set. Also $\{c\}$ is a *gpr* closed set, g^*p-closed set but not a $g^*\ b^\#$-closed set. Again $\{a\}$ is *a* $g^*\ b^\#$-closed set but not a *gpr*-closed set, $g\alpha b$ closed set and a g^*p-closed set. Also $\{b, c\}$ is $g^*b^\#$-closed but not a $g\alpha$-closed set. Again $\{b, d\}$ is a $g^*\ b^\#$-closed set but not a b closed set.

Consider $\tau_4 = \{\Phi, X, \{a\}, \{a, b\}\}$. Here $\{b\}$ is β-closed and *sg*-closed but not a $g^*b^\#$-closed set. Let $B = \{a, c, d\}$. Here B is a $g^*b^\#$-closed set but not a *sg*-closed set and a β-closed set.

The following example shows that the intersection of two $g^*b^\#$-closed sets and the union of two $g^*b^\#$-closed sets need not be a $g^*b^\#$-closed set.

Example 3.5: Let $X = \{a, b, c, d\}$ with the topological space $\tau = \{\Phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$. Here $A = \{b, c\}$ and $B = \{b, d\}$ are $g^*b^\#$-closed sets. But $A \cap B = \{b\}$ is not a $g^*b^\#$-closed set.

Consider the topological space $\tau = \{\Phi, X, \{a\}, \{b\}, \{a, b\}\}$. Here $A = \{a\}$ and $B = \{b\}$ are $g^*b^\#$-closed sets. But $A \cup B = \{a, b\}$ is not a $g^*b^\#$-closed set.

Theorem 3.6: Suppose A is a p-set and a $g^*b^\#$-closed set. Then

(i) A is gp-closed.

(ii) A is gpr-closed.

(iii) A is wg-closed.

(iv) A is mildly g-closed.

(v) A is rwg-closed.

(vi) A is g^*p-closed.

Proof :

(i) Let A be a p-set and a $g^*b^\#$-closed set. Then Theorem 2.22, cl (int (A)) $\subseteq b^\#$- cl (A). Let U be any open set such that $A \subseteq U$. Since every open set is g-open, $b^\#$- cl (A) $\subseteq U$. This implies cl (int (A)) $\subseteq U$. That is pcl (A) = $A \cup$ cl (int (A)) $\subseteq A \cup U = U$. Hence A is gp-closed.

(ii) Let U be any regular open set such that $A \subseteq U$. Sine every regular open set is g-open and since A is $g^*b^\#$-closed, $b^\#$-cl (A) $\subseteq U$. This implies cl (int (A)) $\subseteq U$. That is pcl (A) = $A \cup$ cl (int (A)) $\subseteq U$. Hence A is gpr-closed.

(iii) Let U be any open set such that $A \subseteq U$. Sine every open set is g-open and since A is $g^*b^\#$-closed, $b^\#$-cl (A) $\subseteq U$. This implies cl (int (A)) $\subseteq U$. Hence A is wg-closed.

(iv) Let U be any g-open set such that $A \subseteq U$. Since A is $g^*b^\#$-closed, $b^\#$- cl (A) $\subseteq U$. This implies cl (int (A)) $\subseteq U$. Hence A is mildly g-closed.

(v) Let U be any regular open set such that $A \subseteq U$. Sine every regular open set is g–open and since A is $g^*b^\#$-closed, $b^\#$-cl (A) $\subseteq U$. This implies cl (int (A)) $\subseteq U$. Hence A is rwg-closed.

Let U be a g-open set such that $A \subseteq U$. Sine A is g*$b^\#$- closed, $b^\#$- cl (A) $\subseteq U$. This implies cl (int (A)) $\subseteq U$. That is pcl (A) = $A \cup$ cl (int (A)) $\subseteq U$. Hence A is g^*p-closed.

Theorem 3.7: Suppose A is a p-set, g-open and $g^*b^\#$-closed then A is pre-closed, β-closed and b closed.

Proof : Let A be a p-set, g-open and $g^*b^\#$-closed. Then cl (int (A)) $\subseteq b^\#$-cl (A). Since $A \subseteq A$ and A is $g^*b^\#$-closed, $b^\#$-cl (A) $\subseteq A$. This implies cl (int (A)) $\subseteq A$. Hence A is pre closed. Since every pre closed set is β-closed, A is β-closed and every pre closed set is bclosed, A is bclosed.

Theorem 3.8 : Suppose A is a q-set, g-open and $g^*b^\#$-closed then A is semi-closed, β-closed and bclosed.

Proof : Let A be a q-set, g-open and $g^*b^\#$-closed. Then int (cl (A)) $\subseteq b^\#$-cl (A). Since $A \subseteq A$ and A is $g^*b^\#$-closed, $b^\#$-cl (A) $\subseteq A$. This implies int (cl (A)) $\subseteq A$. Hence A is semi closed. Since every semi closed set is β-closed, A is β-closed and every semi closed set is b closed, A is b closed.

Theorem 3.9 : Suppose A is a p-set, open and $g^*b^\#$-closed then A is closed.

Proof : Let A be a p-set, open and $g^*b^\#$-closed. Then cl (int (A)) $\subseteq$ $b^\#$-cl (A). Since A is open cl (A) $\subseteq$ $b^\#$-cl (A). Since $A \subseteq A$ and A is $g^*b^\#$-closed, $b^\#$-cl (A) $\subseteq A$. This implies cl (A) $\subseteq A$. Hence A is closed.

Theorem 3.10: Let A be a $g^*b^\#$-closed subset of a topological space X. Then $b^\#$-cl (A)$\backslash A$ does not contain any non empty g-closed set.

Proof : Let F be a g–closed subset of $b^\#$- cl (A)$\backslash A$. Then $A \subseteq X\backslash F$. Since $X\backslash F$ is g-open and since A is $g^*b^\#$-closed, we have $b^\#$-cl (A) $\subseteq X\backslash F$ that implies $F \subseteq b^\#$-cl (A) $\cap$ ($X\backslash$ $b^\#$-cl (A))= Φ.

Corollary 3.11: Let A be a $g^*b^\#$-closed set. Then $b^\#$-cl (A) = A if and only if $b^\#$-cl (A)$\backslash A$ is g-closed.

Proof : If $b^\#$-cl (A) = A then $b^\#$-cl (A)$\backslash A$= Φ which is g-closed. Conversely let $b^\#$-cl (A)$\backslash A$ be g-closed. Then by Theorem 3.10, $b^\#$-cl (A)$\backslash A$ does not contain any non empty g-closed set. Since $b^\#$-cl (A)$\backslash A$ is g-closed, $b^\#$- cl (A)$\backslash A$ = Φ. Since $A \subseteq b^\#$-cl (A) it follows that $b^\#$-cl (A) = A.

The converse of the above Theorem need not be true as shown from the following example.

Example 3.12: Let X = {a, b, c, d} with the topological space τ = {Φ, X, {a}, {b}, {a, b}}. Then $b^\#$-closed sets are Φ, X, {a}, {b} and $g^*b^\#$-closed sets are Φ, X, {a}, {b}, {c, d}, {a, c, d} and {b, c, d}. Also g-closed sets are Φ, X, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d} and {b, c, d}. Consider A = {a, b}. It is clear that $b^\#$- cl (A)$\backslash A$= {c, d} contains no non empty g-closed subset in X. But A is not $g^*b^\#$- closed.

Theorem 3.13: If A is $g^*b^\#$-closed and $A \subseteq B \subseteq b^\#$-cl ($A$) then

(i) B is $g^*b^\#$-closed.

(ii) $b^\#$-cl (B)$\backslash B$ contains no non empty g-closed set.

Proof : Let A be a $g^*b^\#$-closed set in X. such that $A \subseteq B \subseteq b^\#$-cl ($A$). Let U be any g-open set in X such that $B \subseteq U$. Then $A \subseteq U$. Since A is $g^*b^\#$-closed, $b^\#$-cl (A) $\subseteq U$. Also $b^\#$-cl (A) = $b^\#$-cl (B). Therefore $b^\#$-cl (B) $\subseteq U$ that implies B is $g^*b^\#$-closed. Since B is $g^*b^\#$-closed using Theorem 3.10, $b^\#$-cl (B)$\backslash B$ contains no nonempty g-closed set.

Theorem 3.14: If A is both g-open and $g^*b^\#$-closed then $b^\#$-cl (A) = A.

Proof : Since A is g-open and $g^*b^\#$-closed, $b^\#$-cl (A) $\subseteq A$. But always $A \subseteq b^\#$-cl (A). Therefore $b^\#$- cl (A) = A.

Theorem 3.15 : For $x \in X$, the set $X\backslash\{x\}$ is $g^*b^\#$-closed or g-open.

Proof : Suppose $X\backslash\{x\}$ is not g-open then X is the only g-open set containing $X\backslash\{x\}$. This implies $b^\#$-cl ($X\backslash\{x\}$) $\subseteq X$. Then $X\backslash\{x\}$ is $g^*b^\#$-closed.

Theorem 3.16 : In a topological space X every g-open set is $b^\#$-closed then every subset of X is $g^*b^\#$- closed.

Proof : Suppose every g-open set is $b^\#$-closed. Let A be a sub set of X such that $A \subseteq U$ whenever U is g-open. But $b^\#$-cl (A) $\subseteq b^\#$-cl (U) = U. Therefore A is $g^*b^\#$-closed.

Theorem 3.17 : Let $A \subseteq Y \subseteq X$ and suppose that A is $g^*b^\#$-closed in X then A is $g^*b^\#$-closed relative to Y.

Proof : Given that $A \subseteq Y \subseteq X$ and A is $g^*b^\#$-closed in X. Let $A \subseteq Y \cap U$ where U is g-open in X. Since A is $g^*b^\#$-closed $A \subseteq U$ implies $b^\#$-cl $(A) \subseteq U$. It follows that $Y \cap b^\#$-cl $(A) \subseteq Y \cap U$. Thus A is $g^*b^\#$-closed relative to Y.

Theorem 3.18 : Suppose that $B \subseteq A \subseteq X$, B is $g^*b^\#$-closed relative to A and that A is both g-open and $b^\#$-closed subset of X then B is $g^*b^\#$-closed set relative to X.

Proof : Let $B \subseteq A \subseteq X$ where A is $b^\#$-closed and g-open. Suppose B is $g^*b^\#$-closed relative to A. Let $B \subseteq G$ where G is g-open in X. Then $B \subseteq G \cap A$. Since $G \cap A$ is g-open in A, $A \cap b^\#$-cl $(B) \subseteq G \cap A$. Since A is $b^\#$-closed, $b^\#$-cl $(B) \subseteq b^\#$-cl $(A) = A$ implies $b^\#$-cl $(B) = b^\#$- cl $(B) \cap A \subseteq G \cap A \subseteq G$. This shows that B is $g^*b^\#$-closed relative to X.

Theorem 3.19 : Every open Interval is $g^*b^\#$-closed in R_1 where R_1 is the set of all real numbers equipped with standard topology.

Proof : Let A be an interval in R_1. If A is empty nothing to prove. If A is an open interval of the form $(-\infty, a)$, (a, ∞) and (a, b), $a < b$ then by Example 5.11 of [20] A is $b^\#$-closed. Therefore by Theorem 3.2, A is $g^*b^\#$-closed.

Theorem 3.20 : In the real line R_1, every singleton set $g^*b^\#$-closed.

Proof : Let $x \in R_1$. Fix an g-open set such that $\{x\} \subseteq G$. Then by Example 5.11 of [20], $b^\#$-cl $(\{x\})$ = the intersection of $b^\#$-closed sets $= \cap\ (x - \in, x + \in) = \{x\} \subseteq G$. Thus every singleton set $g^*b^\#$-closed.

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