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FUNCTIONS OF NEARLY μ PARALINDELOF SPACES

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The purpose of this paper is to introduce and study some μ-paralindelof and product spaces in generalized topological spaces.

KEYWORDS : μ-continuous, almost L perfect μ-open function, nearly μ-paralindelof, strongly μ-*HP* closed.

Introduction

This paper is concerned with the adaptation of the change of topology approach from topological topics to aspects of the theory of generalized topological spaces. This shows that "the change of generalized topology" exhibits some characteristic analogous to change of topology in the topological category. A general application of the change of generalized topology approach occurs when the spaces are ordinary topological spaces. In this case, the generalized topologies are families of distinguished subsets of a topological space which are not topologies but are generalized topologies. Some common examples of generalized topologies that are associated with a given topological space. Consider the collection of all s.o, p.o, β-open, α-open sets in the (ordinary) topology (X, τ). Each collection is a generalized topology on X. In fact, the family of α-open set is a topology. But in general, the other there collections, namely, the family of s.o, p.o and β-open sets are not topologies on X.

In 1992, Blumberg defined what he meant by a real-valued function on Euclidean space being densely approached at a point in its domain. Continuous functions satisfy his condition at each point of their domains. Since then, and particularly in the past four decade, a large number of properties closely related to the notion of continuity of a function have been introduced. The number of properties so large that different authors have used the same term for different concepts and other authors have resorted to exotic terms, sometimes because the natural term has already been pre-empty. It turns out that many of these concepts are not new in the sense that if one is willing to change the topology on the domain and /or the range then the class of functions satisfying a particular property often coincides with the class of continuous functions under the new topologies from their point of view many of the results in the literature concerning such functions are essentially restatements in disguise of familiar properties of continuous functions. Throughout the paper, N stands for natural numbers. The aim of this paper is to introduce and study some μ-paralindelof and product spaces in generalized topological spaces.

Preliminaries

Let X be a set. A subset μ of exp X is called a generalized topology on X and (X, μ) is called a generalized topological space [1] (abbr. GTX) if μ has the following

(i) $\phi \in \mu$

(ii) Any union of elements of μ belongs to μ.

For background material, reference [4, 5] may be perused.

Let (X, μ) be a generalized topological space. $A \subset X$ is called ***C_i - μ_i - closed*** in X if every cover $\mathcal{U}$ of A consisting of sets μ-open in X, has a countable subcollection $\mathcal{V}(\subset \mathcal{U})$ such that $A \subset \bigcup\{i_\mu c_\mu(V) : V \in \mathcal{V}\}$ where $i_\mu c_\mu$ is interior and closure of generalized topological space with respect to X. Indeed if $A \subset X$, then A is $C_i - \mu_i$-closed if and only if every cover consisting of sets μ-regular open in X of A has a countable subcover.

If X is a generalized topological space and $A \subset X$, then A is called a *μ - HP - set* if every μ-open cover $\mathcal{U}$ of A has a countable sub collection $\mathcal{V}(\subset \mathcal{U})$ such that $A \subset \bigcup\{i_\mu c_\mu(V) : V\ \mathcal{V}\}$.

A generalized topological space (X, μ) is called almost μ-paralindelof if every μ-open cover $\mathcal{U}$ of X admits a μ-locally countable collection $\mathcal{V}$ of μ-open subsets of X such that such that $\mathcal{V}$ refines $\mathcal{U}$ and $X = \bigcup\{C_\mu(V): V \in \mathcal{V}\}$.

A function $f : X \to Y$ is called almost μ-continuous if the inverse image of every μ-regular open set in Y is μ-open in X.

A generalized topological space (X, μ) is called μ-lindelof or have the μ-lindelof property if every μ-open cover of X has a countable subcover.

A generalized topological space (X, μ) is called μ - *p* - space if every G_δ - set is μ-open. If it is further *H*-Hausdorff, it is called a *μ - HP - space (i.e.*, every G_δ - set is μ - open).

A μ - H*P* - space (X, μ) is called minimal μ - *HP* if $\mu_1 \subset \mu$ and $(X, \mu_1)$is a μ_1 - *HP*- space imply $\mu_1 = \mu$. A μ-*HP*-space is called μ-*HP*-closed if it is μ-closed in every space in which it is embedded.

A subset A in a generalized topological space (X, μ) is called μ - *HP - closed* if $(A, \mu/_A)$ is μ - *HP* - closed. A space X is called locally μ - *HP* - closed if each point of X has an μ-open *nhd* whose μ-closure is μ -*HP*-closed.

Behavior of nearly μ-paralindelof spaces

Definition 3.1 : Consider a function $f : X \to Y$ where X and Y are generalized topological spaces. F is called almost μ - *L - perfect* if f is a μ-closed and almost μ-continuous surjection such that $f^{-1}(y)$ is μ-lindelof for each $y \in Y$.

Theorem 3.1 : Let X and Y be μ - H*P* - space and $f : X \to Y$ be a μ-closed, almost μ-continuous and almost μ-open surjection such that $f^{-1}(y)$ is a μ - *HP* - set for each $y \in Y$. Then, if X is nearly μ - paralindelof, so is *y*.

Proof : Let $\mathcal{U}$ be a μ-regular open cover of Y. Then $\{f^{-1}(U) : U \epsilon\, \mathcal{U}\}$ is a μ-open cover of X. Take $\mathcal{V} = \{i_\mu c_\mu f^{-1}(U): U \epsilon\, \mathcal{U}\}$. $\mathcal{V}$ is a μ-regular open cover of X and has a μ-regular open refinement $\mathcal{B}$ which is μ - locally countable. Let $= \{f(B): B \epsilon \mathcal{B}\}$, $\mathcal{C}$ is μ-open cover of *Y*, $\mathcal{C}$ is μ-locally countable, $\mathcal{C}$ refines μ..

(i) That $\mathcal{C}$ is μ-open cover of *Y* is obvious.

(ii) Let $y \in Y$. Since X is μ - *HP* and nearly μ-paralindelof, we have *X* is almost μ-regular. Hence $f^{-1}(y)$ is $C_1 - \mu - closed$ for each $y \in Y$. For each $x \in f^{-1}(y)$ there is a μ-open *nhd* W_x of *x* which intersects only countably many members of $\mathcal{B}$. $\{W_x : x \epsilon f^{-1}(y)\}$ is a μ-open cover of $f^{-1}(y)$. Hence there is a countable subset $\mathcal{C} \subset f^{-1}(y)$ such that $f^{-1}(y) \subset W = \{i_\mu c_\mu W_x : x \epsilon \mathcal{C}\}$; also *W* intersects only countably many members of $\mathcal{B}$. Let $V = Y - f(X - W)$. V is μ-open in *Y*. If $V \cap f(B) \neq \phi$ then $B \cap W \neq \phi$. Hence $\mathcal{C}$ is μ-locally countable.

(iii) Since $\mathcal{B}$ refines $\mathcal{V}$, for each $B \in \mathcal{B}$, there is a μ - open set $U \epsilon \mathcal{U}$ such that $B \subset i_\mu c_\mu f^{-1}(U)$. Since *f* is almost μ - continuous, we have $c_\mu f^{-1}(U) \subset . f^{-1}(c_\mu(U))$ so that $B \subset f^{-1}(c_\mu(U))$. Hence $f(B) \subset c_\mu(U)$. Since B is μ - regular open, *f(B)* is μ - open and so that $f(B) \subset c_\mu(U)$. Since B is μ - regular open, *f* (*B*) is μ - open and so that $f(B) \subset i_\mu c_\mu(U) = U$. Thus $\mathcal{C}$ refines μ.

Corollary 3.1 : If *X* and *Y* are μ - *HP* - space and $f : X \to Y$ is an almost *L* - perfect and almost μ - open function of a nearly μ - paralindelof space X onto Y, then *Y* is nearly μ - paralindelof.

Lemma 3.1 : If $f : X \to Y$ is an almost μ-continuous and almost μ-closed bijection of a nearly μ-paralindelof space *X* onto *Y*, then *Y* is nearly μ-pare lindelof.

Proof : Let $\mathcal{U}$ be a μ-regular open cover of *Y*. Then $\mathcal{V} = \{i_\mu c_\mu f^{-1}(U): U \epsilon \mathcal{U}\}$ is a μ-regular open cover of *X*. $\mathcal{V}$ has a μ-regular open refinement $\mathcal{B}$ which is μ-locally countable. Also $f(B) = Y - f(X - B)$, is μ-open for each $B \epsilon \mathcal{B}$. $\{f(B): B \epsilon \mathcal{B}\}$, is a μ-open cover of *Y* with the required properties.

Theorem 3.2: Let $f : X \to Y$ be a μ-continuous, μ-open surjection and let *f* be a μ-closed function such that $f^{-1}(y)$ is C_1 - μ - closed for each $y \in Y$. Then, if *X* is almost μ-paralindelof so is *Y*.

Proof : Let $\mathcal{U}$ be a μ -open cover of Y. Then $\mathcal{V} = \{f^{-1}(U): U \epsilon \mathcal{U}\}$ is a μ-open cover of *X* and it has a μ-open refinement $\mathcal{B}$ such that $\mathcal{B}$ is μ-locally countable and $X = \bigcup\{C_\mu(B): B \epsilon \mathcal{B}\}$. Let $\mathcal{C} = \{f(B): B \epsilon \mathcal{B}\}$, $\mathcal{C}$ is a collection of μ-open subsets of *Y*.

(i) Since $\mathcal{B}$ refines $\mathcal{V}$, for each $B \in \mathcal{B}$, there is a μ-open set $U \epsilon \mathcal{U}$ such that $B \subset f^{-1}(U)$ so that $f(B) \subset U$. Hence $\mathcal{C}$ refines μ.

(ii) Since *f* is μ-continuous, $f(c_\mu(B)) \subset c_\mu f(B)$ so that $Y = \bigcup\{C_\mu f(B): B \epsilon \mathcal{B}\}$.

(iii) Since *f* is μ-closed, $f^{-1}(y)$ is C_1 - μ-closed and $\mathcal{B}$ is μ-locally countable, $\mathcal{C}$ is μ-locally countable.

Corollary 3.2 : Let $f : X \to Y$ be a *L*-perfect μ-open surjection of an almost μ-paralindelof space *X* onto *Y*. Then *Y* is almost μ-paralindelof.

Theorem 3.3 : Let $f : X \to Y$ be a almost μ-continuous and μ-open surjection. *L*et *Y* be almost μ-regular and *f* be μ-closed with $f^{-1}(y)$ is C_1 - μ-closed for each $y \in Y$. Then, if *X* is almost μ-paralindelof and so *Y* is nearly μ-paralindelof.

Proof : Starting with a μ-regular open cover of $\mathcal{U}$ of *Y*, we proceed we did in theorem 4 and then, on applying the fact that an almost μ-regular, almost μ-paralindelof space is nearly μ-paralindelof, the result follows.

Corollary 3.3: Let $f : X \to Y$ be an almost *L*-perfect μ-open function of an almost μ-paralindelof space *X* onto an almost μ-regular space *Y*, then *Y* is nearly μ-paralindelof.

Theorem 3.4: Let $f : X \to Y$ be a μ-continuous and μ-open surjection of an almost μ-regular space *X* onto an almost μ-paralindelof space *Y*. Let *f* be μ-closed such that $f^{-1}(y)$ is a μ-*HP* set for each $y \in Y$. Then *X* is nearly μ-paralindelof.

Proof : Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a μ-regular open cover of *X*. Let $\{V_\alpha : \alpha \in \Delta\}$ be the family of all collections of μ-open subsets of *X* such that $V_\alpha (\alpha \in \Delta)$ is μ-open refinement of $\mathcal{U}$ and that each V_α is μ-locally countable. Then $(X - \cup\{V : V \epsilon V_\alpha\})$. Then $\mathcal{W} = \{W_\alpha : \alpha \in \Lambda\}$ is a μ-open cover of Y. $\mathcal{W}$ admits a μ-locally countable collection *S* of μ-open subsets of *Y* such that *S* refines $\mathcal{W}$ and $Y = \{c_\mu(s) : s \in S\}$. Suppose $s \in S$. Then there is an $\alpha \in \Lambda$ such that $S \subset W_\alpha$ from $\mathcal{V}_1 = \{f^{-1}(s) : s \epsilon S\}$. Form $\varsigma(s) = \{f^{-1}(s) \cap v : v \epsilon V_\alpha\}$ and $\mathcal{V} = \cup \{\varsigma(s) : s \in S\}$. Clearly *V* is μ-locally countable. Also $X = \cup \{c_\mu(p) : p \in V\}$. Thus, since *X* is almost μ-regular, by using the fact that every almost μ-regular, almost μ-paralindelof space is nearly μ-paralindelof, the result follows.

Theorem 3.5: Let $f : X \to Y$ be a almost μ - continuous and μ-open and almost μ-closed surjection such that $f^{-1}(y)$ is a μ - *HP* set for each $y \in Y$ where *X* is almost μ-regular. Then, if *Y* is nearly μ-paralindelof, so is *X*.

Proof : Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a μ-regular open cover of *X*. Since *X* is almost μ-regular, $f^{-1}(y)$ is $C_1 - \mu -$ closed for each $y \in Y$. Hence there is a countable subset $\Delta(y) \subset \Delta$ such that $f^{-1}(y) \subset \{U_\alpha : \alpha \in \Delta\ (y)\}$. Take $P(y) = i_\mu c_\mu(\cup \{U_\alpha : \alpha \in \Delta\ (y)\}) = i_\mu(\cup \{c_\mu U_\alpha : \alpha \in \Delta\ (y)\})$. *P(y)* is a μ-regular open set containing $f^{-1}(y)$. Since *f* is an almost μ-closed surjection, there is a μ - open nbd V_y of y such that $f^{-1}(y) \subset f^{-1}(V_y) \subset P(y)$. Since *f* is μ-open, $f^{-1}\left(c_\mu(V_y)\right) \subset c_\mu f^{-1}(V_y)$ so that $f^{-1}(i_\mu c_\mu V_y) \subset c_\mu\left(f^{-1}(V_y)\right) \subset c_\mu P(y) = \cup \{c_\mu U_\alpha : \alpha \in \Delta\ (y)\}$. The family $V = \{V_y : y \in Y\}$ is a μ-open cover of *Y*. Since *Y* is nearly μ-paralindelof, there is a μ-locally countable collection $\mathcal{W}$ of μ-open subsets of *Y* such that $\mathcal{W}$ refines *V* and $Y = \cup \{i_\mu c_\mu w : w \in W\}$. Let $\mathcal{V}_1 = \left\{f^{-1}\left(i_\mu c_\mu(w)\right) : w \epsilon W\right\}$. $\mathcal{V}_1$ is a collection of μ-open subsets of *X*. If $w \in W$, then there is an $y \in Y$ such that $W \subset V_y$ and $i_\mu c_\mu(w) \subset i_\mu c_\mu(V_y)$ so that $f^{-1}\left(i_\mu c_\mu(w)\right) \subset f^{-1}\left(i_\mu c_\mu(V_y)\right) \cup\{c_\mu(U_\alpha) : \alpha \in \Delta\ (y)\}$.

Form $\{\mathcal{C}(W) = f^{-1}(i_\mu c_\mu(w)) \cap U_\alpha : \alpha \in \Delta\ (y)\}$ and $G = \cup \{\mathcal{C}(W) : w \in W\}$. Each *W* determines *y* and a countable subset $\Delta\ (y)(\subset \Delta)$.

Corollary 3.4: Let $f : X \to Y$ be an almost *L*-perfect μ-open function of an almost μ-regular open space *X* onto a nearly μ-paralindelof space *Y*. Then *X* is nearly μ-paralindelof.

PRODUCT SPACE

In this section, we give some results on products involving nearly μ-paralindelof spaces.

Lemma 4.1 : Let X be a strongly μ - *HP* closed space and *Y* be a μ - *HP* space. Then $X \times \{y\}$ is C_i - μ-closed in $X \times Y$, where $y \in Y$.

Proof : Let $=\{U_\alpha: \alpha \in \Lambda\}$ be a μ-regular in ($X \times Y$) cover of $X \times \{y\}$. For each $(x, y) \in U_\alpha$, there are μ-open *nbds* $P(\alpha, x), Q(\alpha, x)$ of *x* and *y* respectively such that $P(\alpha, x) \times Q(\alpha, x) \subset U_\alpha$. Hence $i_\mu c_\mu(P(\alpha, x) \times Q(\alpha, x)) = i_\mu c_\mu(P(\alpha, x)) \times i_\mu c_\mu(Q(\alpha, x)) \subset U_\alpha$. Thus, if $i_\mu c_\mu(P(\alpha, x)): (x, y) \in U_\alpha = \varsigma_\alpha$ and $\varsigma = \cup \{\varsigma_\alpha: \alpha \in \Delta\}$ ς is a μ-regular open cover of *X*. Clearly, there are *x* (*i*), α (*i*) where each α (*i*) ∈ Δ (*i* ∈ *N*) such that

$$\cup \left\{i_\mu c_\mu \left(P\big(x(i), \alpha(i)\big)\right), \alpha(i): i \in N\right\} = X \text{ such that } \cup \left\{U_{\alpha_i}: i \in N\right\} \supset X \times \{y\}.$$

Lemma 4.2 : Let X be a strongly μ - *HP* closed space and *Y* be a μ - *HP* space. Then the projection $p_2: X \times y \to y$ is almost closed.

Proof : Let $F \subset X \times Y$ be μ-regular closed. Let $y_0 \in y - p_a(F)$. Clearly $X \times \{y_0\} \cap F = \phi$ so that each point (x_0, y_0) has μ-open nhd $U(x) \times U(x_0, y_0)$ which does not intersect *F*. Now $\{(i_\mu c_\mu(U(x))) \times (i_\mu c_\mu(U(x, y_0))): x \in X\}$ is a μ-regular open cover of $X \times \{y_0\}$. Since $X \times \{y_0\}$ is *C* - μ-closed, there is a countable subset *B* ($\subset X$) such that $X \times \{y_0\} \subset \cup \{(i_\mu c_\mu(U(x))) \times (i_\mu c_\mu(U(x, y_0))): x \in B\}$. Take $U = \cap \{(i_\mu c_\mu(U(x, y_0))): x \in B\}$. Then *U* is a μ-regular open *nhd* of y_0 such that $U \cap p_2(F) = \phi$. Thus, $p_2(F)$ is μ-closed in *Y*.

Theorem 4.1 : If *X* is nearly μ-paralindelof and *Y* is strongly *μ-HP* closed, then $X \times Y$ is nearly μ-paralindelof.

Proof : Consider the projection mapping $p_1: X \times Y \to Y$, p_1 is a μ-continuous, μ-open surjection with C_i - μ-closed point inverses by lemma 4.1. Further it is almost μ-closed by lemma 3.2. Again $X \times Y$ is almost μ-regular, then by theorem 3.5, $X \times Y$ is nearly μ-paralindelof.

Corollary 4.1 : If *X* is a μ-lindelof, μ-*HP* space and *Y* is nearly *μ*-paralindelof, then $X \times Y$ is nearly μ-paralindelof.

Lemma 4.3 : Let $\{H_\alpha: \alpha \in \Delta\}$, be a μ-locally countable family of μ-closed subsets of a μ - *P* - space *X* that covers *X*. Then $\{c_\mu i_\mu(H_\alpha): \propto \in \Delta\}$ is a μ-locally countable collection of μ-regular subsets of *X* that covers *X*.

Proof : Let $\mathcal{U} = \{H_\alpha: \alpha \in \Delta\}, \mathcal{V} = \{c_\mu i_\mu(H_\alpha): \propto \in \Delta\}$. Let Δ be a well ordered set and take $\Delta = [0, \xi)$. Define $\mathcal{U}_\beta = \{c_\mu i_\mu(H_\alpha): \alpha \le \beta: \propto \in \Delta\} \cup \{H_\alpha: \beta < \alpha, \propto \in \Delta\}$.

Case (i) : Suppose β = 0 and *x* ∈ *X*. Assume that $x \notin \cup \{H_\alpha: \alpha > 0, \propto \in \Delta\}$. Then $x \in X - \cup \{H_\alpha: \alpha > 0, \propto \in \Delta\}$. Since $\cup \{H_\alpha: \alpha > 0, \propto \in \Delta\}$ is locally countable family of μ-closed sets, in a μ - *P* - space, we have $\cup \{H_\alpha: \alpha > 0, \propto \in \Delta\}$ is μ-closed so that $x \in M \subset H_0$ where $M = X - \cup \{H_\alpha: \alpha > 0, \propto \in \Delta\}$ is μ-open. Thus $\in c_\mu i_\mu(H_0)$. Hence $\mathcal{U}_\beta$ for β = 0 is a cover of *X*.

Case (ii) : Suppose β is any ordinal in Δ. Assume that $\mathcal{U}_\alpha$ is a cover of X for every α < β, if $x \notin \left(\left(\cup \{c_\mu i_\mu(H_\alpha): \alpha < \beta: \propto \in \Delta\}\right) \cup \left(\cup \{H_\alpha: \alpha > 0, \propto \in \Delta\}\right)\right) = Q$, say, then by induction, $x \in X - Q \subset H_\beta$, notice that Q is μ-closed in a μ-space. Thus $x \in M \subset c_\mu i_\mu(H_\beta)$, where $M = X - Q$ is μ-open. In this way, $\mathcal{U}_\beta$ is a cover of *X*. Applying transitive induction, it follows that $\mathcal{U}_\beta$ is a cover of *X* for every β ∈ Δ. Let *x* ∈ *X*. Then there is a nhd M_x of *x* which interests atmost countably many members of $\mathcal{U}$. Then there is a countable subset Σ ⊂ Δ such that $M_x \cap K_\alpha \ne \phi$ if and only if $\alpha \in \Sigma$. Σ has a supremum and denote the same by ∂. Then $x \notin \cup \{H_\alpha: \alpha > \partial\}$ and $x \in c_\mu i_\mu(H_\alpha)$ for some α ≤ ∂. In this way, $\mathcal{V}$ is a cover of *X*. In the last that $\mathcal{V}$ is μ-locally countable is obvious.

Theorem 4.2 : For an almost μ-regular space *X*, the following are equivalent

(i) *X* is nearly μ-paralindelof.

(ii) Every μ-regular open cover $\mathcal{U}$ of *X* has a μ-locally countable refinement consisting of μ-regular open subsets of *X* (covering *X*).

(iii) Every μ-regular open cover $\mathcal{U}$ of *X* has a μ-locally countable family $\mathcal{V}$ of μ-open sets such that $\mathcal{V}$ refines $\mathcal{U}$ and the μ - closures of the members of $\mathcal{V}$ cover *X*.

(iv) Every μ-regular open cover $\mathcal{U}$ of *X* has a μ-locally countable refinement (covering *X*).

(v) Every μ-regular open cover $\mathcal{U}$ of *X* has a μ-locally countable. μ-closed refinement (covering *X*).

(vi) Every μ-regular open cover $\mathcal{U}$ of *X* has a μ-locally countable. μ-regular closed refinement (covering *X*).

Proof : i ⇒ ii Let $\mathcal{U}$ be any μ-regular open cover of *X*. Then there is a μ-locally countable open refinement $\mathcal{V}_1$. Form $\mathcal{V} = \{c_\mu i_\mu(v) \in V\}$. Clearly, $\mathcal{V}$ is a locally countable refinement of μ.

ii ⇒ iii Obvious.

iii ⇒ iv) Let $\mathcal{U}$ be any μ-regular open cover of *X*. Let $x \in X$ and $U \in \mathcal{U}$ such that $x \in U \in \mathcal{U}$. Since *X* is almost μ-regular, there is a μ-open set V_x such that $x \in V_x \subset c_\mu(V_x) \subset U$. Thus $x \in \{i_\mu c_\mu(v_x): x \in X\}$ is a μ-regular open cover of *X*. Then by iii, there is a μ-locally countable family W_1, of μ-open subsets of *X* such that W_1 refines $\mathcal{U}$ and $X = \cup \{c_\mu w: w \in W_1\}$ so that if $W = \{c_\mu w: w \in W_1\}$, W_1 is μ-locally countable μ-closed refinement of $\mathcal{U}$ covering *X*.

v ⇒ iv Obvious

iv ⇒ v Let $\mathcal{U}$ be any μ-regular open cover of *X*. For each $x \in X$, let $U \in \mathcal{U}$ with $x \in \mathcal{U}$. Then there is a μ-open set V_x such that $x \in V_x \subset c_\mu(V_x) \subset U$. Then $\mathcal{V} = \{V_x: x \epsilon X\}$ is a μ-regular open cover of X. $\mathcal{V}$ has, by iv, a μ - locally countable refinement W_1. Let $W = \{c_\mu(w): w \in W\}$. Clearly W is a μ-locally countable μ-closed refinement of $\mathcal{U}$ covering *X*.

v ⇒ vi This follows from an application of lemma 4.3.

vi ⇒ i This follows from the result : Let *X* be almost μ-regular, then *X* is nearly μ-paralindelof if and only if *X* is almost μ-paralindelof.

Lemma 4.4 : Let *X* be μ-*HP* and μ-paralindelof. Let *A* and *B* be two disjoint μ-regular closed subsets of *X*. Then there exist disjoint open sets *U* and *V* such that $A \subset U$ and $B \subset V$.

Proof : Let $x \in A$. Since *X* is almost μ-regular, there is a μ-open set V_x such that $x \in V_x \subset c_\mu(V_x) \subset X - B$. Take V_x to be μ-regular open. Consider $\mathcal{U}' = \{V_x: x \in A\}$. Let $\mathcal{U} = \mathcal{U}' \cup \{X - A\}$. $\mathcal{U}$ is a μ-regular open cover of *X*. Since *X* is nearly μ-paralindelof, $\mathcal{U}$ has a μ-locally countable μ-open refinement $\mathcal{V}$ covering *X*. Let $\mathcal{V}'$ be all those μ-open sets in $\mathcal{V}$ which intersects *A*. Let $U = \cup \mathcal{V}$. Then $A \subset U$. Set $\mathcal{V} = X - \cup \{c_\mu(V'): V' \in \mathcal{V}'\}$.

Lemma 4.5 : Let *X* be a μ-*HP* and nearly μ-paralindelof. Let *A* and *B* are disjoint μ-regular closed subsets of *X*, there is a μ-continuous function $f : X \to [0, 1]$ such that $f(A) \subset \{0\}$ and $f(B) \subset \{1\}$.

Proof : The proof is similar to the standard proof of Uryshon's lemma.

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