

**STUDY OF SERVICE CYCLE IN MULTIPLE VACATION  
MODEL WITH EXHAUSTIVE SERVICE.  
STUDY OF DEPLETION TIME IN DISCREAT-TIME  
QUEUEING SYSTEMS WITHOUT SERVER VACATIONS**

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In this present paper, we study the depletion time in the discreat-time queueing systems without server vacations. The depletion time D (measured in slots) immediately after the  $n^{\text{th}}$  slot is the time it takes the server to empty the system by serving all the messages that are in the system at the time as well as the messages included in the subsequent arrivals during the continuous busy period.

**KEYWORDS** : Depletion, Unfinished work, Derivation, Busy period.

**D**EPLETION TIME

In system without server vacations, the depletion time is a delay cycle with the initial delay given by the unfinished work. Therefore, the PGF  $D(u)$  for D is given by

$$D(u) = U(\Lambda[\theta]) \frac{(1 - \rho) \{ 1 - U \{ u (\Lambda[\theta]) \} \}}{1 - \rho} \quad \dots(1.1.1)$$

Which leads to

$$E[D] = \frac{E[U]}{1 - \rho} = \frac{(\lambda b^{(2)} + \lambda^{(2)} b^{(2)} + \rho(1 - 2\rho))}{2(1 - \rho)^2} \quad \dots(1.1.2)$$

The depletion time is also the residual busy period when the system is not empty. Therefore, we get another derivation of  $D(u)$  as

$$D(u) = 1 - \rho + \rho \frac{U[1 - \theta_g(u)]}{E[\theta_g](1 - u)} \quad \dots(1.1.3)$$

The PGF  $\theta_g(u)$  for  $\theta_g$  and  $E[\theta_g]$  denotes the mean length of a busy period, are given by (1.1.4) & (1.1.5) respectively.

$$\theta_g(u) = \sum_{k=1}^{\infty} [\theta(u)]^k \frac{\lambda(k)}{1 - \lambda(0)} = \frac{\Lambda[\theta(u) - \lambda(0)]}{1 - \lambda(0)} \quad \dots(1.1.4)$$

$$E[\theta_g] = \frac{\lambda E[\theta]}{1 - \lambda(0)} = \frac{\rho}{(1 - \rho)[1 - \lambda(0)]} \quad \dots(1.1.5)$$

Keaping in view (1.1.4), (1.1.5) & (1.1.3), we have,

$$D(u) = 1 - \rho + \frac{\Pi(\Theta(u), \Lambda[\Theta(u)])}{\Theta(u)} \quad \dots (1.1.6)$$

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