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WAITING TIMES DURING A DELAY CYCLE IN DIS CRETE – TIME SYSTEMS WITHOUT VACATION MODELS

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In this present paper, we study Geo^x/G/1 systems. The main aim of this present paper is to study the the waiting times during a delay cycle. We do so by developing the discreat – time version of the analysis for continuous – time system. Assuming that the set o all messages in the *m* – 1 generation is always served before those in the *m*th generation. However, any discipline may be employed with respect to the order of service within each generation.

KEYWORDS : Waiting time, Arrive process, Reccurence, Distribution, Discipline.

WAITING TIMES DURING A DELAY CYCLE :

We consider the waiting time of a message that arrive during a delay cycle with initial delay X_0. We call the initial delay the 0-th generation of the delay cycle, and call the period of time for serving all the messages that arrive during the $m - 1^{\text{th}}$ generation the *m* the generation, where *m* = 1, 2, ………. The length of the *m*-th generation of a delay cycle (measured in slots) is denoted by Θ_m and the PGF for Θ_m is denoted by $\Theta_m(u)$ for *m* = 0, 1, 2, ………. . We then have the recurrence relations as –

$$\Theta_0(u) = \mathrm{B}_0(u) \qquad \dots (1.1.1)$$

$$\Theta_m(u) = \Theta_{m-1}\{ \Lambda\,[B\,(u)\}; m = 1, 2, \dots \qquad \dots (1.1.2)$$

where $\mathrm{B}_0(u)$ is the PGF for X_0.

In what follows, we will consider system with FCFS, LCFS and ROS disiciplines within each generation. Here $\{\Theta_m;\ m = 0, 1, 2, \dots\}$ is invariant within generations. However, the distribution of the waiting time of a Message does not depend on the service disicipline with in generations.

Keaping in view, (1.1.1) and (1.1.2), we have,

$$E\,[\Theta_c] = \sum_{m=0}^{\infty} E[\Theta_m] = \frac{b_0}{1-\rho} \qquad \dots (1.1.3)$$

$$\sum_{m=0}^{\infty} \mathrm{E}[\Theta_m(\Theta_m - 1)] = \frac{\mathrm{b}_0^2}{\mathbf{1}-\boldsymbol{\rho}^{\mathbf{2}}} + \frac{\left(\lambda \boldsymbol{b}^{(\mathbf{2})} + \boldsymbol{\lambda}^{(\mathbf{2})}\boldsymbol{b}^{\mathbf{2}} - 1\right) b_0}{(i-\rho)(\mathbf{1}-\boldsymbol{\rho}^{\mathbf{2}})} \qquad \dots (1.1.4)$$

$$E[\mathrm{S}_c^2] = \frac{\mathrm{b}_0^2}{(\mathbf{1}-\boldsymbol{\rho})^{\mathbf{2}}} + \frac{(\boldsymbol{\lambda b}^{(\mathbf{2})} + \boldsymbol{\lambda}^{(\mathbf{2})}\boldsymbol{b}^{\mathbf{2}} - \rho^{\mathbf{2}})\mathrm{b}_0}{(\mathbf{1}-\boldsymbol{\rho})^{\mathbf{3}}}$$

$$\sum_{m=0}^{\infty} E[\Theta_m(\ \Theta_m - 1)(\ \Theta_m - 2)] = \frac{b_0^3}{\mathbf{1-\boldsymbol{\rho}^3}} + \frac{\left(\lambda\mathbf{b}^{(3)} + \boldsymbol{\lambda}^{(3)}\mathbf{b}^{3} - 1\right)b_0}{(i - \rho)(\mathbf{1-\boldsymbol{\rho}^3})}$$
$$+ \frac{3\{\left(\rho\lambda\mathbf{b}^{(2)} + \rho\boldsymbol{\lambda}^{(2)}\mathbf{b}^{2} - 1\right)\}\,\mathrm{b}_0}{(\mathbf{1-\boldsymbol{\rho}^2})(\mathbf{1-\boldsymbol{\rho}^3})}$$
$$+ \frac{3\rho\{\left(\ \lambda\mathbf{b}^{(2)}\ \right)^2 + (\mathbf{1}+\boldsymbol{\rho}^2)\boldsymbol{\lambda}^{(2)}\mathrm{b}\mathbf{b}^{(2)} + \left.\right)\mathrm{b}_0 + \rho[\ \boldsymbol{\lambda}^{(2)}\mathbf{b}^{2}]\ {}^2\mathrm{b}_0}{(1-\rho)(\mathbf{1-\boldsymbol{\rho}^2})(\mathbf{1-\boldsymbol{\rho}^3})}$$
$$+ \frac{\left((\mathbf{2}+\boldsymbol{\rho}^2) - 3(\ (1+\rho)\left(\lambda\mathbf{b}^{(2)} + \boldsymbol{\lambda}^{(2)}\mathbf{b}^{2} - 1\right)\mathrm{b}_0\ \right)}{(1-\rho)(\mathbf{1-\boldsymbol{\rho}^2})(\mathbf{1-\boldsymbol{\rho}^3})} \quad \ldots (1.1.5)$$

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