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BCI – ALGEBRAS OF GRAPHS

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In this paper we have established that the class of all simple graphs with *n* vertices is a non-singular BCI – algebra. The binary operation on the class of graphs has been defined through binary operation on the class of all adjacency matrices corresponding to the graphs.

KEYWORDS : BCI – algebra , graph, adjacency matrix.

INTRODUCTION

Definition (1.1) : A system $(X; *, 0)$ consisting of a non-empty set X, a binary operation $*$ and a fixed element 0 is called a BCI – algebra if the following conditions are satisfied :

1. (BCI 1) $((x * y) * (x * z)) * (z * y) = 0$
2. (BCI 2) $(x * (x * y)) * y = 0$
3. (BCI 3) $x * x = 0$
4. (BCI 4) $x * y = 0 = y * x \Rightarrow x = y$

for all $x, y, z \in X$.

Definition (1.2) : In a BCI – algebra $(X; *, 0)$ a partial order relation $\leq$ is defined as $x \leq y$ iff $x * y = 0$.

Definition (1.3) : A BCI – algebra $(X, *, 0)$ is called

(a) a BCK – algebra if $0 * x = 0$ for all $x \in X$;

(b) a non-singular BCI – algebra if $0 * x = x$ for all $x \in X$;

Example(1.4) : Let $X = \{0, 1\}$ and let the binary operation $*$ be defined as

*	0	1
0	0	1
1	1	0

Then $(X; *, 0)$ is a non-singular BCI –algebra.

Definition (1.5) : A graph G is a pair (V, E) where V and E are finite sets and the elements of V are called vertices or points or nodes and the elements of set E are called edges or lines or arcs connecting pair of vertices.

Definition (1.6) : A graph which has neither parallel edges nor self -loops is called a simple graph.

Definition (1.7) : Adjacency matrix of a simple graph G, consisting of n vertices in order $v_1, v_2, \ldots\ldots\ldots\ldots v_n$, in an $n \times n$ matrix $A = (x_{ij})$ defied as

$$x_{ij} = \begin{cases} 1, & \text{if there is an edge between } v_i \text{ and } v_j \\ 0, & \text{otherwise.} \end{cases}$$

Note (1.8) : In an adjacency matrix $x_{ii} = 0$ and $x_{ij} = x_{ji}$ for $1 \le i \le n, 1 \le j \le n$.

Note (1.9) : It is important to note that class of adjacency matrices has one – one correspondence with the class of all simple graphs with n vertices taken in order *i.e.*, each simple graph has a representation as an adjacency matrix and for each adjacency matrix there is a unique graph representing it.

Theorem (1.10) : Let $(X; *, 0)$ be a BCI – algebra and let $M(X)$ be the class of all m x n matrix $(a_{ij})_{m \times n}$ with entries $a_{ij} \in X$. For $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$ we define

$$A = B \text{ iff } a_{ij} = b_{ij}, 1 \le i \le m, 1 \le j \le n.$$

Further, we define a binary operation o in $M(X)$ as

$$A \text{ o } B = C = (c_{ij})_{m \times n} \qquad \ldots (1.1)$$

where $c_{ij} = a_{ij} * b_{ij}, 1 \le i \le m, 1 \le j \le n$.

If $\mathbf{0}$ is the zero matrix with all entries 0 then $(M(X); \text{o}, \mathbf{0})$ is a BCI-algebra.

Proof : Let $A, B, \boldsymbol{C} \in M(X)$ where $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$ and $C = (c_{ij})_{m \times n}$.

Then $((A \text{ o } B) \text{ o } (A \text{ o } C)) \text{ o } (C \text{ o } B) = K = (k_{ij})_{m \times n}$.

where

$$k_{ij} = ((a_{ij} * b_{ij}) * (a_{ij} * c_{ij})) * (c_{ij} * b_{ij})$$
$$= 0 \text{ for all } i, j, 1 \le i \le m, 1 \le j \le n.$$

This gives $((A \text{ o } B) \text{ o } (A \text{ o } C)) \text{ o } (C \text{ o } B) = \mathbf{0}$

Other conditions of a BCI–algebra are also satisfied in $M(X)$. Hence $(M(X); \text{o}, \mathbf{0})$ is a BCI–algebra.

Corollary (1.11) : If X is the BCI – algebra considered in example (1.4) and $M(X)$ be class of all $n \times n$ matrixes with entries in $\{0, 1\}$ then $M(X)$ is a non-singular BCI – algebra under the binary operation o defined by (1.1).

BCI – ALGEBRA OF SIMPLE GRAPHS

Theorem (2.1) : Let G^n be the class of all simple graphs with n-vertices arranged in order $v_1, v_2, \ldots\ldots\ldots\ldots v_n$. For $G_1, G_2, \in G^n$, let A_1, A_2 be adjacency matrices representing G_1 and G_2. We define a binary operation $\odot$ in G^n as

$$G_1 \odot G_2 = G_3$$

where G_3 is the graph corresponding to adjacency matrix $A_3 = A_1 \text{ o } A_2$.

If G_o is the null graph in G^n then $(G^n; \odot, G_o)$ is a BCI – algebra.

First we have the following lemma.

Lemma (2.2) : Let L (X) be the class of all $n \times n$ matrices $A = (a_{ij})$, $1 \leq i \leq n$, $1 \leq j \leq n$ such that $a_{ii} = 0$, $a_{ij} = a_{ji} = 0$ or 1. Then L (X) is a sub algebra of M (X).

Proof : Let $A = (a_{ij})$ and $B = (b_{ij})$ be elements of L (X) and let $C = (C_{ij}) = A$ o B.

Then $$c_{ii} = a_{ii} * b_{ii} = 0$$

and $$c_{ij} = a_{ij} * b_{ij} = a_{ji} * b_{ji} = c_{ji}.$$

This means that $C \in L$ (X). So L (X) is a sub algebra of M (X).

Proof of the theorem (2.1) First of all we note that the class of all adjacency matrices corresponding of the graphs in G^n is the class L (X).

Since L (X) is a non singular BCI–algebra, (G^n, $\odot$, G_o) is a non-singular BCI–algebra. where binary operation $\odot$ is extended by binary operation o in L (X).

Note (2.3) : The number of elements in G^n is $2^{n(n-1)/2}$.

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