

## ECCENTRIC AND WIENER INDICES OF NANO PENTACHAIN

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With the invention of Nano-technology, the study of topological indices gained much importance. In this paper we initiate the study of eccentric and Wiener indices of a pentagonal ring. We observe that the eccentric connectivity index is a second degree polynomial in both the structures (Straight chaining and Alternative chaining) where as Wiener index is a third degree polynomial.

**KEYWORDS** : Hyper wiener index, Wiener index, Narumi-Katayama index, pentachains.

### INTRODUCTION

**A**nano pentachain is a closed chain of pentarings in one row. Ivan Gutman and [2] in 2007 presented the study of concatenated 5-cycles in one row and obtained explicit formulas for Schultz and modified Schultz indices of these graphs. Later Lakshmi prasanna [4] and the present author [5] obtained formulas for various topological indices. Recently in 2009, O. Halakoo [3] obtained bounds for Schultz index of pentachains.

The study of eccentric connectivity index gained more importance as the topological models involving this index show a high degree of predictability of pharmaceutical properties. In this paper, we obtained simple formulas for eccentric connectivity index and Wiener indices of nano pentachain.

### PRELIMINARIES

**F**irst we state the definitions of W, WW, S, RW and eccentric indices.

**Definition 2.1.** [7] The Wiener index of a graph  $G = (V, E)$  is defined as  $W(G) = \frac{1}{2} \sum_{v \in V} \sum_{u \in V} d_G(u, v)$  where  $d_G(u, v)$  is the length of shortest path connecting  $u$  and  $v$  in  $G$ .

**Definition 2.2.** [2] The Hyper wiener index of a graph  $G = (V, E)$  is defined as  $WW(G) = \frac{1}{2} \sum_{i < j} (d_{ij} + d_{ij}^2)$ .

**Definition 2.3.** [8] The Narumi-katayama index of a graph  $G$  is defined as  $S = S(G) = \prod_{i=1}^n i$ , where ' $i$ ' is the degree (= number of first neighbours) of the  $i^{\text{th}}$  vertex and  $n$  is the number of the graph.

**Definition 2.4.** [1] The Reverse Wiener index of a graph  $G$  is defined as  $RW(G) = \frac{1}{2}N(N-1)d - W$ , where  $N$  is the number of vertices,  $d$  is the diameter and  $W$  is the Wiener index of the graph  $G$ .

**Definition 2.5.** [6] If  $G = (V, E)$  is a connected graph then the eccentric connectivity index of  $G$  is defined as  $\xi(G) = \sum_{u \in V(G)} \deg(u) \in(u)$ , where  $\in(u) = \max \{d(u, v) / v \in V(G)\}$ .

## NOTATION

**5**-cycles can be concatenated in a chain as shown in figure 1, 2 (we call this case as Straight chaining) or as in figure 3 (we call this case as Alternative chaining).

In Straight chaining case the graph consisting of 5-cycles in a chain is denoted by  $G(a, S)$ . The graph with  $a = 6$  in a chain is as shown below.

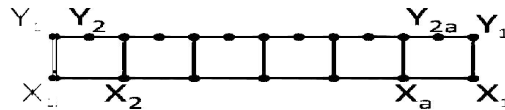


Fig. 1.

The graph with  $a = 7$  in a chain is as shown below.

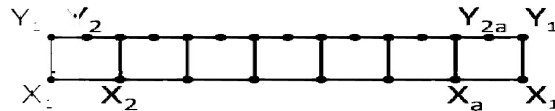


Fig. 2.

In Alternative chaining case the graph consisting of 5-cycles in a chain is denoted by  $G(a, A)$ . The graph with  $a = 6$  in a chain is as shown below.

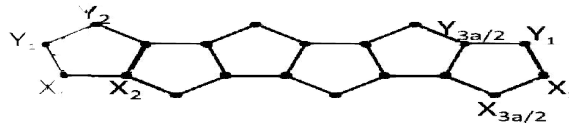


Fig. 3.

## WIENER INDEX

**Theorem 4.1.**  $W(G(a, S)) = \frac{1}{8}(9a^3 + 60a^2 - 101a)$ , if  $a > 3$  and  $a$  is odd.

**Proof:** Let the vertex set of  $G(a, S)$  be  $A \cup B$ , where  $A = \{x_1, x_2, \dots, x_a\}$ ,  $B = \{y_1, y_2, \dots, y_{2a}\}$ .

$$\begin{aligned}
\text{Now } W(G(a, S)) &= \sum_{u, v \in A} d(u, v) + \sum_{u, v \in B} d(u, v) + \sum_{u \in A, v \in B} d(u, v) \\
&= \left\{ a \sum_{i=1}^{\frac{a-1}{2}} i \right\} + \left\{ 2a \sum_{i=1}^4 i + 3a(5) + 4a \sum_{i=6}^{\frac{a+3}{2}} i + 2a \left( \frac{a+5}{2} \right) \right\} + \left\{ a(1) + 4a \sum_{i=2}^{\frac{a+1}{2}} i + a \left( \frac{a+3}{2} \right) \right\} \\
&= \frac{1}{8}(a^3 - a) + \frac{1}{2}(a^3 + 10a^2 - 25a) + \frac{1}{2}(a^3 + 5a^2) \\
&= \frac{1}{8}(9a^3 + 60a^2 - 101a).
\end{aligned}$$

**Theorem 4.2.**  $W(G(a, S)) = \frac{1}{8}(9a^3 + 60a^2 - 100a)$ , if  $a > 4$  and  $a$  is even.

**Proof:** Let the vertex set of  $G(a, S)$  be  $A \cup B$ , where  $A = \{x_1, x_2, \dots, x_a\}$ ,  $B = \{y_1, y_2, \dots, y_{2a}\}$ .

$$\begin{aligned}
\text{Now } W(G(a, S)) &= \sum_{u, v \in A} d(u, v) + \sum_{u, v \in B} d(u, v) + \sum_{u \in A, v \in B} d(u, v) \\
&= \left\{ a \sum_{i=1}^{\frac{a-2}{2}} i + \left( \frac{a}{2} \right)^2 \right\} + \left\{ 2a \sum_{i=1}^4 i + 3a(5) + 4a \sum_{i=6}^{\frac{a+2}{2}} i + \left( \frac{7a}{2} \right) \left( \frac{a+4}{2} \right) + \left( \frac{a}{2} \right) \left( \frac{a+6}{2} \right) \right\} \\
&\quad + \left\{ a(1) + 4a \sum_{i=2}^{\frac{a}{2}} i + (3a) \left( \frac{a+2}{2} \right) \right\} \\
&= \frac{1}{8}(a^3) + \frac{1}{2}(a^3 + 10a^2 - 25a) + \frac{1}{2}(a^3 + 5a^2) \\
&= \frac{1}{8}(9a^3 + 60a^2 - 100a).
\end{aligned}$$

**Theorem 4.3.**  $W(G(a, A)) = \frac{1}{16}(27a^3 + 18a^2 + 48a)$ , if  $\frac{a}{2}$  is odd.

**Proof:** Let the vertex set of  $G(a, A)$  be  $A \cup B$ , where  $A = \{x_1, x_2, \dots, x_{3a/2}\}$ ,  $B = \{y_1, y_2, \dots, y_{3a/2}\}$ .

$$\begin{aligned}
\text{Now } W(G(a, S)) &= \sum_{u, v \in A} d(u, v) + \sum_{u, v \in B} d(u, v) + \sum_{u \in A, v \in B} d(u, v) \\
&= \left\{ \frac{3a}{2} \sum_{i=1}^{\frac{3a-2}{4}} i \right\} + \left\{ \frac{3a}{2} \sum_{i=1}^{\frac{3a-2}{4}} i \right\} + \left\{ a(1) + 4a(2) + 4a(3) + 3a \sum_{i=4}^{\frac{3a-2}{4}} i + \left( \frac{3a}{2} \right) \left( \frac{3a+2}{4} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{64}(27a^3 - 12a) + \frac{1}{64}(27a^3 - 12a) + \frac{1}{32}(27a^3 + 36a^2 + 108a) \\
&= \frac{1}{16}(27a^3 + 18a^2 + 48a).
\end{aligned}$$

**Theorem 4.4.**  $W(G(a, A)) = \frac{1}{16}(27a^3 + 18a^2 + 48a)$ , if  $\frac{a}{2}$  is even.

**Proof:** Let the vertex set of  $G(a, A)$  be  $A \cup B$ , where  $A = \{x_1, x_2, \dots, x_{3a/2}\}$ ,  $B = \{y_1, y_2, \dots, y_{3a/2}\}$ .

$$\begin{aligned}
\text{Now } W(G(a, S)) &= \sum_{u, v \in A} d(u, v) + \sum_{u, v \in B} d(u, v) + \sum_{u \in A, v \in B} d(u, v) \\
&= \left\{ \frac{3a}{2} \sum_{i=1}^{\frac{3a-4}{4}} i + \left(\frac{3a}{4}\right)^2 \right\} + \left\{ \frac{3a}{2} \sum_{i=1}^{\frac{3a-4}{4}} i + \left(\frac{3a}{4}\right)^2 \right\} + \left\{ a(1) + 4a(2) + 4a(3) + 3a \sum_{i=4}^{\frac{3a}{4}} i \right\} \\
&= \frac{1}{64}(27a^3) + \frac{1}{64}(27a^3) + \frac{1}{32}(27a^3 + 36a^2 + 96a) \\
&= \frac{1}{16}(27a^3 + 18a^2 + 48a).
\end{aligned}$$

## HYPER WIENER INDEX

**Theorem 5.1.**  $WW(G(a, S)) = \frac{1}{16}(3a^4 + 39a^3 + 189a^2 - 599a)$ , where  $a > 3$  and  $a$  is odd.

**Proof:** Let the vertex set of  $G(a, S)$  be  $A \cup B$ , where  $A = \{x_1, x_2, \dots, x_a\}$ ,  $B = \{y_1, y_2, \dots, y_{2a}\}$ .

$$\begin{aligned}
\text{Now } WW(G(a, S)) &= \frac{1}{2} \left( \sum_{u, v \in A} d(u, v) + \sum_{u, v \in B} d(u, v) + \sum_{u \in A, v \in B} d(u, v) + \sum_{u, v \in A} d^2(u, v) \right. \\
&\quad \left. + \sum_{u, v \in B} d^2(u, v) + \sum_{u \in A, v \in B} d^2(u, v) \right) \\
&= \frac{1}{2} \left[ \left\{ a \sum_{i=1}^{\frac{a-1}{2}} i \right\} + \left\{ 2a \sum_{i=1}^4 i + 3a(5) + 4a \sum_{i=6}^{\frac{a+3}{2}} i + 2a \left(\frac{a+5}{2}\right) \right\} + \left\{ a(1) + 4a \sum_{i=2}^{\frac{a+1}{2}} i + a \left(\frac{a+3}{2}\right) \right\} \right. \\
&\quad \left. + \left\{ a \sum_{i=1}^{\frac{a-1}{2}} i^2 \right\} + \left\{ 2a \sum_{i=1}^4 i^2 + 3a(5)^2 + 4a \sum_{i=6}^{\frac{a+3}{2}} i^2 + 2a \left(\frac{a+5}{2}\right)^2 \right\} \right]
\end{aligned}$$

$$\begin{aligned}
& \left\{ a(1)^2 + 4a \sum_{i=2}^{\frac{a+1}{2}} i^2 + a \left( \frac{a+3}{2} \right)^2 \right\} \\
&= \frac{1}{2} \left[ \frac{1}{8}(a^3 - a) + \frac{1}{2}(a^3 + 10a^2 - 25a) + \frac{1}{2}(a^3 + 5a^2) + \frac{1}{24}(a^4 - a^2) \right. \\
&\quad \left. + \frac{1}{6}(a^4 + 15a^3 + 77a^2 - 375a) + \frac{1}{12}(2a^4 + 15a^3 + 40a^2 + 3a) \right] \\
&= \frac{1}{16}(3a^4 + 39a^3 + 189a^2 - 599a).
\end{aligned}$$

**Theorem 5.2.**  $WW(G(a, S)) = \frac{1}{16}(3a^4 + 39a^3 + 190a^2 - 600a)$ , where  $a > 4$  and  $a$  is even.

**Proof:** Let the vertex set of  $G(a, S)$  be  $A \cup B$ , where  $A = \{x_1, x_2, \dots, x_a\}$ ,  $B = \{y_1, y_2, \dots, y_{2a}\}$ .

$$\begin{aligned}
\text{Now } WW(G(a, S)) &= \frac{1}{2} \left( \sum_{u, v \in A} d(u, v) + \sum_{u, v \in B} d(u, v) + \sum_{u \in A, v \in B} d(u, v) + \sum_{u, v \in A} d^2(u, v) \right. \\
&\quad \left. + \sum_{u, v \in B} d^2(u, v) + \sum_{u \in A, v \in B} d^2(u, v) \right) \\
&= \frac{1}{2} \left\{ \left[ a \sum_{i=1}^{\frac{a-2}{2}} i + \left( \frac{a}{2} \right)^2 \right] + \left[ 2a \sum_{i=1}^4 i + 3a(5) + 4a \sum_{i=6}^{\frac{a+2}{2}} i + \left( \frac{7a}{2} \right) \left( \frac{a+4}{2} \right) + \left( \frac{a}{2} \right) \left( \frac{a+6}{2} \right) \right] \right\} \\
&\quad + \left\{ a(1) + 4a \sum_{i=2}^{\frac{a}{2}} i + (3a) \left( \frac{a+2}{2} \right) \right\} + \left\{ a \sum_{i=1}^{\frac{a-2}{2}} i^2 + \left( \frac{a}{2} \right) \left( \frac{a}{2} \right)^2 \right\} \\
&\quad + \left\{ 2a \sum_{i=1}^4 i^2 + 3a(5)^2 + 4a \sum_{i=6}^{\frac{a+2}{2}} i^2 + \left( \frac{7a}{2} \right) \left( \frac{a+4}{2} \right)^2 + \left( \frac{a}{2} \right) \left( \frac{a+6}{2} \right)^2 \right\} \\
&\quad + \left\{ a(1)^2 + 4a \sum_{i=2}^{\frac{a}{2}} i^2 + (3a) \left( \frac{a+2}{2} \right)^2 \right\} \\
&= \frac{1}{2} \left[ \frac{1}{8}(a^3) + \frac{1}{2}(a^3 + 10a^2 - 25a) + \frac{1}{2}(a^3 + 5a^2) + \frac{1}{24}(a^4 + 2a^2) \right. \\
&\quad \left. + \frac{1}{6}(a^4 + 15a^3 + 77a^2 - 375a) + \frac{1}{12}(2a^4 + 15a^3 + 40a^2) \right] \\
&= \frac{1}{16}(3a^4 + 39a^3 + 190a^2 - 600a).
\end{aligned}$$

**Theorem 5.3.**  $WW(G(a, A)) = \frac{1}{64}(27a^4 + 81a^3 + 60a^2 + 460a)$ , where  $\frac{a}{2}$  is odd.

**Proof:** Let the vertex set of  $G(a, A)$  be  $A \cup B$ , where  $A = \{x_1, x_2, \dots, x_{3a/2}\}$ ,  $B = \{y_1, y_2, \dots, y_{3a/2}\}$ .

$$\begin{aligned} \text{Now } WW(G(a, S)) &= \frac{1}{2} \left( \sum_{u,v \in A} d(u,v) + \sum_{u,v \in B} d(u,v) + \sum_{u \in A, v \in B} d(u,v) + \sum_{u,v \in A} d^2(u,v) \right. \\ &\quad \left. + \sum_{u,v \in B} d^2(u,v) + \sum_{u \in A, v \in B} d^2(u,v) \right) \\ &= \frac{1}{2} \left[ \left\{ \frac{3a}{2} \sum_{i=1}^{\frac{3a-2}{4}} i \right\} + \left\{ \frac{3a}{2} \sum_{i=1}^{\frac{3a-2}{4}} i \right\} + \left\{ a(1) + 4a(2) + 4a(3) + 3a \sum_{i=4}^{\frac{3a-2}{4}} i + \left( \frac{3a}{2} \right) \left( \frac{3a+2}{4} \right) \right\} \right. \\ &\quad \left. + \left\{ \frac{3a}{2} \sum_{i=1}^{\frac{3a-2}{4}} i^2 \right\} + \left\{ \frac{3a}{2} \sum_{i=1}^{\frac{3a-2}{4}} i^2 \right\} + \{a(1)^2 + 4a(2)^2 + 4a(3)^2 \right. \\ &\quad \left. + 3a \sum_{i=4}^{\frac{3a-2}{4}} i^2 + \left( \frac{3a}{2} \right) \left( \frac{3a+2}{4} \right)^2 \right] \\ &= \frac{1}{2} \left[ \frac{1}{64}(27a^3 - 12a) + \frac{1}{64}(27a^3 - 12a) + \frac{1}{32}(27a^3 + 36a^2 + 108a) \right. \\ &\quad \left. + \frac{1}{128}(27a^4 - 12a^2) + \frac{1}{128}(27a^4 - 12a^2) + \frac{1}{64}(27a^4 + 54a^3 + 60a^2 + 728a) \right] \\ &= \frac{1}{64}(27a^4 + 81a^3 + 60a^2 + 460a). \end{aligned}$$

**Theorem 5.4.**  $WW(G(a, A)) = \frac{1}{64}(27a^4 + 81a^3 + 60a^2 + 448a)$ , where  $\frac{a}{2}$  is even.

**Proof:** Let the vertex set of  $G(a, A)$  be  $A \cup B$ , where  $A = \{x_1, x_2, \dots, x_{3a/2}\}$ ,  $B = \{y_1, y_2, \dots, y_{3a/2}\}$ .

$$\begin{aligned} \text{Now } WW(G(a, S)) &= \frac{1}{2} \left( \sum_{u,v \in A} d(u,v) + \sum_{u,v \in B} d(u,v) + \sum_{u \in A, v \in B} d(u,v) + \sum_{u,v \in A} d^2(u,v) \right. \\ &\quad \left. + \sum_{u,v \in B} d^2(u,v) + \sum_{u \in A, v \in B} d^2(u,v) \right) \\ &= \frac{1}{2} \left[ \left\{ \frac{3a}{2} \sum_{i=1}^{\frac{3a-2}{4}} i \right\} + \left\{ \frac{3a}{2} \sum_{i=1}^{\frac{3a-2}{4}} i \right\} + \left\{ a(1) + 4a(2) + 4a(3) + 3a \sum_{i=4}^{\frac{3a-2}{4}} i + \left( \frac{3a}{2} \right) \left( \frac{3a+2}{4} \right) \right\} \right. \end{aligned}$$

$$\begin{aligned}
& + \left\{ \frac{3a}{2} \sum_{i=1}^4 i^2 \right\} + \left\{ \frac{3a}{2} \sum_{i=1}^4 i^2 \right\} + \{a(1)^2 + 4a(2)^2 + 4a(3)^2 \\
& + 3a \sum_{i=4}^{\frac{3a-2}{4}} i^2 + \left( \frac{3a}{2} \right) \left( \frac{3a+2}{4} \right)^2 \} \\
& = \frac{1}{2} \left[ \frac{1}{64} (27a^3) + \frac{1}{64} (27a^3) + \frac{1}{32} (27a^3 + 36a^2 + 96a) \right. \\
& \quad \left. + \frac{1}{128} (27a^4 + 24a^2) + \frac{1}{128} (27a^4 + 24a^2) + \frac{1}{64} (27a^4 + 54a^3 + 24a^2 + 704a) \right] \\
& = \frac{1}{64} (27a^4 + 81a^3 + 60a^2 + 448a).
\end{aligned}$$

## S INDEX

**Theorem 6.1.**  $S(G(a, S)) = 12a^2$ , if  $a \geq 2$ .

**Proof:** The total number of vertices in this graph is  $3a$ .

The number of vertices of degree 2 is  $a$  where as the number of vertices of degree 3 is  $2a$ .

$$\text{Hence } S(G(a, S)) = \prod_{i=1}^n i = (a)(2)(2a)(3) = 12a^2.$$

**Theorem 6.1.**  $S(G(a, A)) = 12a^2$ , if  $a \geq 2$ .

**Proof:** The total number of vertices in this graph is  $3a$ .

The number of vertices of degree 2 is  $a$  where as the number of vertices of degree 3 is  $2a$ .

$$\text{Hence } S(G(a, A)) = \prod_{i=1}^n i = (a)(2)(2a)(3) = 12a^2.$$

## REVERSE WIENER INDEX

**Theorem 7.1.**  $RW(G(a, S)) = \frac{1}{8}(9a^3 + 24a^2 + 71a)$ , where  $a > 3$  and  $a$  is odd.

**Proof:** The total number of vertices in this graph  $N$  is  $3a$ .

The diameter in this graph  $d$  is  $\frac{a+5}{2}$ .

And the Wiener index of this graph is  $W(G(a, S)) = \frac{1}{8}(9a^3 + 60a^2 - 101a)$ .

$$\begin{aligned} \text{Now } RW(G(a, S)) &= \frac{1}{2}N(N-1)d - W. \\ &= \frac{1}{2}(3a)(3a-1)\left(\frac{a+5}{2}\right) - \frac{1}{8}(9a^3 + 60a^2 - 101a) \\ &= \frac{1}{8}(9a^3 + 24a^2 + 71a). \end{aligned}$$

**Theorem 7.2.**  $RW(G(a, S)) = \frac{1}{8}(9a^3 + 42a^2 + 64a)$ , where  $a > 4$  and  $a$  is even.

**Proof:** The total number of vertices in this graph  $N$  is  $3a$ .

The diameter in this graph  $d$  is  $\frac{a+6}{2}$ .

And the Wiener index of this graph is  $W(G(a, S)) = \frac{1}{8}(9a^3 + 60a^2 - 100a)$ .

$$\begin{aligned} \text{Now } RW(G(a, S)) &= \frac{1}{2}N(N-1)d - W. \\ &= \frac{1}{2}(3a)(3a-1)\left(\frac{a+6}{2}\right) - \frac{1}{8}(9a^3 + 60a^2 - 100a) \\ &= \frac{1}{8}(9a^3 + 42a^2 + 64a). \end{aligned}$$

**Theorem 7.3.**  $RW(G(a, A)) = \frac{1}{16}(27a^3 - 60a)$ , where  $\frac{a}{2}$  is odd.

**Proof:** The total number of vertices in this graph  $N$  is  $3a$ .

The diameter in this graph  $d$  is  $\frac{3a+2}{4}$ .

And the Wiener index of this graph is  $W(G(a, A)) = \frac{1}{16}(27a^3 + 18a^2 + 48a)$ .

$$\begin{aligned} \text{Now } RW(G(a, A)) &= \frac{1}{2}N(N-1)d - W. \\ &= \frac{1}{2}(3a)(3a-1)\left(\frac{3a+2}{4}\right) - \frac{1}{16}(27a^3 + 18a^2 + 48a) \\ &= \frac{1}{16}(27a^3 - 60a). \end{aligned}$$

**Theorem 7.4.**  $RW(G(a, A)) = \frac{1}{16}(27a^3 - 36a^2 - 48a)$ , where  $\frac{a}{2}$  is even.

**Proof:** The total number of vertices in this graph  $N$  is  $3a$ .



The diameter in this graph  $d$  is  $\frac{3a}{4}$ .

And the Wiener index of this graph is  $W(G(a, A)) = \frac{1}{16}(27a^3 + 18a^2 + 48a)$ .

$$\begin{aligned} \text{Now } RW(G(a, A)) &= \frac{1}{2}N(N-1)d - W. \\ &= \frac{1}{2}(3a)(3a-1)\left(\frac{3a}{4}\right) - \frac{1}{16}(27a^3 + 18a^2 + 48a) \\ &= \frac{1}{16}(27a^3 - 36a^2 - 48a). \end{aligned}$$

## ECCENTRIC CONNECTIVITY INDEX

**Theorem 8.1.**  $\xi(G(a, S)) = 4a^2 + 17a$ , where  $a > 3$  and  $a$  is odd.

**Proof:** The total number of vertices in this graph is  $3a$ .

In the first row the number of vertices of degree 2 is  $a$ , the number of vertices of degree 3 is 0.

In the second row the number of vertices of degree 2 is 0, the number of vertices of degree 3 is  $a$ .

In the third row the number of vertices of degree 2 is 0, the number of vertices of degree 3 is  $a$ .

In this graph the eccentricity of every vertex in the first row is  $\frac{a+5}{2}$ .

The eccentricity of every vertex in the second row is  $\frac{a+5}{2}$ .

And the eccentricity of every vertex in the second row is  $\frac{a+3}{2}$ .

$$\text{Then } \xi(G(a, S)) = 2a\left(\frac{a+5}{2}\right) + 3a\left(\frac{a+5}{2}\right) + 3a\left(\frac{a+3}{2}\right) = 4a^2 + 17a.$$

**Theorem 8.2.**  $\xi(G(a, S)) = 4a^2 + 15a$ , where  $a > 4$  and  $a$  is even.

**Proof:** The total number of vertices in this graph is  $3a$ .

In the first row the number of vertices of degree 2 is  $a$ , the number of vertices of degree 3 is 0.

In the second row the number of vertices of degree 2 is 0, the number of vertices of degree 3 is  $a$ .

In the third row the number of vertices of degree 2 is 0, the number of vertices of degree 3 is  $a$ .

In this graph the eccentricity of every vertex in the first row is  $\frac{a+6}{2}$ .

The eccentricity of every vertex in the second row is  $\frac{a+4}{2}$ .

And the eccentricity of every vertex in the second row is  $\frac{a+2}{2}$ .

$$\text{Then } \xi(G(a, S)) = 2a\left(\frac{a+6}{2}\right) + 3a\left(\frac{a+4}{2}\right) + 3a\left(\frac{a+2}{2}\right) = 4a^2 + 15a.$$

**Theorem 8.3.**  $\xi(G(a, A)) = 6a^2 + 4a$ , where  $\frac{a}{2}$  is odd.

**Proof:** The total number of vertices in this graph is  $3a$ .

In the first row the number of vertices of degree 2 is  $a$ , the number of vertices of degree 3 is 0.

In the second row the number of vertices of degree 2 is 0, the number of vertices of degree 3 is  $a$ .

In the third row the number of vertices of degree 2 is 0, the number of vertices of degree 3 is  $a$ .

In this graph the eccentricity of every vertex in the first row is  $\frac{3a+2}{4}$ .

The eccentricity of every vertex in the second row is  $\frac{3a+2}{4}$ .

And the eccentricity of every vertex in the second row is  $\frac{3a+2}{4}$ .

$$\text{Then } \xi(G(a, A)) = 2a\left(\frac{3a+2}{4}\right) + 3a\left(\frac{3a+2}{4}\right) + 3a\left(\frac{3a+2}{4}\right) = 6a^2 + 4a.$$

**Theorem 8.4.**  $\xi(G(a, A)) = 6a^2$ , where  $\frac{a}{2}$  is even.

**Proof:** The total number of vertices in this graph is  $3a$ .

In the first row the number of vertices of degree 2 is  $a$ , the number of vertices of degree 3 is 0.

In the second row the number of vertices of degree 2 is 0, the number of vertices of degree 3 is  $a$ .

In the third row the number of vertices of degree 2 is 0, the number of vertices of degree 3 is  $a$ .

In this graph the eccentricity of every vertex in the first row is  $\frac{3a}{4}$ .

The eccentricity of every vertex in the second row is  $\frac{3a}{4}$ .

And the eccentricity of every vertex in the second row is  $\frac{3a}{4}$ .

$$\text{Then } \xi(G(a, A)) = 2a\left(\frac{3a}{4}\right) + 3a\left(\frac{3a}{4}\right) + 3a\left(\frac{3a}{4}\right) = 6a^2.$$

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