CONVECTIVE INSTABILITY OF MHD MICROPOLAR FLUID LAYER SATURATING A POROUS MEDIUM

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This paper deals with the theoretical investigation of convective instability of a electrically non-conducting, incompressible MHD micropolar fluid layer heated from below in the presence of porous medium. A dispersion relation is obtained for a flat fluid layer, contained between two free boundaries using a linear stability analysis theory and normal mode analysis method. The influence of various parameters like medium permeability magnetic field, coupling parameter, micropolar heat conduction parameter and micropolar coefficient has been analyzed on the onset of stationary convection and results are depicted graphically. The principle of exchange (PES) is found valid. In this paper we obtained that the magnetic field produces the oscillatory modes and found that the oscillatory modes will be stable when $\varepsilon < \frac{t_2}{2l_5}$ and $Q < \frac{l_1}{P_r l_7}$.

Keywords: Convective Instability; Micropolar Fluid; Horizontal Magnetic Field; Porous Medium.

INTRODUCTION

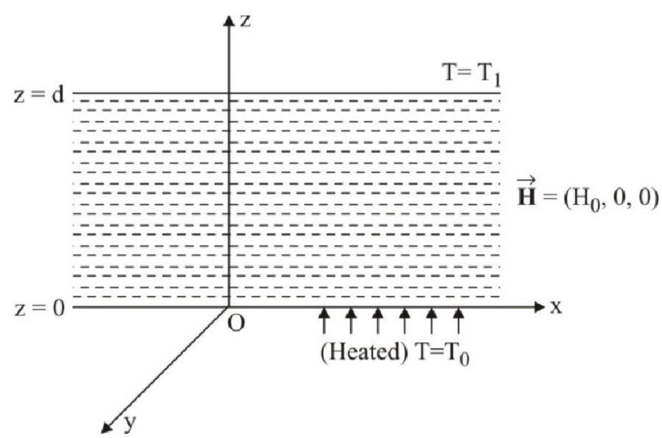
The onset of convection instability of a fluid layer heated from below has been studied by many researchers. Bénard [3] in 1900 did an experiment of a fluid layer heated from below and observed a thermal instability. The theoretical analysis of Bénard's experiment has been given by Rayleigh [4] and this analysis has also received a considerable importance due to its relevance in various fields such as chemical and industrial engineering, soil mechanics, geophysics etc. The main objectives of the various studies related to the convective instability, in particular, is to determine the critical Rayleigh number at which the onset of instability sets in either as stationary convection or through oscillations.

The Rayleigh-Bénard convection in micropolar fluids heated from below has been extensively studied by Ahmadi [2], Datta and Sastry [1], Bhattacharyya and Jena [9], L.E. Payne and B. Straughan [5]. The common results of all these studies are found that the stationary convection is the preferred mode of instability and the microrotation has a stability effect on the onset of Rayleigh-Bénard convection. An excellent review as well as large number of new developments are given by Chandrasekhar [8] in his celebrated book on hydrodynamic and hydromagnetic stability. In these methods of stability study a linear theory is usually employed *i.e.*, the equations governing the disturbances are linearized and then the grow or decay of the disturbances is studied. The effect of a magnetic field on the onset of convection in a horizontal micropolar fluid layer heated from below has also been investigated by several researchers. The extension of micropolar flows to include magneto-hydrodynamics effects is of interest in regard to various engineering applications such as in the design of the cooling systems for nuclear reactors, MHD electrical power generation, shock tubes, pump, flow meters etc. The effects of through flow and magnetic field on the onset of Bénard convection in a horizontal layer of micropolar fluid confined between two rigid, isothermal and microrotation free, boundaries have been studied by Narasimha Murty [10]. Z Alloui and P. Vasseur [11] studied onset of Rayleigh-Bénard MHD convection in a micropolar fluid.

Sharma and Kumar [6, 7] also studied the effect of magnetic field on the micropolar fluids heated from below in a non-porous and porous medium, they found that in the presence of various coupling parameters, the magnetic field has a stabilizing effect whereas the medium permeability has destabilizing effect on stationary convection.

Mathematical formulation

Consider an infinite, horizontal, electrically non-conducting, incompressible micropolar fluid layer of thickness d. This layer is heated from below such that the lower boundary is held at constant temperature $T = T_0$ and the upper boundary is held at fixed temperature $T = T_1$ therefore, a uniform temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained. The physical configuration is one of infinite extent in x and y directions bounded by the planes $z = 0$ and $z = d$. The whole system is acted on by gravity force $\overline{\mathbf{g}}(0, 0, -g)$.



A uniform magnetic field $\vec{\mathbf{H}} = (H_0, 0, 0)$ is applied along x-direction. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field can be neglected in comparison with the applied magnetic field.

The governing equations, which describe the system behavior following Boussinesq approximation, are as follows

The equation of continuity for an incompressible fluid is

$$\nabla.\vec{q} = 0 \qquad \dots(1)$$

The equation of momentum, following Darcy law, is given by

$$\frac{\rho_0}{\in}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right] = -\nabla p - \rho g \hat{e_z} + (\mu+\zeta)\nabla^2 \vec{q} - \left(\frac{\zeta+\mu}{\kappa}\right)\vec{q} + \zeta\, \nabla\times\vec{N} + \frac{\mu_e}{4\pi}\left(\nabla\times\vec{H}\right)\times\vec{H} \qquad \ldots(2)$$

The equation of internal momentum is given by

$$\rho_0 j\left[\frac{\partial \vec{N}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{N}\right] = \left(\alpha+\acute{\beta}\right)\nabla\left(\nabla.\vec{N}\right) + \acute{\gamma}\,\nabla^2\vec{N} + \zeta\left(\frac{1}{\in}\nabla\times\vec{q} - 2\vec{N}\right) \qquad \ldots(3)$$

The equation of energy is given by

$$\left[\rho_0 C_v \in + \rho_s C_s(1-\in)\right]\frac{\partial T}{\partial t} + \rho_0 C_v(\vec{q}.\nabla)T = \chi_T\nabla^2 T + \delta\left(\nabla\times\vec{N}\right).\nabla T \qquad \ldots.(4)$$

And the equation of state is

$$\rho = \rho_0\left[1-\alpha(T-T_0)\right] \qquad \ldots(5)$$

where $\vec{\mathbf{q}}$, $\vec{\mathbf{N}}$, p, ρ, ρ_o, ρ_s, μ, ζ, μ_e, κ, j, α', β', γ', T, t, χ_T, δ, α, T_o, C_v, C_s and $\hat{e}_z$ denote respectively fluid velocity, microrotation, pressure, fluid density, reference density, fluid viscosity, coupling viscosity coefficient, magnetic permeability, microinertia coefficient, micropolar viscosity coefficients, specific heat at constant volume, temperature, time, thermal conductivity, micropolar heat conduction coefficient, coefficient of thermal expansion, reference temperature and unit vector along *z*-direction.

The Maxwell's equations become

$$\in\frac{\partial \vec{H}}{\partial t} = \nabla\times\left(\vec{q}\times\vec{H}\right) + \in\gamma_m\nabla^2\vec{H} \qquad \ldots(6)$$

$$\nabla.\vec{H} = 0 \qquad \ldots(7)$$

where γ_m is the magnetic viscosity.

BASIC STATE OF THE PROBLEM

The basic state of the problem is assumed to be

$$\vec{q} = \overrightarrow{q_b} = (0,0,0),\ \vec{N} = \overrightarrow{N_b} = (0,0,0), \vec{H} = \overrightarrow{H_b} = (H_0, 0,0), p = p_b(z), \rho = \rho_b(z)$$

Using above equations the equations (1)-(7) yield

$$\frac{dp_b}{dz} + \rho_b g = 0 \qquad \ldots(8)$$

$$T = -\beta z + T_0 \qquad \ldots(9)$$

$$\rho = \rho_0(1+\alpha\beta z) \qquad \ldots(10)$$

PERTURBATION EQUATIONS

Let $\vec{q}, \vec{N}, \acute{\rho}, \theta, \vec{h}$ be represent the perturbations in $\vec{q}, \vec{N}, \rho, T, \vec{H}$ then the new variables become

$$\vec{q} = \overrightarrow{q_b} + \vec{q},\ \vec{N} = \overrightarrow{N_b} + \vec{N},\ \rho = \rho_b + \acute{\rho},\ T = T_b + \theta,\ \vec{H} = \overrightarrow{H_b} + \vec{h}$$

Using these new variables and using equations (8), (9), (10) the equations (1)-(7) become

$$\nabla.\vec{q} = 0 \quad ...(11)$$

$$\frac{\rho_0}{\in}\left[\frac{\partial \vec{q}}{\partial t} + \frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right] = -\nabla \acute{p} + (\mu+\zeta)\nabla^2\vec{q} - \frac{(\mu+\zeta)}{\kappa}\vec{q} - \acute{\rho} g\hat{e_z} + \zeta\nabla\times\vec{N}$$

$$+\frac{\mu_e}{4\pi}(\nabla\times\vec{h})\times\overrightarrow{H_b} + \frac{\mu_e}{4\pi}(\nabla\times\vec{h})\times\vec{h} \quad ...(12)$$

$$\rho_0 j\left[\frac{\partial \vec{N}}{\partial t} + \frac{1}{\in}(\vec{q}.\nabla)\vec{N}\right] = (\acute{\alpha}+\acute{\beta})\nabla(\nabla.\vec{N}) + \acute{\gamma}\nabla^2\vec{N} + \zeta\left(\frac{1}{\in}\nabla\times\vec{q} - 2\vec{N}\right) \quad ...(13)$$

$$[\rho_0 C_v \in + \rho_s C_s(1-\in)]\frac{\partial\theta}{\partial t} + \rho_0 C_v(\vec{q}.\nabla)\theta + \rho_0 C_v(\vec{q}.\nabla)\, T_b$$

$$= \chi_T\nabla^2\theta + \delta(\nabla\times\vec{N}).\nabla\theta + \delta(\nabla\times\vec{N}).\nabla T_b \quad ...(14)$$

$$\acute{\rho} = -\rho_0\alpha\theta \quad ...(15)$$

$$\in\frac{\partial\vec{h}}{\partial t} = (\overrightarrow{H_b}.\nabla)\vec{q} + \in\gamma_m\nabla^2\vec{h} \quad ...(16)$$

$$\nabla.\vec{h} = 0 \quad ...(17)$$

Using the following non-dimensional variables

$$x = x^*d\,, y = y^*d\,, z = z^*d\,, \vec{q} = \frac{K_T}{d}\overrightarrow{q^*}\,, \vec{N} = \frac{K_T}{d^2}\overrightarrow{N^*}\,, t = \frac{\rho_0 d^2}{\mu}t^*\,, \theta = \beta d\theta^*,$$

$$\acute{p} = \frac{\mu K_T}{d^2}p^*\,, \vec{h} = H_0\overrightarrow{h^*}\,, K_T = \frac{\chi_T}{\rho_0 C_v}$$

and dropping the stars , the equations (11)-(17) become

$$\nabla.\vec{q} = 0 \quad ...(18)$$

$$\frac{1}{\in}\frac{\partial\vec{q}}{\partial t} = -\nabla p + R\theta\hat{e_z} + (1+K)\nabla^2\vec{q} - \frac{(1+K)}{K_1}\vec{q} + K\nabla\times\vec{N} + (\nabla\times\vec{h})\times\hat{e_x} \quad ...(19)$$

$$\bar{J}\frac{\partial\vec{N}}{\partial t} = \acute{C}\nabla(\nabla.\vec{N}) - C\nabla\times(\nabla\times\vec{N}) + K\left(\frac{1}{\in}\nabla\times\vec{q} - 2\vec{N}\right) \quad ...(20)$$

$$EP_r\frac{\partial\theta}{\partial t} = \nabla^2\theta + W - \bar{\delta}\xi \quad ...(21)$$

$$\in P_r\frac{\partial\vec{h}}{\partial t} = \frac{\partial\vec{q}}{\partial x} + \frac{\in P_r}{P_m}\nabla^2\vec{h} \quad ...(22)$$

$$\nabla.\vec{h} = 0 \quad ...(23)$$

where $R = \frac{\rho_0 g\alpha\beta d^4}{\mu K_T}$ is the thermal Rayleigh number, $QR = \frac{\mu_e H_0^2 d^2}{4\pi\mu K_T}$ is the Chandrasekhar number, $K = \frac{\zeta}{\mu}$, $\bar{J} = \frac{k}{d^2}$, $\acute{C} = \frac{\acute{\alpha}+\acute{\beta}+\acute{\gamma}}{\mu d^2}$, $C = \frac{\acute{\gamma}}{\mu d^2}$, $P_r = \frac{\mu}{\rho_0 K_T}$ is the Prandtl number

$P_m = \frac{\mu}{\rho_0\gamma_m}$ is the magnetic Prandtl number, $\bar{\delta} = \frac{\delta}{\rho_0 C_v d^2}$, $\xi = (\nabla\times\vec{N})_z$,

$W = \vec{q}.\hat{e_z}\ , E = \left[\in + \frac{\rho_s C_v (1-\in)}{\rho_0 C_v}\right]$

Boundary conditions

We consider that both the boundaries of the problem are free and perfectly heat conducting, thus

$$W = 0 = \frac{\partial^2 W}{\partial z^2}\ , \theta = 0\ , \vec{N} = 0\ , \xi = 0 \text{ at } z = 0 \text{ and } z = 1 \qquad \ldots (24)$$

Dispersion relations

Using curl operator on equations (18) to (23) and applying normal mode given by $[\,W\,, \xi\,, \theta\,, h_z\,] = [\,W(z), G(z), \Theta(z), B(z)]e^{(ik_x + jk_y + \sigma t)}$ and eliminating $\Theta\,, G\,, B$, we have

$$\left[\frac{\sigma}{\in}(D^2 - a^2) - (1+K)(D^2-a^2)^2 + \left(\frac{1+K}{K_1}\right)(D^2-a^2)\right][\bar{j}\sigma - C(D^2-a^2) + 2K]$$

$$[EP_r\sigma - (D^2-a^2)]\left[\in P_r\sigma - \frac{\in P_r}{P_m}(D^2-a^2)\right]W$$

$$+Ra^2\left[\in P_r\sigma - \frac{\in P_r}{P_m}(D^2-a^2)\right]\left[\bar{j}\sigma - C(D^2-a^2) + 2K + \frac{\bar{\delta}K}{\in}(D^2-a^2)\right]W$$

$$+\frac{K^2}{\in}(D^2-a^2)^2\left[\in P_r\sigma - \frac{\in P_r}{P_m}(D^2-a^2)\right][EP_r\sigma - (D^2-a^2)]W$$

$$+Q{k_x}^2(D^2-a^2)[EP_r\sigma - (D^2-a^2)][\bar{j}\sigma - C(D^2-a^2) + 2K]W = 0 \ldots (25)$$

where $a = \sqrt{{k_x}^2 + {k_y}^2}$ and $D = \frac{d}{dz}$

Boundary conditions (24) become $W = 0 = D^2W$ at $z = 0$ and $z = 1$ therefore $D^{2n}\ W = 0$ at $z = 0$ and $z = 1$, where n is a positive integer.

Thus, $W = W_0 \sin \pi z$, where W_0 is a constant.

Substituting for W in equation (25), we have

$$\left[\frac{\sigma b}{\in} + (1+K)b^2 + \left(\frac{1+K}{K_1}\right)b\right][\bar{j}\sigma + Cb + 2K][EP_r\sigma + b]\left[\in P_r\sigma + \frac{\in P_r b}{P_m}\right]$$

$$-Ra^2\left[\in P_r\sigma + \frac{\in P_r b}{P_m}\right]\left[\bar{j}\sigma + Cb + 2K - \frac{\bar{\delta}Kb}{\in}\right] - \frac{K^2b^2}{\in}\left[\in P_r\sigma + \frac{\in P_r b}{P_m}\right][EP_r\sigma + b]$$

$$+Q{k_x}^2 b[EP_r\sigma + b][\bar{j}\sigma + Cb + 2K] = 0 \qquad \ldots (26)$$

where $b = a^2 + \pi^2$.

Stationary convection

For the stationary marginal state we set $\sigma = 0$ in (26) and we obtain

$$R = \frac{b^2\left(b+\frac{1}{K_1}\right)(1+K)(Cb+2K) - \frac{K^2b^3}{\in} + \frac{Qk_x{}^2P_m b}{\in P_m}(b+2K)}{a^2\left(Cb+2K-\frac{\bar{\delta}Kb}{\in}\right)} \quad \ldots (27)$$

In the non-porous medium and in the absence of magnetic field and coupling parameter equation (27) reduces to

$$R = \frac{b^3}{a^2}\left[\frac{b(1+K)C+2K+K^2}{(Cb+2K)}\right]$$

Which is the same as given by Goodarz Ahmadi [2].

In the absence of magnetic field and in non-porous medium equation (27) reduces to

$$R = \frac{b^3}{a^2}\left[\frac{C(1+K)b+2K+b^2}{\left(C-\bar{\delta}K\right)b+2K}\right]$$

Which is the same as proposed by C.E. Payne and B. Straughan and Y. Qin and P.N. Kaloni.

Equation (27) can also be written as

$$R = \frac{b^4(1+K)+b^3\left[(1+K)\left(2A+\frac{1}{K_1}\right)-\frac{KA}{\in}\right]+b^2\left[\frac{2A}{K_1}(1+K)+\frac{Qk_x{}^2P_m}{\in P_r}\right]+\frac{2AQk_x{}^2P_m b}{\in P_r}}{a^2\left[2A+b\left(1-\frac{\bar{\delta}A}{\in}\right)\right]} \quad \ldots (28)$$

where $A = \frac{K}{C}$ is the micropolar coefficient.

In order to investigate the effect of medium permeability K_1, coupling parameter K, micropolar coefficient A, heat conduction parameter $\bar{\delta}$ and magnetic field Q, we examine the behaviour of $\frac{dR}{dK_1}, \frac{dR}{dK}, \frac{dR}{dA}, \frac{dA}{d\bar{\delta}}$ and $\frac{dR}{dQ}$.

From equation (28), we have

$$\frac{dR}{dK_1} = -\frac{b^2(1+K)(2A+b)}{K_1{}^2a^2\left[2A+b\left(1-\frac{\bar{\delta}A}{\in}\right)\right]} \Rightarrow \frac{dR}{dK_1} < 0 \text{ when } \bar{\delta} < \frac{\in}{A}$$

Thus, R decreases as K_1 increases when $\bar{\delta} < \frac{\in}{A}$ hence the medium permeability has destabilizing effect when $\bar{\delta} < \frac{\in}{A}$.

From equation (28), we have

$$\frac{dR}{dK} = \frac{b^4+b^3\left[2A+\frac{1}{K_1}-\frac{A}{\in}\right]+b^2\left(\frac{2A}{K_1}\right)}{a^2\left[2A+b\left(1-\frac{\bar{\delta}A}{\in}\right)\right]}$$

$$\Rightarrow \quad \frac{dR}{dK} > 0 \quad \text{when} \quad \bar{\delta} < \frac{\in}{A} \quad \text{and} \quad K_1 < \frac{\in}{A}$$

Thus, R increases as K increases when $\bar{\delta} < \frac{\in}{A}$ and $K_1 < \frac{\in}{A}$. Hence the coupling parameter has stabilizing effect.

$$\frac{dR}{dA} = \frac{b}{a^2}\left[\frac{b^5\left[\frac{\bar{\delta}(1+K)}{\in}\right] + b^4\left[\frac{\bar{\delta}(1+K)}{\in K_1} - \frac{K}{\in}\right] + b^3\left[\frac{\bar{\delta} Q {k_x}^2 P_m}{\in^2 P_m} - 2AK\right]}{\left[2A + b\left(1 - \frac{\bar{\delta}A}{\in}\right)\right]^2}\right]$$

$$\Rightarrow \quad \frac{dR}{dA} > 0 \quad \text{when} \quad K < \frac{\bar{\delta}}{K_1} \quad \text{and} \quad Q > \frac{2AK \in^2 P_m}{\bar{\delta} {k_x}^2 P_m}$$

Thus, R increases as A increases when $K < \frac{\bar{\delta}}{K_1}$ and $Q > \frac{2AK \in^2 P_m}{\bar{\delta} k_x^2 P_m}$ hence the micropolar coefficient A has stabilizing effect when $K < \frac{\bar{\delta}}{K_1}$ and $Q > \frac{2AK \in^2 P_m}{\bar{\delta} k_x^2 P_m}$.

Again from equation (28), we have

$$\frac{dR}{d\bar{\delta}} = \frac{Ab}{\in}\left[\frac{b^4(1+K) + b^3\left[(1+K)\left(2A + \frac{1}{K_1}\right) - \frac{KA}{\in}\right] + b^2\left[\frac{2A(1+K)}{K_1} + \frac{Q{k_x}^2 P_m}{\in P_m}\right] + \frac{2AQ{k_x}^2 P_m b}{\in P_m}}{a^2\left[2A + b\left(1 - \frac{\bar{\delta}A}{\in}\right)\right]^2}\right]$$

$$\Rightarrow \quad \frac{dR}{d\bar{\delta}} > 0 \quad \text{when} \quad \in > \frac{1}{2}$$

Thus, R increases as $\bar{\delta}$ increases when $\in > \frac{1}{2}$, hence the heat conduction parameter has stabilizing effect when $\in > \frac{1}{2}$.

Again from equation (28), we have

$$\frac{dR}{dQ} = \frac{b{k_x}^2 P_m (2A + b)}{\in P_r a^2\left[2A + b\left(1 - \frac{\bar{\delta}A}{\in}\right)\right]}$$

$$\Rightarrow \quad \frac{dR}{dQ} > 0 \quad \text{when} \quad \bar{\delta} < \frac{\in}{A}$$

Thus, R increases as Q increases when $\bar{\delta} < \frac{\in}{A}$, hence the magnetic field has stabilizing effect when $\bar{\delta} < \frac{\in}{A}$.

CASE OF OVERSTABILITY:

Equation (26) may be written as

$$\frac{lbEP_rP_m\sigma^4}{\in}+\sigma^3\left[\frac{lb^2P_m}{\in}+\frac{lbEP_r}{\in}+EP_rP_mlb^2(1+K)+\frac{EP_rP_mlb(1+K)}{K_1}+\frac{EP_rP_mb(b+2A)}{\in}\right]$$

$$+\sigma^2\left[\frac{lb^3}{\in}+EP_rP_mb^2(1+K)(b+2A)+\frac{EP_rP_mb(1+K)(b+2A)}{K_1}+lb^3P_m(1+K)\right.$$

$$+\frac{lb^2P_m(1+K)}{K_1}+\frac{b^2P_m(b+2A)}{\in}+EP_rlb^2(1+K)+\frac{EP_rlb(1+K)}{K_1}+\frac{EP_rb(b+2A)}{\in}$$

$$\left.-Ra^2lP_m-\frac{KAb^2EP_rP_m}{\in}+\frac{lEQ{k_x}^2P_mb}{\in}\right]+\sigma\left[lb^4(1+K)+\frac{lb^3(1+K)}{K_1}+\frac{b^3(b+2A)}{\in}\right.$$

$$+b^3P_m(1+K)(b+2A)+\frac{b^2P_m(1+K)(b+2A)}{K_1}+EP_rb^2(1+K)(b+2A)$$

$$+\frac{EP_rb(1+K)(b+2A)}{K_1}-Ra^2\left[lb+P_m\left(b+2A-\frac{\bar{\delta}Ab}{\in}\right)\right]-\frac{KAEP_rb^3}{\in}-\frac{KAP_mb^3}{\in}$$

$$\left.+\frac{lb^2Q{k_x}^2P_m}{\in P_r}+\frac{EQ{k_x}^2P_mb(b+2A)}{\in}\right]+b^4(1+K)(b+2A)+\frac{b^3(1+K)(b+2A)}{K_1}$$

$$-Ra^2b\left(b+2A-\frac{\bar{\delta}Ab}{\in}\right)-\frac{KAb^4}{\in}+\frac{Q{k_x}^2P_mb^2(b+2A)}{\in P_r}=0 \quad \ldots(29)$$

where $l=\dfrac{J}{C}$, $A=\dfrac{K}{C}$

Putting $\sigma=i\sigma_i$ in equation (29) and separating real and imaginary parts and then eliminating Ra^2, we have

$$A_0s^2+A_1s+A_2=0 \quad \ldots(30)$$

where ${\sigma_i}^2=s$

As $s={\sigma_i}^2$ which is always positive, therefore both the roots of equation (30) must be positive so that the sum of the roots will be positive. But from equation (30), the sum of the roots is $-\left(\dfrac{A_1}{A_0}\right)$ thus, the sufficient conditions for non-existence of over-stability are given by

$A_0<0$ and $A_1<0$ which give

$$\bar{\delta}<\frac{\in}{A},\in>\frac{1}{2},A>\frac{1}{2},\qquad \frac{EP_r}{2}<P_m<min.\left\{EP_r,\frac{1}{2(1+K)}\right\}$$

and $$max.\left\{\frac{1}{\in},EP_r\right\}<l<min.\left\{\in P_r,\frac{P_m}{K_1},\frac{2AP_r}{Q{k_x}^2}\right\}$$

Hence PES is valid.

NATURE OF OSCILLATORY MODES

Multiplying both sides of equation (25) by *W** (complex conjugate of *W*) and integrate with respect to *z* from *z* = 0 to *z* = 1, we have

$$\left(\frac{\sigma}{\in}+\frac{1+K}{K_1}\right)I_1+(1+K)I_2-Ra^2\left[\sigma^* EP_r I_5+I_6+\bar{\delta}\int_0^1 G^*\Theta dz\right]-(\in \bar{j}\sigma^*+2K\in)I_3$$
$$+\in CI_4+\frac{QP_r}{\in P_m}[\sigma^* P_m I_7+I_8]=0 \qquad \ldots(31)$$

where $I_1=\int_0^1[|DW|^2+a^2|W|^2]dz$, $\quad I_2=\int_0^1[|D^2W|^2+2a^2|DW|^2+a^4|W|^2]dz$

$$I_3=\int_0^1|G|^2dz, \qquad I_4=\int_0^1[|DG|^2+a^2|G|^2]dz,$$
$$I_5=\int_0^1|\Theta|^2dz, I_6=\int_0^1[|D\Theta|^2+a^2|\Theta|^2]dz, \qquad I_7=\int_0^1[|DB|^2+a^2|B|^2]dz,$$
$$I_8=\int_0^1[|D^2B|^2+2a^2|DB|^2+a^4|B|^2]dz$$

Putting $\sigma=\sigma_r+i\sigma_i$ and $\sigma^*=\sigma_r-i\sigma_i$ in equation (31) and separating real and imaginary parts, we have

The real part in the absence of micropolar heat conduction parameter is given by

$$Ra^2=\frac{\sigma_r\left[\frac{I_1}{\in}-\in \bar{j}I_3+\frac{QP_rI_7}{\in}\right]+(1+K)\left(\frac{I_1}{K_1}+I_2\right)-2K\in I_3+\in CI_4+\frac{QP_rI_8}{\in P_m}}{\sigma_r EP_r I_5+I_6} \qquad \ldots(32)$$

The imaginary part in the absence of micropolar heat conduction parameter is given by

$$\sigma_i\left[\frac{I_1}{\in}+Ra^2EP_rI_5+\in \bar{j}I_3-\frac{QP_rI_7}{\in}\right]=0 \qquad \ldots(33)$$

In the absence of magnetic field equation (33) reduces to

$$\sigma_i\left[\frac{I_1}{\in}+Ra^2EP_rI_5+\in \bar{j}I_3\right]=0 \qquad \ldots(34)$$

Since I_1, I_3, I_5 are all positive, then $\sigma_i=0$. Hence in the absence of magnetic field and heat conduction parameter oscillatory modes do not exist and PES is valid. Hence magnetic field produces oscillatory modes.

For oscillatory modes equation (33) reduces to

$$Ra^2=\frac{\frac{QP_rI_7}{\in}-\frac{I_1}{\in}-\in \bar{j}I_3}{EP_rI_5} \qquad \ldots(35)$$

Eliminating Ra^2 between equations (32) and (35), we have

$$\sigma_r=-\frac{\in}{2EP_rI_1I_5}\left[\frac{(1+K)EP_rI_1I_5}{K_1}+EP_rI_2I_5+KEP_rI_5(I_2-2\in I_5)\right.$$
$$\left.+\in CEP_rI_4I_5+\frac{QP_r{}^2EI_5I_8}{\in P_m}+\frac{I_6}{\in}(I_1-QP_rI_7)+\in \bar{j}I_3I_6\right]$$

$$\Rightarrow \sigma_r<0 \text{ when } \in<\frac{I_2}{2I_5} \text{ and } Q<\frac{I_1}{P_rI_7}$$

Hence oscillatory modes if exist will be stable when $\in < \frac{I_2}{2I_5}$ and $Q < \frac{I_1}{P_r I_7}$.

Conclusion

1. The medium permeability has destabilizing effect when $\bar{\delta} < \frac{\in}{A}$.

2. The coupling parameter has stabilizing effect when $\bar{\delta} < \frac{\in}{A}$ and $K_1 < \frac{\in}{A}$.

3. The micropolar coefficient has stabilizing effect when $K < \frac{\bar{\delta}}{K_1}$ and $Q > \frac{2AK \in^2 P_m}{\bar{\delta} k_x^2 p_m}$.

4. The heat conduction parameter has stabilizing effect when $\in > \frac{1}{2}$.

5. The magnetic field has stabilizing effect when $\bar{\delta} < \frac{\in}{A}$.

6. The oscillatory modes if exist will be stable when $\in < \frac{I_2}{2I_5}$ and $Q < \frac{I_1}{P_r I_7}$.

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