SLIGHTLY $\hat{\alpha}g$ CONTINUOUS FUNCTIONS

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In this paper, slightly $\hat{\alpha}g$ continuity is introduced and studied. Furthermore, basic properties and preservation theorems of slightly $\hat{\alpha}g$ continuous functions are investigated and relationships between slightly $\hat{\alpha}g$ continuous function and graphs are investigated.

KEYWORDS: clopen, $\hat{\alpha}g$ open, $\hat{\alpha}g$ continuity, slightly continuity, slightly $\hat{\alpha}g$ continuity.

INTRODUCTION

Many different forms of continuous functions have been introduced over the years. Some of them are pre continuity [1, 4], semi continuity [3], $\hat{\alpha}g$ continuity [12] and slightly continuity [2, 6]. Various interesting problems arise when one considers continuity. Its importance is significant in various areas of mathematics and related sciences. In this paper, slightly $\hat{\alpha}g$ continuity is introduced and studied.

Throughout the present paper, X and Y are always topological spaces. Let A be a subset of X. We denote interior and closure of A by int (A) and cl (A) respectively.

PRELIMINARIES

Definition 2.1: A subset A of a topological space X is said to be

(1) pre open [4] if $A \subset \text{int}(\text{cl}(A))$

(2) semi open [3] if $A \subset \text{cl}(\text{int}(A))$

(3) α open [5] if $A \subset \text{int}(\text{cl}(\text{int}(A)))$

A subset A of a topological space X is said to be $\hat{\alpha}$ generalized closed ($\hat{\alpha}g$ closed) [12] if int (cl (int (A))) $\subset U$ whenever $A \subset U$ and U is open in X.

The complement of pre open (semi open, α open) set is pre closed (semi closed, α closed). The complement of $\hat{\alpha}g$ closed set is $\hat{\alpha}g$ open.

The family of all open(respy $\hat{\alpha}g$ open, clopen, $\hat{\alpha}g$ clopen) sets of X is denoted by $O(X)$ ($\hat{\alpha}gO(X)$, $CO(X)$, $\hat{\alpha}gCO(X)$).

Definition 2.2: A function $f : X \to Y$ is $\hat{\alpha}g$ continuous [12] if $f^{-1}(V)$ is $\hat{\alpha}g$ open in X for each open set V of Y.

Definition 2.3: A function $f : X \to Y$ is slightly continuous [2] if $f^{-1}(V)$ is open in X for each clopen set V of Y.

SLIGHTLY $\hat{\alpha}g$ CONTINUOUS FUNCTIONS

In this section the idea of slightly $\hat{\alpha}g$ continuous function is introduced and characterizations and relationships of $\hat{\alpha}g$ continuous function and basic properties of slightly $\hat{\alpha}g$ continuous functions are studied.

Definition 3.1 : A function $f : X \to Y$ is called slightly $\hat{\alpha}g$ continuous if $f^{-1}(V)$ is $\hat{\alpha}g$ open in X for each clopen set V of Y.

Theorem 3.2 : The following statements are equivalent for a function $f : X \to Y$:

(1) f is slightly $\hat{\alpha}g$ continuous

(2) for every clopen set V of Y, $f^{-1}(V)$ is $\hat{\alpha}g$ closed in X

(3) for every clopen set V of Y, $f^{-1}(V)$ is $\hat{\alpha}g$ clopen in X

Proof : Straight forward.

Theorem 3.3 : Let $f : X \to Y$ be a function and $g : X \to Y$ be the graph of the function f, defined by $g(x) = (x, f(x))$, for every x in X. If g is slightly $\hat{\alpha}g$ continuous, then f is slightly $\hat{\alpha}g$ continuous.

Proof : Let $V \in CO(Y)$. Then $X \times V \in CO(X \times Y)$. As g is slightly $\hat{\alpha}g$ continuous, $f^{-1}(V) = g^{-1}(X \times V) \in \hat{\alpha}g\, O(X)$. Then f is slightly $\hat{\alpha}g$ continuous.

Definition 3.4 : A function $f : X \to Y$ is called:

(1) $\hat{\alpha}g$ irresolute [12] if $f^{-1}(V)$ $\hat{\alpha}g$ open for every $\hat{\alpha}g$ open set V of Y.

(2) $\hat{\alpha}g$ open [12] if $f(A)$ is $\hat{\alpha}g$ open in Y for every $\hat{\alpha}g$ open set A of X.

Theorem 3.5 : Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then the following are true:

(1) if f is $\hat{\alpha}g$ irresolute and g is slightly $\hat{\alpha}g$ continuous, then $gof : X \to Z$ is slightly $\hat{\alpha}g$ continuous

(2) if f is $\hat{\alpha}g$ irresolute and g is $\hat{\alpha}g$ continuous, then $gof : X \to Z$ is slightly $\hat{\alpha}g$ continuous

(3) if f is $\hat{\alpha}g$ irresolute and g is slightly continuous, then $gof : X \to Z$ is slightly $\hat{\alpha}g$ continuous

Proof : (1) Let V be clopen in Z. As g is slightly $\hat{\alpha}g$ continuous, $g^{-1}(V)$ is $\hat{\alpha}g$ open. Hence $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\hat{\alpha}g$ open, since f is $\hat{\alpha}g$ irresolute. So gof is slightly $\hat{\alpha}g$ continuous.

(2) and (3) can be got similarly.

Theorem 3.6 : Let $f : X \to Y$ and $g : Y \to Z$ be functions. If f is $\hat{\alpha}g$ open and surjective and $gof : X \to Z$ is slightly $\hat{\alpha}g$ continuous then g is slightly $\hat{\alpha}g$ continuous.

Proof : Let $V \in CO(Z)$. As gof is slightly $\hat{\alpha}g$ continuous, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\hat{\alpha}g$ open in X. Since f is $\hat{\alpha}g$ open and surjective, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is $\hat{\alpha}g$ open in Y. Hence g is slightly $\hat{\alpha}g$ continuous.

Combining the previous two theorems, we have the following result.

Theorem 3.7 : Let $f : X \to Y$ be surjective, $\hat{\alpha}g$ irresolute and $\hat{\alpha}g$ open and $g : Y \to Z$ be a function. Then $gof : X \to Z$ is slightly $\hat{\alpha}g$ continuous if and only if g is $\hat{\alpha}g$ slightly continuous.

Definition 3.8:

(i) A filter base Λ is said to be $\hat{\alpha}g$ convergent to a point $x \in X$ if for any $U \in \hat{\alpha}g\ O(X)$ containing x, there exists $B \in \Lambda$ such that $B \subset U$.

(ii) A filter base Λ is said to be coconvergent to a point $x \in X$ if for any $U \in CO(X)$ containing x, there exists $B \in \Lambda$ such that $B \subset U$.

Theorem 3.9 : If a function $f : X \to Y$ is slightly $\hat{\alpha}g$ continuous, then for each $x \in X$ and each filter base Λ in X $\hat{\alpha}g$ converging to x, the filter base $f(\Lambda)$ is coconvergent to $f(x)$.

Proof : Let $x \in X$ and Λ be any filter base in X, $\hat{\alpha}g$ converging to x. Since f is slightly $\hat{\alpha}g$ continuous, then for any $V \in CO(Y)$ containing $f(x)$, $f^{-1}(V)$ is $\hat{\alpha}g$ open containing x. As Λ is converging to x, there exists $B \in \Lambda$ such that $B \subset f^{-1}(V)$. Hence $f(B) \subset V$. So the filter base $f(\Lambda)$ is is coconvergent to $f(x)$.

Definition 3.10: A space X is $\hat{\alpha}g$ connected [12] if X is not the union of two disjoint nonempty $\hat{\alpha}g$ open sets.

Theorem 3.11: Let $f : X \to Y$ be slightly $\hat{\alpha}g$ continuous surjective function and X be $\hat{\alpha}g$ connected. Then Y is a connected space.

Proof : Let Y be not connected. Then there exists two disjoint nonempty open sets U and V such that $Y = U \cup V$. So, U and V are clopen sets in Y. Hence $f^{-1}(U)$ and $f^{-1}(V)$ are $\hat{\alpha}g$ open in X. As f is surjective and U and V are disjoint, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint nonempty $\hat{\alpha}g$ open sets whose union is X, which is a contradiction. Hence Y is connected.

Definition 3.12 : A topological space X is called hyperconnected [11] if every nonempty open subset of X is dense in X. It is well known that every hyperconnected space is connected but not conversely.

The following example shows that slightly $\hat{\alpha}g$ continuous surjection does not necessarily preserve hyper connectedness.

Example 3.13: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, X\}$ and $\sigma = \{\varphi, \{b\}, \{c\}, \{b, c\}, \varphi\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is slightly $\hat{\alpha}g$ continuous and surjective. (X, τ) is hyperconnected but (X, σ) is not hyperconnected.

COVERING PROPERTIES

In this section, the relation between slightly $\hat{\alpha}g$ continuous function and compactness are studied.

Definition 4.1 : A space X is said to be mildly compact [7] (respy. $\hat{\alpha}g$ compact [12]) if every clopen cover (respy. $\hat{\alpha}g$ open cover) of X has a finite sub cover.

A subset A of a space X is said to be mildly compact (respy. $\hat{\alpha}g$ compact) relative to X if every cover of A by clopen (respy. $\hat{\alpha}g$ open) sets of X has a finite sub cover.

A subset A of a space X is said to be mildly compact (respy. $\hat{\alpha}g$ compact) if the subspace A is mildly compact (respy. $\hat{\alpha}g$ compact)

Theorem 4.2 : If a function $f : X \to Y$ is slightly $\hat{\alpha}g$ continuous and K is $\hat{\alpha}g$ compact relative to X, then $f(K)$ is mildly compact in Y.

Proof: Let $\{H_\alpha : \alpha \in I\}$ be any cover of $f(K)$ by clopen sets of the subspace $f(K)$. For each $\alpha \in I$, there exists a clopen set K_α of Y such that $H_\alpha = K_\alpha \cap f(K)$. For every $x \in K$, there exists $\alpha_x \in I$ such that $f(x) \in K_{\alpha x}$ and $f^{-1}(K_{\alpha x}) \in \hat{\alpha}g\ O(X)$. Since the family $\{f^{-1}(K_{\alpha x}) : x \in K\}$

is a cover of K by $\hat{\alpha}g$ open sets of X, there exits a finite subset K_0 of K such that $K \subset \cup \{f^{-1}(K_{\alpha x}) : x \in K_0\}$. Hence we have $f(K) \subset \cup \{K_{\alpha x} : x \in K_0\}$. Thus $f(K) = \cup \{H_{\alpha x} : x \in K_0\}$ and hence $f(K)$ is mildly compact.

Corollary 4.3 : If $f : X \to Y$ is slightly $\hat{\alpha}g$ continuous, surjective and X is $\hat{\alpha}g$ compact, then Y is mildly compact.

Definition 4.4 : A space X is said to be:

(1) mildly countably compact [7] if every clopen countable cover of X has a countable sub cover

(2) mildly Lindelof [7] if every clopen cover of X has a countable sub cover

(3) countably $\hat{\alpha}g$ compact if every countable $\hat{\alpha}g$ open cover of X has a finite sub cover

(4) $\hat{\alpha}$g Lindelof if every $\hat{\alpha}g$ open cover of X has a countable sub cover

(5) $\hat{\alpha}$g closed compact if every $\hat{\alpha}g$ closed cover of X has a finite sub cover

(6) countably $\hat{\alpha}g$ closed compact if every countable $\hat{\alpha}g$ closed cover of X has a finite sub cover

(7) $\hat{\alpha}g$ closed Lindelof if every $\hat{\alpha}g$ closed cover of X has a countable sub cover.

Theorem 4.5: Let $f : X \to Y$ be a slightly $\hat{\alpha}g$ continuous surjection. Then the following statements hold:

(1) if X is $\hat{\alpha}g$ Lindelof, then Y is mildy Lindelof

(2) if X is countably $\hat{\alpha}g$ compact, then Y is mildly countably compact.

Proof : (1) Let $\{V_\alpha : \alpha \in I\}$ be any clopen cover of Y. As f is slightly $\hat{\alpha}$g continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a $\hat{\alpha}g$ open cover of X. Since X is $\hat{\alpha}g$ Lindelof, there exists a countable subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Hence $Y = \cup \{V_\alpha : \alpha \in I_0\}$. So Y is mildly Lindelof.

The proof of (2) is similar.

Theorem 4.6 : Let $f : X \to Y$ be a slightly $\hat{\alpha}g$ continuous surjection. Then the following statements hold:

(1) if X is $\hat{\alpha}g$ closed compact, then Y is mildly compact

(2) if X is $\hat{\alpha}g$ closed Lindelof, then Y is mildly compact

(3) if X is countably $\hat{\alpha}g$ closed compact, then Y is mildly countably compact.

Proof : Similar to the proof of the above theorem.

Separation axioms

In this section, the relation between slightly $\hat{\alpha}g$ continuous function and separation axioms are investigated.

Definition 5.1 : A space X is said to be :

(i) $\hat{\alpha}g$ T_1 [12] if for each pair of distinct points x and y of X, there exist $\hat{\alpha}g$ open sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

(ii) $\hat{\alpha}g$ T_2 ($\hat{\alpha}g$ Hausdorff) [12] if for each pair of distinct points x and y of X, there exist disjoint $\hat{\alpha}g$ open sets U and V in X containing x and y respectively.

(iii) clopen T_1 [7], if for each pair of distinct points x and y of X, there exist clopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

(iv) clopen T_2 (clopen Hausdorff or ultra Hausdorff) [7] if for each pair of distinct points x and y of X, there exist disjoint clopen sets containing x and y respectively.

Remark 5.2 : The following implications hold for a topological space X :

(1) clopen T_1 implies T_1

(2) T_1 implies $\hat{\alpha}g$ T_1

None of these implications is reversible.

Example 5.3 : Let R be the real numbers with the finite complement topology τ. Then (R, τ) is T_1 but not clopen T_1.

Example 5.4 : Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a, b\}, X\}$. (X, τ) is $\hat{\alpha}g$ T_1 but not T_1.

Theorem 5.5 : If $f : X \to Y$ be a slightly $\hat{\alpha}g$ continuous injection and Y is clopen T_1, then X is $\hat{\alpha}g$ T_1.

Proof : Let Y be f clopen T_1. Let x and y be distinct points of X. There exist V, $W \in CO(Y)$ such that $f(x) \in V, f(y) \notin V$ and $f(y) \in W, f(x)$ W. Since f is slightly $\hat{\alpha}g$ continuous $f^{-1}(V), f^{-1}(W) \in \hat{\alpha}g\,O(X)$ such that $x \in f^{-1}(V), y \notin f^{-1}(V)$ and $y \in f^{-1}(W), x \notin f^{-1}(W)$. Hence X is $\hat{\alpha}g$ T_1.

Theorem 5.6 : If $f : X \to Y$ be a slightly $\hat{\alpha}g$ continuous injection and Y is clopen T_2, then X is $\hat{\alpha}g$ T_2.

Proof : Let Y be clopen T_2. Let x and y be distinct points of X. There exist disjoint clopen sets U and V containing $f(x)$ and $f(y)$ respectively. Since f is slightly $\hat{\alpha}g$ continuous $f^{-1}(U)$, $f^{-1}(V) \in \hat{\alpha}g\,O(X)$. $f^{-1}(U), f^{-1}(V)$ are disjoint $\hat{\alpha}g$ open sets containing x and y respectively. Hence X is $\hat{\alpha}g$ T_2.

Definition 5.7 : A space is called clopen regular (respy. $\hat{\alpha}g$ regular) if for each clopen (respy. $\hat{\alpha}g$ closed) set F and each point $x \notin F$, there exist disjoint open sets U and V such that $F \subset U$ and $x \in V$.

Definition 5.8 : A space is called clopen normal (respy. $\hat{\alpha}g$ normal) if for every pair of clopen (respy. $\hat{\alpha}g$ closed) sets F_1 and F_2, there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Theorem 5.9 : If f is slightly $\hat{\alpha}$g continuous injective open function from a $\hat{\alpha}g$ regular space X onto a space Y, then Y is clopen regular.

Proof : Let F be clopen in Y and $y \in Y$ be such that $y \notin F$. Let $y = f(x)$. As f is slightly $\hat{\alpha}g$ continuous, $f^{-1}(F)$ is $\hat{\alpha}g$ closed in X. Take $G = f^{-1}(F)$. We have $x \notin G$. Since X is $\hat{\alpha}g$ regular, there exist disjoint open sets U and V such that $G \subset U$ and $x \in V$. We have $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$ such that $f(U)$ and $f(V)$ are disjoint open sets. So Y is clopen regular.

Theorem 5.10 : If f is slightly $\hat{\alpha}g$ continuous injective open function from a $\hat{\alpha}g$ normal space X onto a space Y, then Y is clopen normal.

Proof : Let F_1 and F_2 be disjoint clopen subsets of Y. Since f is slightly $\hat{\alpha}g$ continuous $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are $\hat{\alpha}g$ closed sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. We have $U \cap V = \varphi$. Since X is $\hat{\alpha}g$ normal, there exist disjoint open sets A and B such that $U \subset A$ and $V \subset B$. We obtain that $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$ such that $f(A)$ and $f(B)$ are disjoint open sets. Thus Y is clopen normal. For a function $f : X \to Y$ the subset $G(f) = \{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f.

Definition 5.11: A graph $G(f)$ of a function $f: X \to Y$ is said to be strongly $\hat{\alpha}g$ co-closed if for each $(x, y) \in (X \times Y) - G(f)$, there exists $U \in \hat{\alpha}g\ CO(X)$ containing x and $V \in CO(Y)$ containing y such that $(U \times V) \cap G(f) = \varphi$.

Lemma 5.12 : [12] A graph $G(f)$ of a function $f: X \to Y$ is strongly $\hat{\alpha}g$ co-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exists $U \in \hat{\alpha}g\ CO(X)$ containing x and $V \in CO(Y)$ containing y such that $f(U) \cap Y = \varphi$.

Theorem 5.13 : If $f: X \to Y$ is slightly $\hat{\alpha}g$ continuous and Y is clopen T_1, then $G(f)$ is strongly $\hat{\alpha}g$ co-closed $X \times Y$.

Proof : Let $(x, y) \in (X \times Y) - G(f)$. Then $f(x) \neq y$ and there exists clopen set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is slightly $\hat{\alpha}g$ continuous, then $f^{-1}(V) \in \hat{\alpha}g\ CO(X)$ containing x. Take $U = f^{-1}(V)$. We have $f(U) \subset V$. Hence $f(U) \cap (Y - V) = \varphi$ and $Y - V \in CO(Y)$ containing y. This shows that $G(f)$ is strongly $\hat{\alpha}g$ co-closed in $X \times Y$.

Corollary 5.14 : If $f: X \to Y$ is slightly $\hat{\alpha}g$ continuous and Y is clopen Hausdorff, then $G(f)$ is strongly $\hat{\alpha}g$ co-closed in $X \times Y$.

Theorem 5.15 : Let $f: X \to Y$ has a strongly $\hat{\alpha}g$ co-closed graph $G(f)$. If f is injective, then X is $\hat{\alpha}g\ T_1$.

Proof : Let x and y be distinct points of X. Then $(x, f(y)) \in (X \times Y) - G(f)$. By Lemma 5.12, there exists $\hat{\alpha}g$ clopen set U of X and $V \in CO(Y)$ such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \varphi$. Hence $U \cap f^{-1}(V) = \varphi$ and $y \notin U$. This implies that X is $\hat{\alpha}g\ T_1$.

Theorem 5.16 : Let $f: X \to Y$ has a strongly $\hat{\alpha}g$ co-closed graph $G(f)$. If f is surjective $\hat{\alpha}g$ open function, then Y is $\hat{\alpha}g\ T_2$.

Proof: Let y_1 and y_2 be distinct points of Y. Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X \times Y) - G(f)$. By definition, there exists $\hat{\alpha}g$ clopen set U of X and $V \in CO(Y)$ such that $(x, y_2) \in U \times V$ and $(U \times V) \cap G(f) = \varphi$. Then we have $f(U) \cap V = \varphi$. Since f is $\hat{\alpha}g$ open, $f(U)$ is $\hat{\alpha}g$ open such that $f(x) = y_1 \in f(U)$. This implies Y is $\hat{\alpha}g\ T_2$.

RELATIONSHIPS

Definition 6.1 : A function $f: X \to Y$ is said to be:

(1) semi continuous [3] if $f^{-1}(V)$ is semi open for each open set V of Y.

(2) pre continuous [1, 4] if $f^{-1}(V)$ is pre open for each open set V of Y.

(3) strongly $\hat{\alpha}g$ irresolute if $f^{-1}(V)$ is open for each $\hat{\alpha}g$ open set V of Y.

The following diagram holds.

(i) Pre continuous → $\hat{\alpha}g$ continuous → slightly $\hat{\alpha}g$ continuous

(ii) Slightly continuous → slightly $\hat{\alpha}g$ continuous.

(iii) Strongly $\hat{\alpha}g$ irresolute → $\hat{\alpha}g$ irresolute → $\hat{\alpha}g$ continuous.

(iv) Semi continuity → $\hat{\alpha}g$ continiuity.

Example 6.2 : Let $X = \{a, b, c\}$, $\tau = \sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$

Define f : $(X, \tau) \to (X, \sigma)$ to be the identity function. f is $\hat{\alpha}g$ irresolute but not strongly $\hat{\alpha}g$ irresolute, as $\{a, c\}$ is $\hat{\alpha}g$ open, $f^{-1}(\{a, c\}) = \{a, c\}$ is not open.

Example 6.3 : Let $X = \{a, b, c\}$ $\tau = \sigma = \{\varphi, \{a\}, \{a, b\}, X\}$

Define $f : (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = b$. f is $\hat{\alpha}g$ continuous but not $\hat{\alpha}g$ irresolute, as $\{b\}$ is $\hat{\alpha}g$ open $f^{-1}(\{b\}) = \{b, c\}$ is not $\hat{\alpha}g$ open.

Example 6.4 : Let $X = \{a, b, c\}$, $\tau = \sigma = \{\varphi, \{a\}, X\}$

Define $f : (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = b$, $f(\mathrm{b}) = a$, $f(c) = c$. f is $\hat{\alpha}g$ continuous but not pre continuous, as $\{a\}$ is open $f^{-1}(\{a\}) = \{b\}$ is not pre open.

Example 6.5 : Take X, τ, σ, f as in the above example.

f is $\hat{\alpha}g$ continuous but not semi continuous, as $\{a\}$ is open $f^{-1}(\{a\}) = \{b\}$ is not semi open.

Example 6.6 : Let $X = \{a, b, c\}$ $\tau = \sigma = \{\varphi, \{a\}, \{a, b\}, X\}$

Define $f : (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = a$. f is slightly $\hat{\alpha}g$ continuous but not $\hat{\alpha}g$ continuous, as $\{a\}$ is open, $f^{-1}(\{a\}) = \{b, c\}$ is not $\hat{\alpha}g$ open.

Example 6.7 : Let $X = \{a, b, c\}$ $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$, $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$

Define $f : (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$. f is slightly $\hat{\alpha}g$ continuous but not slightly continuous, as $\{b\}$ is clopen, $f^{-1}(\{b\}) = \{b\}$ is not open.

References

1. Blumberg, N., New properties of real functions,*Trans. Amer. Math. Soc.,* **24**, 113-128 (1922).
2. Jain, R.C., The role of regularly open sets in general topology, *Ph.D. Thesis*, Meerut University, Institute of Advanced Studies, Meerut, India (1980).
3. Levine, N., Semi open sets and semi continuity in topological spaces, *Amer. Math. Monthly,* **70,** 36-41 (1963).
4. Mashour, A.S, Abd-El-Monsef, M.E. and El-Deeb, S.N., On pre continuous and weak pre continuous mappings, *Proc. Math. Phys. Soc. Egypt,* **53**, 47-53 (1982).
5. Njastad, O., On some classes of nearly open sets, *Pacific J. Math.,* **15**, 961-970 (1965).
6. Singal, A.R. and Jain, R.C., Slightly continuous mappings, *J. Indian Math. Soc.,* **64**,195-203 (1997).
7. Staum, R., The algebra of bounded continuous functions into a non Archimedian field, *Pacific J. Math*., **50**,169-185 (1974).
8. Ekici, E. and Caldas, M., Slightly γ continuous functions, *Bol. Soc. Paran. Mat.,* **22**, No. **2**, 63-74(2004).
9. Hatir, E. and Noiri, T., On $\delta - \beta$ continuous functions, *Choas. Solitons and Fractals,* **42**, 205-211 (2009).
10. Chakraborty, U.S., Slightly $\delta - \beta$ continuous functions, *International Journal of Pure and Applied Mathematics,* Vol. **74**, No. **3**, 393-403 (2012).
11. Steen, L.A. and Seebach, Jr. J.A., Counter examples in topology, *Holt, Rinenhart and Winston*, *Newyork* (1970).
12. Senthilkumaran, V., On $\hat{\alpha}$ generalized closed set and its properties in topological spaces, *Ph. D. Thesis*, submitted to Bharathidasan University, Tiruchirappalli, India.

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