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# EVALUATION OF A STUDENT BY USING NORMALIZED HAMMING DISTANCE

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The conventional evaluation method to a student is a process designed to evaluate the qualitative aspects but, in fact, its final result is a grade that values the quantitative aspect. intuitionistic fuzzy sets may be used for evaluation as it contains the membership degree (*i.e.* the marks of the questions answered by the student correctly), the non membership degree (*i.e.* the marks allocated to the questions answered wrongly by the student) and the hesitation degree (which is the mark allocated to the questions the student do not attempt).

**KEYWORDS :** Intuitionistic fuzzy sets, distance measures in intuitionistic fuzzy sets, normalized Hamming distance.

## INTRODUCTION

In real life most of the information that we have to deal with is mostly uncertain. Intuitionistic fuzzy sets introduced by Atanassov [1, 2] has played an important role in the analysis of uncertainty of data. Distance measure between intuitionistic fuzzy sets is an important concept in fuzzy mathematics because of its wide applications in real world. Most of the applications of intuitionistic fuzzy sets are carried out using distance measure approach. Szmidt and Kacpryzk proposed different distance measures between intuitionistic fuzzy sets [5, 6, 7]. We shall make use of normalized Hamming distance for evaluation of a student in an examination.

## PRELIMINARIES

Let $X$ be a nonempty set. A fuzzy set $A$ in $X$ is defined as a set of ordered pairs $A = \{< x, \mu_A(x) > : x \in X\}$, where $\mu_A(x) : X \to [0,1]$ is the grade of belongingness of $x$ into $A$ [8]. Thus the grade of non belongingness of $x$ into $A$ is equal to $1 - \mu_A(x)$. However, while expressing the degree of membership of any given element in a fuzzy set, the degree of non membership is not always expressed as a complement to 1. Therefore Atanassov (1983) suggested a generalization of fuzzy set, called an intuitionistic fuzzy (IF) set [1, 2]. An IF set $A$ in $X$ is given by a set of ordered triples $A = \{< x, \mu_A(x), \nu_A(x) > : x \in X\}$, where $\mu_A(x)$, $\nu_A(x) : X \to [0, 1]$ denote membership and non-membership functions for each element $x \in X$ to $A \subset X$, respectively, such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$.

**2.1. Definition** (Coker 1997) [3] An intuitionistic fuzzy topology on a non empty set $X$ is a family $\tau$ of intuitionistic fuzzy sets in $X$ satisfying the following axioms

($T_1$) $0_{\sim}, 1_{\sim} \in \tau$, $T_2$) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

($T_3$) $\cup G_{\text{i}} \in \tau$ for any arbitrary sub family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space.

**2.2 Different distance measures in intuitionistic fuzzy sets**

Distance measure between intuitionistic fuzzy sets describes the difference between intuitionistic fuzzysets and can be considered as a dual concept of similarity measure. Szmidt and Kacprzyk [5, 6, 7] proposed following four distance measures between *A* and *B*.

The Hamming distance measures in intuitionistic fuzzy sets *A*, *B* is defined by

$$d_H(A, B) = \frac{1}{2} \sum_{i=1}^{n} (\mu_A(x_i) - \mu_B(x_i)| + | \nu_A(x_i) - \nu_B(x_i) | + | \pi_A(x_i) - \pi_B(x_i) | )$$

The Euclidean distance between two intuitionistic fuzzy sets *A*, *B* is defined by

$$d_E(A, B) = \sqrt{\frac{1}{2}} \sum_{i=1}^{n} (\mu_A(x_i) - \mu_B(x_i) | + | \nu_A(x_i) - \nu_B(x_i) | + | \pi_A(x_i) - \pi_B(x_i) | )$$

The normalized Hamming distance between two intuitionistic fuzzysets *A*, *B* is defined by

$$d_{n-H}(A, B) = \frac{1}{2n} \sum_{i=1}^{n} (\mu_A(x_i) - \mu_B(x_i) | + | \nu_A(x_i) - \nu_B(x_i) | + | \pi_A(x_i) - \pi_B(x_i) | )$$

The normalized Euclidean distance between two intuitionistic fuzzysets *A*, *B* is defined by

$$d_{n-E}(A, B) = \sqrt{\frac{1}{2n}} \sum_{i=1}^{n} (\mu_A(x_i) - \mu_B(x_i) | + | \nu_A(x_i) - \nu_B(x_i) | + | \pi_A(x_i) - \pi_B(x_i) | )$$

Ejegwa, Onoja and Emmanuel in [4] showed that the normalized Hamming distance gives the best distance measure between two intuitionistic fuzzysets *A* and *B* because the distance is the shortest or smallest.

# METHODOLGY APPLIED FOR THE EVALUATION OF A STUDENT

Suppose in an examination there are three subjects and four students. Three subjects are Mathematics, English and Science. Four students are $S_1$, $S_2$, $S_3$ and $S_4$. Each subject is of 100 marks containing five groups of equal marks. In each group there will be some basic questions which are expected that a average student can answer, some harder questions and will be doubt about some questions which are simple or not. Hence in each subject group wise question distribution are in intuitionistic fuzzy set.

The marks of the four students $S_1$, $S_2$, $S_3$ and $S_4$ in those three subjects in each group will be also in intuitionistic fuzzysets as it contains the membership degree (*i.e.* the marks of the questions answered by the student correctly), the non-membership degree (*i.e.* the marks allocated to the questions answered wrongly by the student) and the hesitation degree (which is the mark allocated to the questions the student do not attempt).

**Table 1** shows subjects and different groups' distribution

**Table 1**

| | **Group A** | **Group B** | **Group C** | **Group D** | **Group E** |
|---|---|---|---|---|---|
| Mathematics | (.4, .3) | (.3, .5) | (.6, .2) | (.5, .5) | (.6, .4) |
| English | (.3, .5) | (.4, .4) | (.5, .4) | (.6, .4) | (.3, .5) |
| Science | (.4, .4) | (.5, .3) | (.4, .3) | (.3, .5) | (.3, .6) |

where, number of basic questions in a particular group = 20 × Membership function of that group

Number of hard questions in that particular group = 20 × Non-membership function of that group

Number of doubt full questions about their simplicity in that particular group = 20 × (1 – (Membership function of that group + Non-membership function of that group)).

Suppose after evaluation students obtained the following marks in different groups in different subjects as shown in the tables below.

**Table 2 (in Mathematics)**

| Student | Group A | Group B | Group C | Group D | Group E |
|---|---|---|---|---|---|
| $S_1$ | (.4, .4) | (.3, .7) | (.5, .4) | (.5, .2) | (.3, .4) |
| $S_2$ | (.3, .5) | (.7, .3) | (.5, .3) | (.4, .6) | (.4, .4) |
| $S_3$ | (.7, .2) | (.4, .5) | (.4, .4) | (.3, .5) | (.6, .4) |
| $S_4$ | (.5, .4) | (.4, .5) | (.3, .5) | (.5, .3) | (.5,. 1) |

**Table 3 (in English)**

| Student | Group A | Group B | Group C | Group D | Group E |
|---|---|---|---|---|---|
| $S_1$ | (.5, .4) | (.4, .5) | (.3, .5) | (.5, .3) | (.4, .3) |
| $S_2$ | (.7, .2) | (.4, .5) | (.4, .4) | (.3, .5) | (.6, .4) |
| $S_3$ | (.3, .5) | (.7, .3) | (.5, .3) | (.4, .6) | (.4, .4) |
| $S_4$ | (.4, .4) | (.3, .7) | (.5, .4) | (.5, .2) | (.3, .4) |

**Table 4 (in Science)**

| Student | Group A | Group B | Group C | Group D | Group E |
|---|---|---|---|---|---|
| $S_1$ | (.3, .5) | (.7, .3) | (.5, .3) | (.4, .6) | (.4, .4) |
| $S_2$ | (.5, .4) | (.4, .5) | (.3, .5) | (.5, .3) | (.4, .3) |
| $S_3$ | (.4, .4) | (.3, .7) | (.5, .4) | (.5, .2) | (.3, .4) |
| $S_4$ | (.7, .2) | (.4, .5) | (.4, .4) | (.3, .5) | (.6, .4) |

where marks that a student got in a particular group answering correctly ($M_1$) = 20 × Membership function of that group

Marks of the questions that a student answer wrongly in a particular group ($M_2$) = 20 × Non-membership function of that group

Marks of the questions which the student did not answer for his or her doubtfulness in a particular group = 20 – ($M_1 + M_2$).

Using normalized Hamming distance between each student and each subject we get the following table

**Table 5**

| Student | Mathematics | English | Science |
|---|---|---|---|
| $S_1$ | .12 | .12 | .1 |
| $S_2$ | .17 | .17 | .14 |
| $S_3$ | .11 | .11 | .15 |
| $S_4$ | .15 | .1 | .14 |

where $d_{n-H}(S_i, S_j^*)$ is the normalized Hamming distance between $i^{th}$ student and $j^{th}$ subject, $i = 1, 2, 3, 4$ and $j = 1, 2, 3$ and $S_1^*$ = Mathematics, $S_2^*$ = English, $S_3^*$ = Science.

The shortest distance gives the best evaluation. Thus the student $S_1$ is best in Science and the student $S_3$ is best in Mathematics and $S_4$ is best in English. Now taking simple mean for each student we get

Mean distance for the student $S_1$ = .1133 (approx)

Mean distance for the student $S_2$ = .16 (approx)

Mean distance for the student $S_3$ = .1233(approx)

Mean distance for the student $S_4$ = .13 (approx)

From these mean distances we can say that the student $S_1$ is the best student.

## Conclusion

Evaluation of a student by the application of intuitionistic fuzzy sets is of great significance. Through this method weakness of a particular student can be find out easily as well as result can be analyzed as a whole also.

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