HEAT AND MASS TRANSFER IN THE ROTATIONAL FLOW OF A VISCO-ELASTIC FLUID DUE TO ROTATION OF AN INFINITE POROUS DISK IN THE PRESENCE OF HEAT SOURCES AND CHEMICAL REACTION

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This paper deals with the study of heat and mass transfer in the rotational flow of a visco-elastic fluid due to rotation of an infinite porous disk in the presence of heat sources, and chemical reaction. Embracing all the physical situations of this problem, the constitutive equations of momentum, energy and concentration have been developed under prescribed boundary conditions. Walters' B' liquid model has been used in deriving the equation of motion and Von-Karman's similarity transformations have been followed for reduction of this equation. Velocity, temperature and concentration profiles have been drawn. The values of Skin-friction, rates of heat transfer and the concentration gradients have been entered in tables. It is observed that drag on the disk is larger in case of non-Newtonian fluid than Newtonian fluid. Rise in the Permeability parameter raises the rate of rotational flow of the visco-elastic fluid. The temperature of the fluid is influenced by the Reynolds number, Prandtl number and the species concentration is highly effected by Schmidt number and the chemical reaction parameter.

KEYWORDS : Heat and mass transfer, rotational flow, Walters' B' liquid, chemical reaction.

Introduction

Several researchers have investigated the flow about a rotating disk ever since it was incepted by Von Karman [1] and later studies more thoroughly by Cochran [2]. The results of such investigations have immerse practical utility especially when the fluid is non-Newtonian. Quite a number of fluids of industrial importance have their constitutive equations as Proposed by Walters' B' model. Consequently, a number of researchers have devoted themselves to the study of heat transfer problems relating to the flow of non-Newtonian fluids in several geometric shapes and for different physical situations. Heat transfer in the flow of second order fluids over an enclosed rotating disk has been studied by Sharma and Bhatia [3]. Bhatnagar and Bhatnagar [4] have analysed the problem of heat transfer due to a sphere steadily rotating in an infinitely extending non-Newtonian fluid. Again, in 1970, Bhatnagar [5] alone has discussed a problem on heat transfer in a visco-elastic fluid flowing around a steadily rotating and thermally insultaed sphere. Sharma and Bhatnagar [6] have studied the problem of low Reynolds number heat transfer from fluids has also been discussed by Rao and Kuloor [7]. Kalpur and Tyagi [8] have analysed the problem of heat transfer in some flows of a certain class of non-Newtonian fluids. Krishnan and Pandya [9] have studied the problem of heat transfer in a non-Newtonian flow in jacketed agitated vessels. Combined laminar free and forced convection heat transfer in non-Newtonian fluids has been analysed by Kubair and Pei [10]. Pandian and Raja Rao [11] have discussed the problem of heat transfer in non-Newtonian liquids in agitated vessels. Rajasekharan, Kubair and Kuloor [12] have studied the problem of heat transfer in non-Newtonian laminar flows through coiled pipes. Mahalingam, Telton and coulson [13] have discussed the problem of heat transfer in Laminar flow of non-Newtonian fluids. Rath and Bastia [14] have investigated the problem of steady flow of Visco-elastic liquid (Walters' B') about a porous rotating disk. They have not analysed the heat transfer aspect of the flow. Dash and Biswal [15] have analysed the effects of heat transfer in the free convection flow of an elastic-viscous fluid past an exponentially accelerated vertical plate. They have also studied heat and mass transfer effects of free convection flow of a visco-elastic fluid inside a porous vertical channel with heat sources [16]. Free convective flow and heat transfer in a visco-elastic fluid inside a porous vertical channel with constant suction and heat sources has been analysed by the same authors [17]. Biswal and Mahalik [18] have investigated the heat transfer in the free convection flow of a visco-elastic fluid inside porous vertical channel with constant suction and heat sources. Unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started porous flat plate with heat sources/sinks have been analysed by them [19]. Also Biswal [20] alone has studied the problem of unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started porous wall. Mishra, Ray and Biswal [21] have analysed the problem of heat transfer in the rotational flow of a visco-elastic liquid due to rotation of an infinite porous disk in the presence of heat sources.

In this paper, our aim is to investigate the problem of heat and mass transfer in the rotational flow of a visco-elastic liquid due to rotation of an infinite porous disk in the presence of heat sources and chemical reaction.

Formulation

The flow under consideration takes place when a porous disk of infinite radius has been rotating about a fixed axis through its centre, for sufficiently large time, with a constant angular velocity ω. The constitutive equation for a fluid of Walters '*B*' model is

$$P^{ik} = -p\,\delta^{ik} + 2\mu g^{ik} - 2K_o g^{ik}, \qquad \ldots(1)$$

$$\overset{\nabla}{g}^{ij} = g^{ij}{}_{,K}V^K - g^{ik}V^j{}_{,k} - g^{kj}V^i{}_{,K} + g^{ij}V^K{}_{,K}, \qquad \ldots(2)$$

where P^{ik} is the stress tensor, g^{ij} the rate of strain tensor given by

$$2g^{ik} = V_{i,K} + V_{K,i} \qquad \ldots(3)$$

and V is a velocity vector. A comma denotes covariant differentiation with respect to the symbol following it. We have also supposed that a non-Newtonian fluid of Walters '*B*' model occupies the space $0 < Z < \infty$ where the disk is in the plane $z = 0$. Assuming that the constant velocity of suction (or injection as the case may be) at the disk is $W = W_0$, we have to solve the Navier–Stokes equations of motion of the fluid in cylindrical polar co-ordinate system with U,

V, W as the radial, azimuthal and axial velocity component respectively. The equation of continuity, energy and concentration are also solved. Taking $\frac{\partial}{\partial\theta} = 0$ for axial symmetry, the boundary conditions applicable are

$$\left.\begin{aligned} &U = 0,\ V = r\omega,\ W = W_0,\ at\ \ z = 0 \\ \text{and} \qquad &U = 0,\ V = 0,\ W = 0,\ as\ \ z \longrightarrow \infty \end{aligned}\right\} \qquad \ldots(4)$$

Equation of Motion

The equations of motion are

$$\frac{U^2}{r^2\omega^2}\sqrt{\frac{V}{W}} - \frac{U^2}{r^2\omega^2}\sqrt{\frac{V}{W}} + \frac{W}{\sqrt{\nu\omega}} - \frac{1}{r\omega}.\frac{\partial U}{\partial z}\sqrt{\frac{V}{W}} = \frac{1}{r\omega}\frac{\partial^2 U}{\partial z^2}\sqrt{\frac{V}{W}} - R_C$$

$$\left[\left(\frac{W}{\sqrt{\nu\omega}}\right)\sqrt{\frac{V}{W}}\frac{1}{r\omega}.\frac{\partial^3 U}{\partial z^3} - \frac{2}{\sqrt{VW}}\frac{\partial W}{\partial Z}\sqrt{\frac{V}{W}}\frac{1}{r\omega}\frac{\partial U}{\partial z}\sqrt{\frac{V}{W}}\right] \qquad \ldots(5)$$

$$\frac{2}{r\omega}\sqrt{\frac{V}{W}}\frac{\partial U}{\partial z}\frac{V}{r\omega}\sqrt{\frac{V}{W}} + \frac{1}{r\omega}\sqrt{\frac{V}{W}}\frac{\partial U}{\partial z}.\frac{W}{\sqrt{\nu\omega}}\sqrt{\frac{V}{W}} = \frac{1}{r\omega}$$

$$\sqrt{\frac{V}{W}}\frac{\partial^2 U}{\partial z^2} - R_C\left[\left(\frac{W}{\sqrt{r\omega}}\right)\sqrt{\frac{V}{W}}.\frac{1}{r\omega}\sqrt{\frac{V}{W}}\frac{\partial^3 U}{\partial z^3} - 6\frac{1}{r\omega}\sqrt{\frac{V}{W}}\frac{\partial U}{\partial z}.\frac{1}{\partial\omega}\sqrt{\frac{V}{W}}\frac{\partial U}{\partial z}\right]$$

$$+\ 2R_C\sqrt{\frac{V}{W}}U\ \frac{1}{r\omega}\sqrt{\frac{V}{W}} \qquad \ldots(6)$$

$$\frac{1}{\mu\omega}\sqrt{\frac{V}{W}}\frac{\partial P}{\partial Z} = \frac{1}{\sqrt{\nu\omega}}\sqrt{\frac{V}{W}}\frac{\partial^2 W}{\partial Z^2} - \frac{W}{\sqrt{\nu\omega}} \times \frac{1}{\sqrt{\nu\omega}}\sqrt{\frac{V}{W}}\frac{\partial W}{\partial Z}$$

$$+\ 2R_C\left[\left(\frac{W}{\sqrt{\nu\omega}}\right)\sqrt{\frac{V}{W}} \times \frac{1}{r\omega}\sqrt{\frac{V}{W}}\frac{\partial^2 U}{\partial z^2} + 6\,\frac{U}{r\omega}\sqrt{\frac{V}{W}} \times \frac{1}{r\omega}\sqrt{\frac{V}{W}}\frac{\partial U}{\partial z}\right] \qquad \ldots(7)$$

Equation of Continuity :

$$\frac{W}{\sqrt{\nu\omega}}\sqrt{\frac{V}{W}}\frac{\partial W}{\partial Z} = -2\frac{U}{r\omega}\sqrt{\frac{V}{W}} \qquad \ldots(8)$$

After simplification, the equations, (5), (6), (7) and (8) become,

$$\frac{\partial^2 U}{\partial z^2} - \frac{W}{\sqrt{\nu\omega}}\frac{\partial U}{\partial z} - R_C\left[\frac{W}{\sqrt{\nu\omega}}\frac{\partial^3 U}{\partial z^3} - \frac{2}{\sqrt{\nu\omega}}\frac{\partial W}{\partial Z}\sqrt{\frac{V}{W}}\frac{\partial U}{\partial z}\right] = 0 \qquad \ldots(9)$$

$$\frac{\partial^2 U}{\partial z^2} - \left[\frac{2}{r\omega} + \frac{W}{\sqrt{r\omega}}\right]\sqrt{\frac{V}{W}}\frac{\partial U}{\partial z} - R_C\sqrt{\frac{V}{W}}\left[\frac{W}{\sqrt{r\omega}}\frac{\partial^3 U}{\partial z^3} - \frac{6}{r\omega}\left(\frac{\partial U}{\partial z}\right)^2\right] + 2\ R_C\left(\sqrt{\frac{V}{\omega}}\right)U = 0 \quad \ldots(10)$$

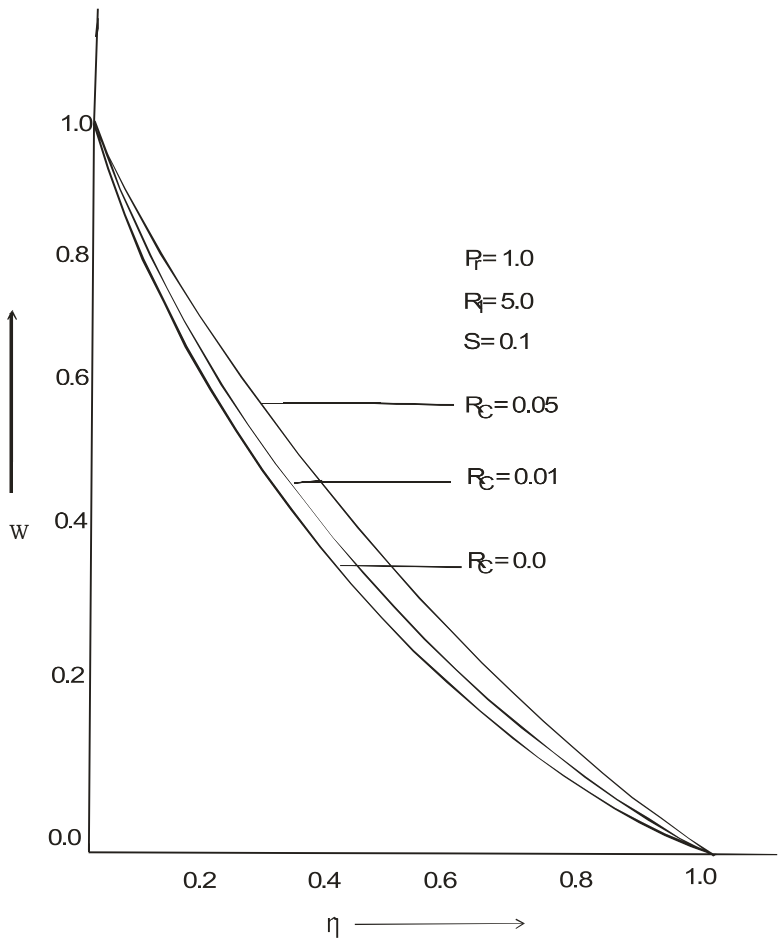

Fig. 1. Effect of R_C on velocity field.

$$\frac{1}{\mu\omega}\frac{\partial P}{\partial Z} = \frac{1}{\sqrt{\nu\omega}}\frac{\partial^2 W}{\partial Z^2} - \frac{W}{\nu\omega}.\frac{\partial W}{\partial Z} + 2\,R_C\left[\frac{W}{\sqrt{r\omega}}.\frac{1}{r\omega}\sqrt{\frac{V}{W}}\frac{\partial^2 U}{\partial z^2} + \frac{6}{(r\omega)^2}\sqrt{\frac{V}{W}}\,U\frac{\partial U}{\partial z}\right] \quad \ldots(11)$$

and
$$\frac{W}{\sqrt{\nu\omega}}\frac{\partial W}{\partial Z} = -\frac{2}{r\omega}\,U\,, \quad \ldots(12)$$

Using Von-Karman Similarity transformations,

$$\left.\begin{aligned} &\eta = z\left(\frac{W}{\nu}\right)^{\frac{1}{2}}, \quad U = r\omega F(\eta) \\ &V = r\,\omega\,G(\eta), \quad W = (\nu\omega)^{\frac{1}{2}}\;H(\eta) \\ &P = \mu\,\omega\,p(\eta) \end{aligned}\right\} \quad \ldots(13)$$

The above equations (9) – (12) are reduced to

$$\left.\begin{aligned} &F^2 - G^2 + HF' = F'' - R_C(HF''' - 2H'F') \\ &2FG + HG' = G'' - R_C(HG''' - 6F'G') + 2R_C R\,FG \\ &P' = H'' - HH' + 2R_C(HF'' + 6FF') \\ &H' = -2F \end{aligned}\right\} \quad \ldots(14)$$

where a Prime denotes differentiation with respect to η.

The corresponding boundary conditions become,

$$\left.\begin{aligned} &F(0) = 0,\ G(0) = 1,\ H(0) = K^*,\ P(0) = 0 \\ &F(\infty) = 0,\ G(\infty) = 0,\ H(\infty) = 0,\ P(\infty) = 0 \end{aligned}\right\} \quad \ldots(15)$$

The non-dimensional parameters are

$R_C = \dfrac{K_0 W^2}{\mu\nu}$, non- Newtonian Parameter,

$R = \dfrac{\nu}{r\omega}$, rotation parameter,

$K^* = \dfrac{W_0}{(\nu\omega)^{\frac{1}{2}}}$, Permeability Parameter

where μ is the co-efficient of viscosity and ν is the kinematic

Viscosity $\left(\nu = \dfrac{\mu}{\rho}\right)$, ω is the angular velocity.

Equation of Energy

The equation of energy in the present problem is,

$$(-W_0)\frac{\partial T}{\partial Z} = \frac{K}{\rho C_\rho}\frac{\partial^2 T}{\partial Z^2} + S^*(T - T_0) \quad \ldots(16)$$

where K is the co-efficient of thermal conductivity, W_0 is the injection velocity and $(-W_0)$ is the suction velocity, ρ is the density of the fluid, C_ρ is the specific heat at constant pressure (for a gas), S^* is the dimensional source/sink parameter.

The boundary conditions are

$$\left.\begin{aligned} &T = T_0 \quad \text{when } z = 0 \\ &T \to T_\infty, \quad \text{when } z \to \infty \end{aligned}\right\} \quad \ldots(17)$$

Introducing the non-dimensional parameters

$$\left.\begin{aligned} &\theta = \frac{T - T_\infty}{T_0 - T_\infty}, \quad P_r = \frac{\nu\rho C_\rho}{\in K}, \\ &R_1 = \frac{W_0}{\nu}\sqrt{\frac{V}{\omega}}, \quad S = \frac{4\,S^*\nu}{W_0^2}, \end{aligned}\right\} \quad \ldots(18)$$

where θ is the non-dimensional temperature, P_r is the PrandtL temperature, R_1 is the Reynolds number, S is the non-dimensional source/sink parameter, the equation (16) becomes

$$\theta'' + P_r R_1 \theta' = \frac{-P_r R_1 S \theta}{4} \qquad \ldots(19)$$

where the primes denote differentiation with respect to Z.

or, $$\theta'' + P_r R_1 \theta' + \frac{P_r R_1 S \theta}{4} = 0 \qquad \ldots(20)$$

The modified boundary conditions are

$$\left.\begin{array}{ll} \theta = 1 & \text{at } z = 0 \\ \theta \to 0 & \text{as } z \to \infty \end{array}\right\} \qquad \ldots(21)$$

Equation of Concentration

The equation of concentration with chemical reaction term is given by

$$-W_0 \frac{\partial C'}{\partial Z} = D \frac{\partial^2 C'}{\partial Z^2} + \lambda^*, \qquad \ldots(22)$$

where D is the chemical molecular diffusivity, λ^* is the chemical reaction term.

$$\lambda^* = -K'(C' - C'_\infty)^n \qquad \ldots(23)$$

where K' is the reaction rate constant and n is the order of the reaction as hold by Aris (22).

The boundary conditions are

$$\left.\begin{array}{ll} C' = C'_0 & \text{when } z = 0 \\ C' \longrightarrow C'_\infty & \text{when } z \longrightarrow \infty \end{array}\right\} \qquad \ldots(24)$$

Introducing the non-dimensional parameters.

$$C = \frac{C' - C'_\infty}{C'_0 - C'_\infty},\ S_C = \frac{\nu}{D} \text{ and } K_1 = \frac{\nu K'}{V^2} \qquad \ldots(25)$$

where C is the non-dimensional concentration

S_C is the Schmidt number and K_1 is the chemical reaction parameter.

Equation (22) becomes

$$\frac{\partial^2 C}{\partial Z^2} + R_1 S_C \frac{\partial C}{\partial Z} - K_1 S_C C = 0, \qquad \ldots(26)$$

With the modified boundary condition

$$\left.\begin{array}{ll} C = 1 & \text{at } z = 0 \\ C = C_\infty & \text{as } z \longrightarrow \infty \end{array}\right\} \qquad \ldots(27)$$

Solutions of the equations of motion.

As the non-Newtonian character of a fluid is a deviation of small extent from the Newtonian behavior of fluids in general, we can write the velocity and pressure functions as

$$\left.\begin{aligned} F &= \sum_{n=0}^{\infty} R_c^n F_n(\eta), \quad G = \sum_{n=0}^{\infty} R_c^n G_n(\eta) \\ H &= \sum_{n=0}^{\infty} R_c^n H_n(\eta), \quad P = \sum_{n=0}^{\infty} R_c^n P_n(\eta) \end{aligned}\right\} \qquad \ldots(28)$$

where $n = 0, 1, 2, 3, \ldots\ldots\ldots\infty$

For small values of R_C, say $R_c << 1$, we can neglect the terms involving higher powers of R_c (*i.e.* $n \geq 2$) in the above expressions as there terms will provide little contribution to the flow characteristics. Then equation (28) takes the form

$$\left.\begin{aligned} F &= F_0 + R_c F_1 \\ G &= G_0 + R_c G_1 \\ H &= H_0 + R_c H_1 \\ \text{and } P &= P_0 + R_c P_1 \end{aligned}\right\} \qquad \ldots(29)$$

Introducing (29) in (14) and (15) and equating the co-efficients of like powers of R_c, we obtain two sets of equations with the modified boundary conditions.

First set :

$$\left.\begin{aligned} F_0'' &= F_0^2 - G_0^2 + H_0 F_0' \\ G_0'' &= 2F_0 G_0 + H_0 G_0' \\ H_0' &= -2F_0 \\ P_0' &= H_0'' - H_0 H_0' \end{aligned}\right\} \qquad \ldots(30)$$

With the boundary conditions

$$\left.\begin{aligned} F_0(0) = 0,\ G_0(0) = 1,\ H_0(0) = K^*,\ P_0(0) = 0 \\ F_0(\infty) = 0,\ G_0(\infty) = 0,\ H_0(\infty) = 0,\ P_0(\infty) = 0 \end{aligned}\right\} \qquad \ldots(31)$$

Second set :

$$\left.\begin{aligned} F_1'' &= 2F_0F_1 - 2G_0G_1 + H_1F_0' + H_0F_1' \\ &\quad + H_0F_0''' - 2H_0'F_0'' \\ G_1'' &= 2(F_0F_1 - F_1G_0) + H_0G_1' + H_1G_0' \\ &\quad + H_0G_0''' - 6F_0'G_0' - 2\ R F_0 G_0 \\ H_1' &= -2F_1 \\ P_1' &= -H_0H_0''' + 3H_0'H_1' + H_1'' - H_0H_1' - H_1H_0' \end{aligned}\right\} \qquad \ldots(32)$$

With the boundary conditions,

$$\left.\begin{aligned} F_1(0) = 0,\ G_1(0) = 0,\ H_1(0) = 0,\ P_1(0) = 0, \\ F_1(\infty) = 0,\ G_1(\infty) = 0,\ H_1(\infty) = 0,\ P_1(\infty) = 0 \end{aligned}\right\} \qquad \ldots(33)$$

We can immediately see that first is a set of simultaneous non-linear differential equations while the second set (and as a matter of fact any subsequent set if obtained) is a linear one. One of the most suitable method for the solution of such sets of equations is the Runge-Kutta technique of numerical integration as indicated in Ralston and Wilf [23]. We have integrated the equations accordingly carrying out the computations with the step length as 0.1. We have found the axial velocity component with the variations of the fluid parameters.

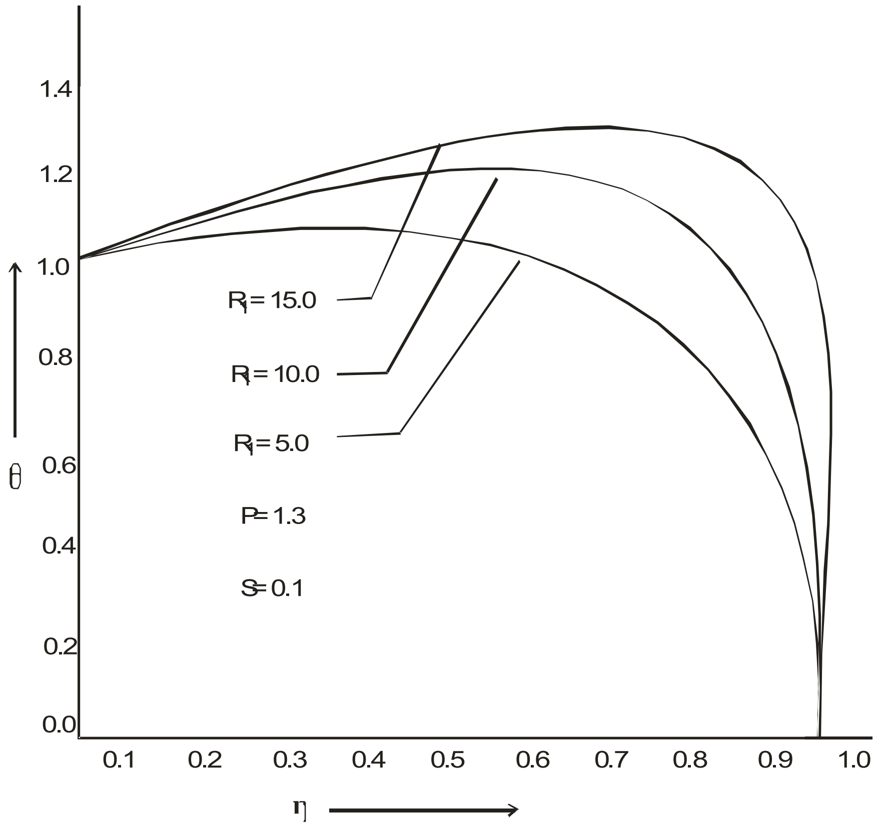

Fig. 2. Effect of R_1 on Temperature Field.

Skin – Friction

The skin-friciton τ near the disk is given by

$$\tau = \left.\frac{\partial w}{\partial \eta}\right|_{\eta=0} + R_c \left.\frac{\partial^2 W}{\partial \eta^2}\right|_{\eta=0} \qquad \ldots(34)$$

Solution of the energy equation

Temperature ' θ ' of the fluid is obtained by solving the energy equation (20).

$$\theta = A_2 e^{-a_1 R_1^Z} - A_3\, e^{-a_2 R_2^z}\,, \qquad \ldots(35)$$

where the contsants involved are

$$a_1 = \frac{1}{2}\left[P_r - \sqrt{P_r^2 - P_r S}\right]$$

$$a_2 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 - P_r S}\right]$$

$$a_1 - a_2 = -\sqrt{P_r^2 - P_r S}$$

$$A_1 = e^{-(a_1 - a_2)R_1}$$

$$G_1 = 1 - e^{-(a_1 - a_2)R_1} = 1 - A_1$$

$$A_2 = \frac{1}{G_1} \text{ and } A_3 = \frac{A_1}{G_1}$$

Heat transfer

The rate of heat transfer near the disk is given by

$$Nu = -\left.\frac{\partial \theta}{\partial z}\right|_{z=0} = R_1(a_1 A_2 - a_2 A_3) \qquad \ldots(36)$$

Pressure gradient

The pressure gradient is given by

$$\frac{\partial P}{\partial \eta} = \left[2\frac{\partial W}{\partial \eta}.\frac{\partial^2 W}{\partial \eta^2} + \frac{1}{\eta}\left(\frac{\partial W}{\partial \eta}\right)^2\right], \qquad \ldots(37)$$

Solution of the Concentration equation:

Solving the second order homogeneous differential equation (26) with the aid of the boundary condition (27), we obtain

$$C = e^{-\alpha_1 z}, \qquad \ldots(38)$$

where $$\alpha_1 = \frac{1}{2}\left[R_1 S_c + \sqrt{S_c^2 R_1^2 + 4K_1 S_c}\right]$$

Concentration gradient :

The concentration gradient is given by

$$CG = -\left.\frac{\partial C}{\partial z}\right|_{z=0} \text{ at the plate}$$

$$= \left[\frac{R_1 S_c}{1 - e^{-R_1 S_c}}\right] e^{R_1 S_c} \qquad \ldots(39)$$

Results and discussion

The characteristics of the non-Newtonian flow, temperature, heat transfer, concentration of the species and the concentration gradient are revealed from the profiles drawn and the tables framed with the numerical values of the Skin-friciton, the Nusselt number and the concentration gradient. The effects of R_c (non-Newtonian Parameter), R_1 (Reynolds number), R (rotation Parameter), P_r (Prandtl number) , S (non-dimensional source strength), S_c (Schmidt number) and K^* (non-dimensional permeability parameter) are studied with the aid of graphs and tables.

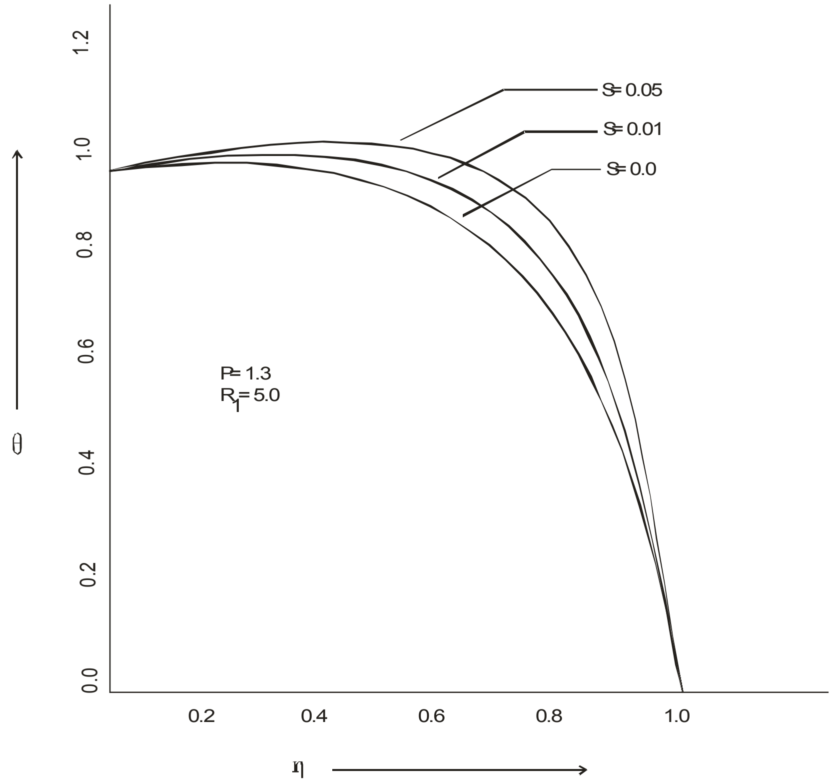

Fig. 3. Effect of *S* on Temperature field.

Velocity distribution :

The effects of the non-Newtonian parameter on the axial velocity component W are revealed from the profiles of Fig. 1 It is observed that the axial velocity rises at every point of the flow field with the increase of R_c. However, the nature of the velocity curves remains the same for both Newtonian ($R_c = 0$) and non-Newtonian ($R_c > 0$) fluids. The velocity component W gradually falls with the distance ' η ' from the porous disk.

The effects of rotation parameter (R) and the permeability parameter (K^*) on the azimuthal and axial velocity components V and W respectively are revealed from the numerical values of $-G(0)$ and $H(0)$ entered on the table 1.

It is observed from this table that the increase in K^*, increases the values of $-G(0)$, *i.e.*, the velocity component V rises, keeping the

Table 1. Effects of R and K* on the azimuthal velocity and axial velocity field.

K*	R	–G(0)	H(0)
–0.02	0.1	0.624	0.656
	0.2		0.646

	0.3		0.634
0.0	0.01	0.615	0.665
	0.05		0.659
	0.10		0.653
0.03	0.10	0.607	0.650
	0.20		0.638
	0.30		0.626

Value of – G (0) and K^* constant, the values of H (0) decreases with the increase of the rotation parameter R, *i.e.*, the axial velocity W falls with the rise of R.

Temperature distribution

Figure 2 exhibits the effects of the Reynolds number on the temperature of the fluid.

The striking feature of the profiles is that the fall of temperature is delayed with the rise in the value of R_1. It is also marked that Reynolds number increase the temperature at any point of the fluid.

Fig. 3 exhibits the uniform fall of temperature is marked with the decreasing value of the source parameter S.

CONCENTRATION DISTRIBUTION

Fig. 4 shows the profiles of concentration of the species with the variations of the Schmidt number S_c and the chemical reaction parameter K_1. Generally concentration falls with the distance from the porous disk and finally becomes zero at large distances from the disk. It is observed that the concentration decreases at all points from the disk with the increase in the value of S_c from 2.13 to 5.0 without chemical reaction. But the chemical reaction parameter reduces the concentration further.

Skin-friction

The value of the skin friction τ for different values of R_c are entered in table 2. It is noticed that the skin-friction decrease with the increase in the value of R_c. But, in the absence of heat source, opposite effect has been marked by Padhy [24].

Table 2. Effect of R_c on the Skin-friction τ when S = 0.1, P_r = 1.0, R = 5.0, K^* = 1.0, R_1 = 5.0

R_c	τ
0.0	–2.2596964
0.05	–2.4290450
0.10	–2.5779325

Rate of heat transfer

Numerical values of the Nusselt number are entered in the table-3. The rate of heat transfer is given by the Nussett number (Nu). It is observed that the rate of heat transfer decreases with the source strength. The Reynolds number R_1 increases the rate of heat transfer for $S = 0$ and decreases it for $S > 0$.

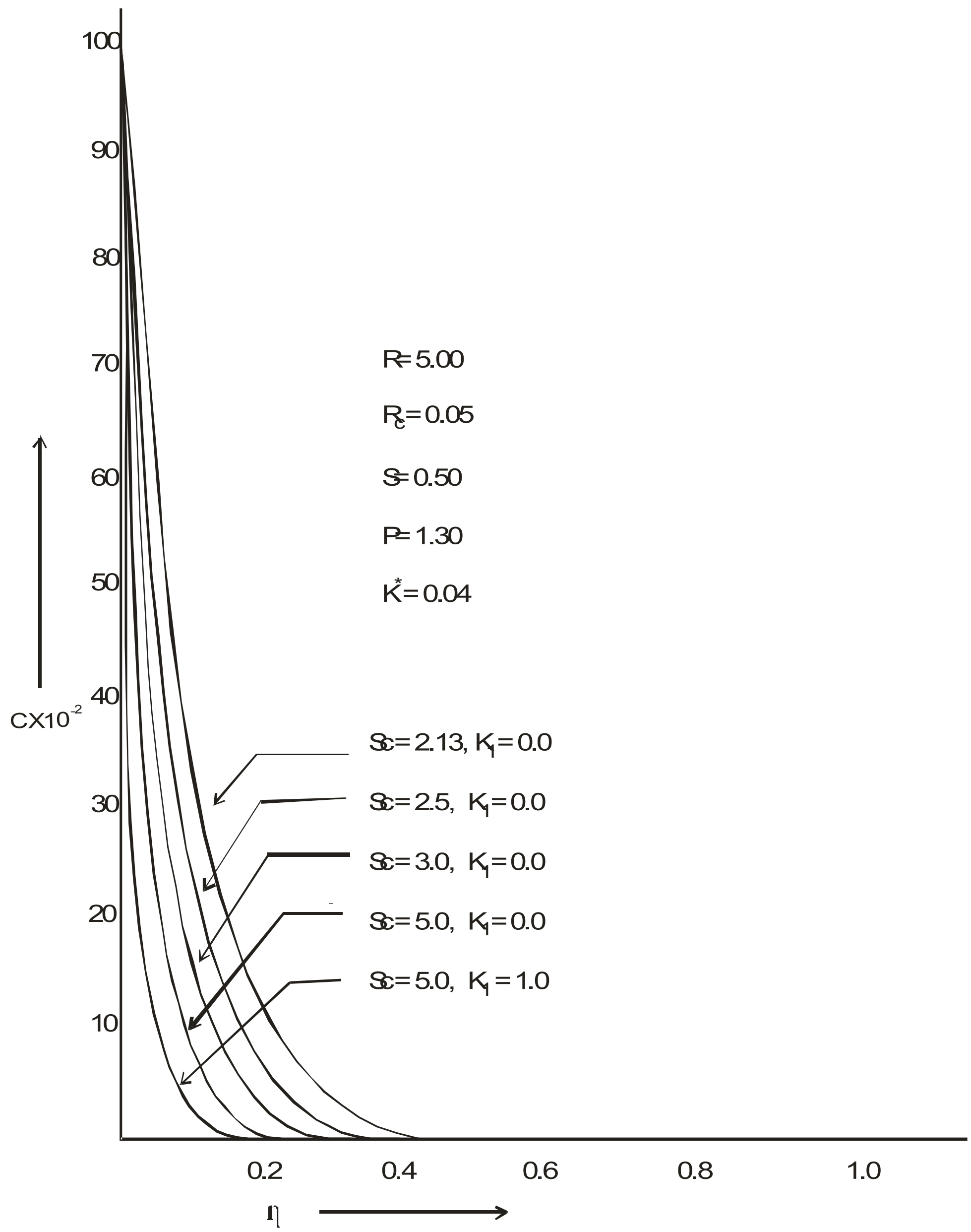

Fig. 4. Effect of K_1 and S_c on concentration.

Table 3. Values of the rate of heat transfer for $P_r = 1.3$, $R_c = 0.05$

S	R_1	*Nu*
0.0	5.0	–0.00878
	10.0	–0.00032
	15.0	–0.00021
0.1	5.0	–0.13875
	10.0	–0.29542

	15.0	–0.37825
0.5	5.0	–0.58456
	10.0	–1.16348
	15.0	–1.85277
1.0	5.0	–1.23461
	10.0	–2.24346
	15.0	–3.19876

The effect of the Prandtl number P_r on the rate of heat transfer is predicted from the numerical values of the Nusselt number entered in the table 4.

Table 4. Effect of P_r on the Nusselt number for $R_c = 0.05$, $R_1 = 5.0$

P_r	Nu
1.3	6.3846
9.0	44.8746
12.0	59.8747

From the above table 4, it is marked that the rate of heat transfer rises with the increase in the value of the Prandtl number P_r.

Concentration gradient

The effect of the Schmidt number S_c on the concentration gradient CG is predicted from the numerical values of CG entered in the table 5.

Table 5. Effect of S_c on the concentration gradient for $R_c = 0.05$, $R_1 = 5.0$, $K_1 = 1.0$

S_c	*CG*
2.13	10.65
2.5	12.5
3.0	15.0
5.0	25.0

Numerical values of the concentration gradient CG entered in the table 5 show that the concentration gradient increases with the increases of the Schmidt number S_c.

Conclusions

Following Predictions are resulted from the above theoretical investigation of non-Nowtonian flow of Walters '*B*' fluid about a rotating disk.

(i) Drag on the disk is larger in the Visco-elastic fluid than that in the Viscous fluid.

(ii) Drag is prominent in the Visco-elastic case due to any small variation in the rotation of the disk.

(iii) The Visco-elastic fluid rotates slower than Newtonian fluid as observed by Rath and Bastia (25) without heat sources. But the velocity of the fluid enhances with the rise of R_c.

(iv) The Visco-elastic effect decreases with an increase in either the speed of rotation or of suction at the disk.

(v) Increase in the permeability parameter K^* increases the rate of rotational flow of the non-Newtonian fluid.

(vi) The Reynolds number R_1raises the temperature at any point of the fluid.

(vii) The Skin-friction decrease, with the increase of R_c.

(viii) The rate of heat transfer decreases with the source strength.

(ix) Reynolds number increases the rate of heat transfer in the absence of the source. But, reverse effect in marked when the source is present.

(x) The visco-elastic parameter helps in lowering the pressure everywhere in the fluid region excepts very near the disk where a slight rise in pressure is marked.

(xi) The rate of heat transfer rises with the rise of the thermal Prandtl number P_r.

(xii) The species concentration falls at all distances from the Porous disk with increase in the value of the Schmidt number S_c.

(xiii) The chemical reaction parameter reduces the concentration further.

(xiv) The concentration gradient increases with the increase of S_c.

References

1. Karman, Th. Von, Uber Laminar and turbulente, *Reibung angew Mth. Mech.*, **I**, 233 (1921).
2. Cochran, W.G., *Proc. Camb. Phil. Soc*., **30**, 365 (1931).
3. Sharma, S.K. and Bhatia, S.P.S., *J. Sci. Engg. Res*., **10**, 135 (1966).
4. Bhatnagar, R.K. and Bhatnagar, K.S., *PNISI*, **35**, 295 (1969).
5. Bhatnagar, R.K., *Rheol. Acta,* **9**, 419 (1970).
6. Sharma, O.P. and Bhatnagar, R.K., *ZAMM,* 55 (1975).
7. Rao, M.S. and Kuloor, N.R., *Ind. J. Technol*., **2**, 113 (1964).
8. Kapur, J.N. and Tyagi, V.P., *Math, Seminar*, **4**, 58 (1966).
9. Krishnan, R.M. and Pandya, S.B., *Ind. Chem. Eng*., **2**, 42 (1966).
10. Kubair, V.G. and Pei, D.C.T., *Int. J. Heat Mass Transfer*, **11**, 855 (1968).
11. Pandian, J.R. and Raja Rao, M., *Ind. Chem. Engrs.*, **12**, 29 (1970).
12. Rajasekharan, S., Kubair, V.G. and Kuloor, N.R., *Int. J. Heat Mass Transfer*, **13**, 1583 (1970).
13. Mahalingam, R., L.O. Tilton and Coulson, J.M., *Chem. Engg. Sci*., **30**, 921 (1975).
14. Rath,m R.S. and Bastia, S.N., *Proc. Ind. Acad. Sci*., **87A**, No. **9**, 227 (1978).
15. Dash, G.C. and Biswal, S., *AMSE J*., **21**, No. **4**, 13 (1989).
16. Dash, G.C. and Biswal, S., *AMSE J*., **21**, No. **4**, 25 (1989).
17. Dash, G.C. and Biswal, S., *AMSE J*., **46**, 3, 01 (1992).
18. Biswal, S. and Mahalik, S., *Acta Ciencia Indica*, **33** P, No. **2**, 155 (2007).
19. Biswal, S. and Mahalik, S., *Acta Ciencia Indica,* **33**P, No. **2**, 259 (2007).
20. Biswal, S., *Acta Ciencia Indica*, **35** P, No. **2**, 211 (2009).
21. Mishra, A., Ray, G.S. and Biswal, S., *Ultra Scientist*, **23(3) B**, 515 (2011).
22. Aris, R., *Introduction to the analysis of Chemical reactors*, Prentice Hall, E.Cliffs, N.J. (1965).
23. Ralston, A. and Wilt, W., *Mathematical Methods for Digital Computers*, John Willey and Sons, 117 (1960).
24. Padhy, S., *Ph.D. Thesis*, Utkal University, Bhubaneswar, India, P. 188 (1978).
25. Rath, R.S. and Bastia, S.N., *Ind. J. Pure and Appl. Math,* **7**, No. **8**, 880 (1976).

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