HEAT AND MASS TRANSFER OF A Cu-WATER NANOFLUID WITH HEAT SOURCE AND OHMIC HEATING OVER A STRETCHING SHEET

HITESH KUMAR

Department of Mathematics and Statistics, Jai Narain Vyas University, Jodhpur

RECEIVED : 22 February, 2018

A two dimensional forced convection boundary layer flow and heat transfer characteristics of nanofluid Cu-water over a linearly stretching sheet in the presence of heat source and ohmic heating is presented in this paper. The governing equations are solved using MATLAB, results for the velocity, temperature, skin friction and Nusselt number are obtained. The effects of various parameters on the flow characteristics are displayed graphically and physical interpretation is presented.

KEYWORDS : MHD, Nanofluid, Heat source, Ohmic heating, Stretching Sheet.

AMS Subject Classification : 80A20, 76R05.

INTRODUCTION

Convectional heat transfer fluids, including oil, water, and ethylene glycol mixture are poor heat transfer fluids. But the thermal conductivity of these fluids play important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface [Muthtamilselvan *et al.* (2010)]. In view of the rising demands of modern technology, including chemical production, power station, and microelectronics, there is a need to develop new types of fluids that will be more effective in terms of heat exchange performance. Numerous methods have been tried to improve the thermal conductivity of these fluids by suspending nano/micro-sized particle materials in liquids. Recently several researchers including Tiwari – Das (2007), Ho *et al.* (2008), Abu-Nada (2008), Oztop – Abu-Nada (2008), Abu-Nada–Oztop (2009), Congedo *et al.* (2009), Aminossadati – Ghasemi (2009), Ghasemi – Aminossadati (2009, 2010), Ahmad – Pop (2010), etc. studied on the modeling of natural convection heat transfer in nanofluids. Hamad (2011) obtained the analytical solutions for convective flow and heat transfer of an viscous incompressible nanofluid past a semi-infinite vertical stretching sheet in the presence of magnetic field.

Heat and mass transfer in the laminar boundary layer flow over a stretching sheet is an important type of flow due to its application such as polymer engineering, metallurgy etc. Wang (1989) analyzed the free convection on a vertical stretching surface by employing Runge-Kutta Fehlberg algorithm. Elbashbeshy – Bazid (2004) considered the flow and heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection. Nazar *et al.* (2004) investigated the unsteady mixed convection boundary layer flow in the region of stagnation-point on a vertical surface in a fluid-saturated porous medium.

Common heat transport fluids such as water, ethylene glycol, end engine oil have limited heat transfer capabilities due to their low heat transfer properties. In contrast, metals have thermal conductivities up to three times higher than these fluids, so it is natural that it would be desired to combine the two substances to produce a heat transfer medium that behaves like a fluid, but has the thermal conductivity of a metal. A mixture of nano-size particle suspended in a base fluid is named nanofluid and was firstly reported by Choi (1995). The broad ranges of current and future applications involving nanofluids have been given by Wong – Leone (2010). Many researchers [Khanafer *et al.* (2003), Maiga *et al.* (2005), Jou – Tzeng (2006) and Hwang *et al.* (2007)] have studied and reported results on convective heat transfer in nanofluids considering various flow conditions in different geometries. A comprehensive study of convective transport in nanofluids was made by Buongiorno (2006). Kuznetsov – Nield (2010) presented a similarity solution of natural convective boundary-layer flow of a nanofluid past a vertical plate.

Steady boundary layer flow and heat transfer for different types of nanofluids near a vertical plate with heat generation effects was studied by Rana– Bhargava (2011)]. Chamkha – Aly (2011) presented a nonsimilar solution of boundarylayer flow of a nanofluid near a porous vertical plate with magnetic field and heat generation/absorption effects numerically. Recently Mukhopadhyay – Layek (2011) studied the boundary layer flow and heat transfer of a fluid through a porous medium towards a stretching sheet in presence of heat generation or absorption.

The intention of the present study is to analyze the effects of magnetic field, heat source, volume fraction and Prandtl number over Cu-Water nanofluid velocity, temperature, skin friction and Nusselt number, over a linearly stretching sheet.

FORMULATION AND SOLUTION OF PROBLEM

Consider a two dimensional forced convection boundary layer flow of an incompressible viscous nanofluid over a stretching sheet. The fluid flow is in the influence of ohmic heating, uniform transverse magnetic field of strength B_0 (applied parallel to the y-axis) and heat source. It is assumed that the induced magnetic field and the external electric field are negligible, the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. The flow is considered to be in x-direction which is chosen along the sheet and the y-axis normal to it. The sheet issues from a thin slit at the origin, it is assumed that the speed of a point on the plate is proportional its distance from the slit. Since the length of the sheet is supposed to be large and fluid flow extends to infinity, therefore all physical variables are independent of x and hence functions of y only. The general equations governing the nanofluid flow are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \ldots (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_{nf}}\left[\mu_{nf}\frac{\partial^2 u}{\partial y^2}-\sigma B_0^2 u\right] \qquad \ldots (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{1}{(\rho c_p)_{nf}}\left[\mu_{nf}\left(\frac{\partial u}{\partial y}\right)^2+Q\ T-T_\infty\ +\sigma B_0^2 u^2\right] \qquad \ldots (3)$$

$$u = u_w = cx,\ v = 0,\ T = T_w,\ at\ y = 0$$

$$u \to 0,\ T \to T_\infty \quad as\ y \to \infty \qquad \dots (4)$$

here $y,\ u,\ v,\ T,\ T_\infty,\ T_w,\ \sigma,\ Q,\ B_0$ and c_p are horizontal coordinate, axial velocity, transverse velocity, temperature of the fluid, far field temperature, wall temperature, electrical conductivity, heat sink coefficient, magnetic field coefficient and specific heat, here c is a constant.

The nanofluid properties such as the density ρ_{nf}, the dynamic viscosity μ_{nf}, thermal diffusivity α_{nf}, heat capacitance and the thermal conductivity k_{nf} are defined in terms of fluid and nanoparticles properties as in Aminossadati – Ghasemi (2009).

$$\rho_{nf} = 1-\phi\ \rho_f + \phi\rho_n,\quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},\quad \frac{k_{nf}}{k_f} = \frac{k_n + 2k_f - 2\phi(k_f - k_n)}{k_n + 2k_f + 2\phi(k_f - k_n)},$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_n,\quad \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

where, ρ_f is the density of fluid, ρ_n is the density of nanoparticles, ϕ is defined as the volume fraction of the nanoparticles, μ_f is the dynamic viscosity of fluid, $(\rho c_p)_f$ is the thermal capacitance of fluid, $(\rho c_p)_n$ is the thermal capacitance of nanoparticles, and k_f and k_n are the thermal conductivities of fluid and nanoparticles, respectively.

In order to solve eqs. (1) - (3), assuming the dimensionless parameters as follows:

$$u = cx\ f'(\eta),$$

$$v = -\sqrt{\vartheta c}\ f(\eta), \qquad \dots (5)$$

$$\eta = \sqrt{\frac{c}{\vartheta}}\ y$$

where f is a dimensionless stream function and η is the similarity variable.

On applying the similarity transformation parameters, nanofluid parameters and below mensioned non dimensional parameters, the eqs. (1) to (3) with the boundary conditions (4) changes to the following:

$$f''' + A_1 f f'' - A_1 f'^2 - A_2 f' = 0 \qquad \dots (6)$$

$$\theta'' + B_1 f\theta' + B_2 f''^2 + B_3 f'^2 + \beta\theta = 0 \qquad \dots (7)$$

$$f = 0,\ f' = 1,\ \theta = 1,\ at\ \eta = 0$$

$$f' \to 0,\ \theta \to 0,\ at\ \eta \to \infty \qquad \dots (8)$$

where, Prandtl number is $\Pr = \dfrac{\vartheta_f (\rho c_p)_f}{k_f}$, Eckert number is $Ec = \dfrac{c^2 x^2}{(c_p)_f (T_w - T_\infty)}$, magnetic field parameter is $M = \dfrac{\sigma B_0^2}{c \rho_f}$, injection parameter is $m = \dfrac{V_0}{\sqrt{\vartheta_f c}}$, dimensionless temperature $\theta = \dfrac{T - T_\infty}{T_w - T_\infty}$, and heat absorption parameter $\beta = \dfrac{\vartheta_f Q}{k_{nf} c}$

$$A_1 = \left[(1-\phi) + \phi \frac{\rho_n}{\rho_f}\right] 1-\phi^{\ 2.5}, \quad A_2 = \frac{1}{K} + 1-\phi^{\ 2.5} M, \quad B_1 = \left[(1-\phi) + \phi \frac{(\rho c_p)_n}{(\rho c_p)_f}\right]\left(\frac{k_f}{k_{nf}}\right)\Pr,$$

$$B_2 = \frac{Ec \Pr}{(1-\phi)^{2.5}}\left(\frac{k_f}{k_{nf}}\right), \; B_3 = M \Pr Ec \left(\frac{k_f}{k_{nf}}\right).$$

The skin friction can be presented as follows

$$C_f = \frac{2\mu_{nf}}{\rho_f u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \text{or} \quad \sqrt{\mathrm{Re}_x} C_f = \frac{2}{(1-\phi)^{2.5}} f''(0) \; . \qquad \ldots(9)$$

and the Nusselt number is written as

$$Nu = \frac{x k_{nf}}{k_f (T_w - T_\infty)}\left(-\frac{\partial T}{\partial y}\right)_{y=0} \quad \text{or} \quad \sqrt{\mathrm{Re}_x} Nu = -\frac{k_{nf}}{k_f}\theta'(0) \qquad \ldots(10)$$

where $\mathrm{Re}_x = \dfrac{u_w x}{\vartheta_f}$ is the local Reynolds number.

RESULTS AND DISCUSSION

The governing equations (7) and (8) with boundary conditions (9) are solved using MATLAB by perturbation analysis. Graphical results of velocity, temperature, skin friction and Nusselt number of Cu-Water nanofluid, are presented in figs. 2 to 10. Effects of variables like volume fraction, magnetic parameter, Prandtl number and heat source parameter are studied in detail.

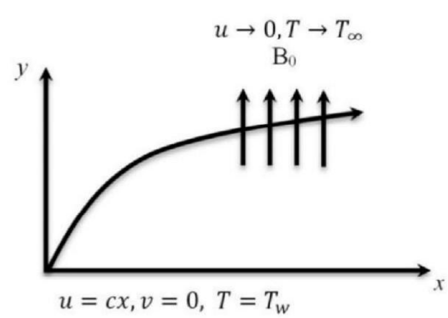

Fig. 1. Physical Model and Coordinate System

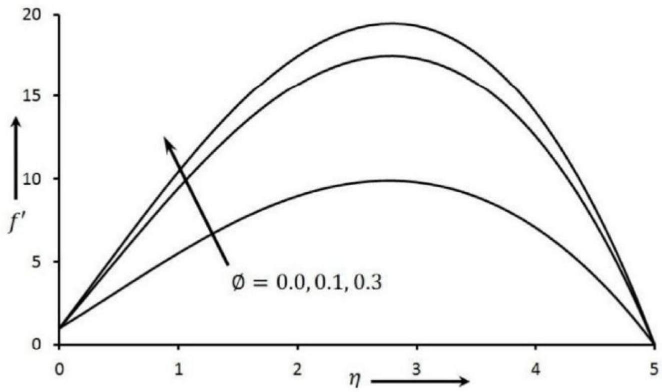

Fig. 2. Dimensionless velocity of the nanofluid (Cu-Water) verses η for different values of ϕ when $M = 0.2$, $\Pr = 1.0$, $Ec = 0.1$ **and** $\beta = 0.1$.

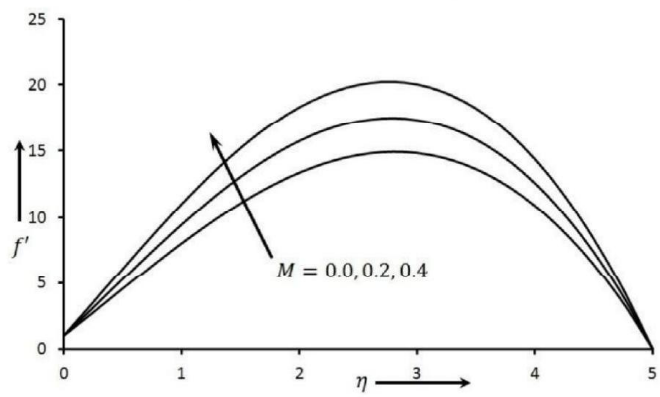

Fig. 3. Dimensionless velocity of the nanofluid (Cu-Water) verses η for different values of M when $\phi = 0.1$, $\Pr = 1.0$, $Ec = 0.1$ **and** $\beta = 0.1$.

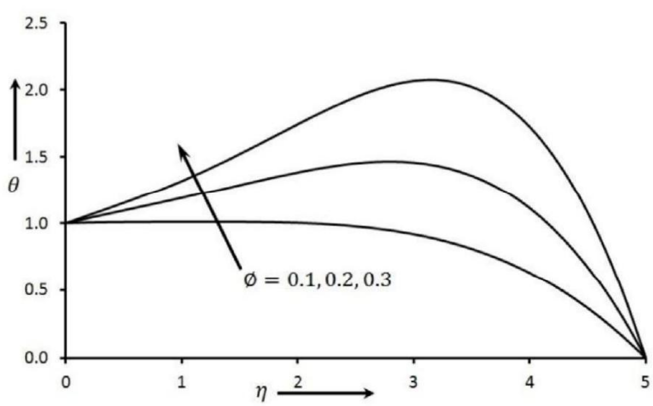

Fig. 4. Dimensionless temperature of the nanofluid (Cu-Water) verses η for different values of ϕ when $M = 0.2$, $\Pr = 1.0$, $Ec = 0.1$ **and** $\beta = 0.1$.

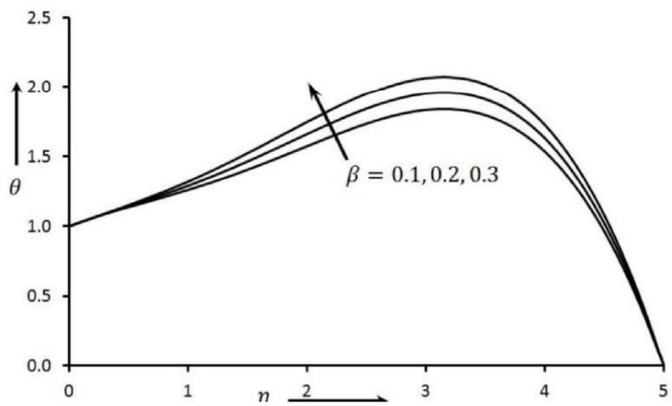

Fig. 5. Dimensionless temperature of the nanofluid (Cu-Water) verses η **for different values of** ϕ **when** $\phi = 0.1$, $M = 0.2$, $\Pr = 1.0$ **and** $Ec = 0.1$.

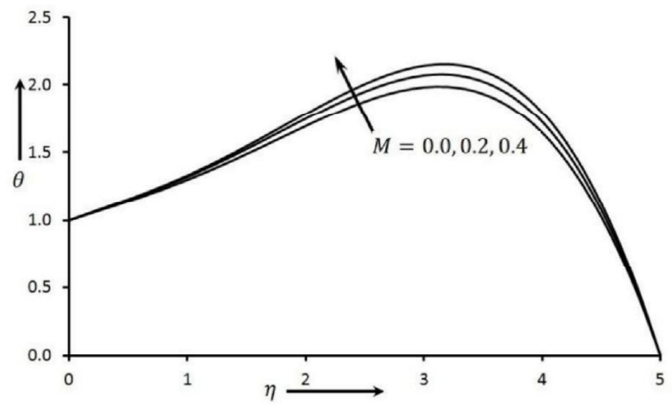

Fig. 6. Dimensionless temperature of the nanofluid (Cu-Water) verses η **for different values of** M **when** $\phi = 0.1$, $\Pr = 1.0$, $Ec = 0.1$ **and** $\beta = 0.1$.

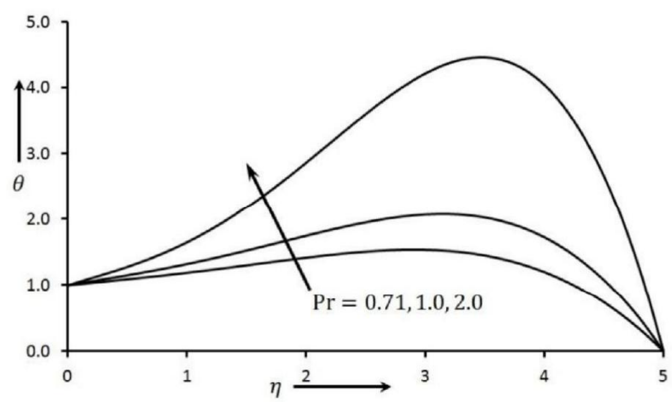

Fig. 7. Dimensionless temperature of the nanofluid (Cu-Water) verses η **for different values of Pr when** $\phi = 0.1$, $M = 0.2$, $Ec = 0.1$ **and** $\beta = 0.1$.

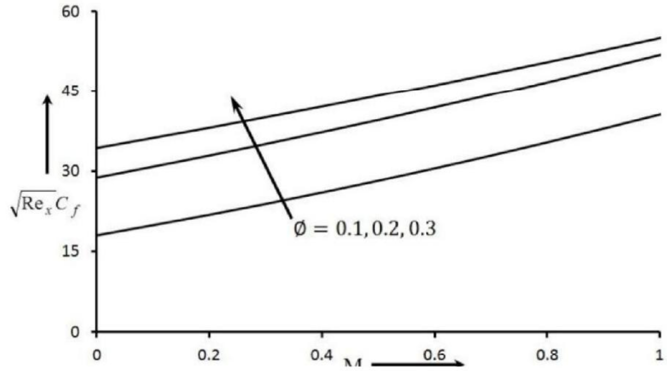

Fig. 8. Skin friction coefficient of the nanofluid (Cu-Water) verses M for different values of ϕ when $\mathrm{Pr} = 1.0$, $Ec = 0.1$ **and** $\beta = 0.1$.

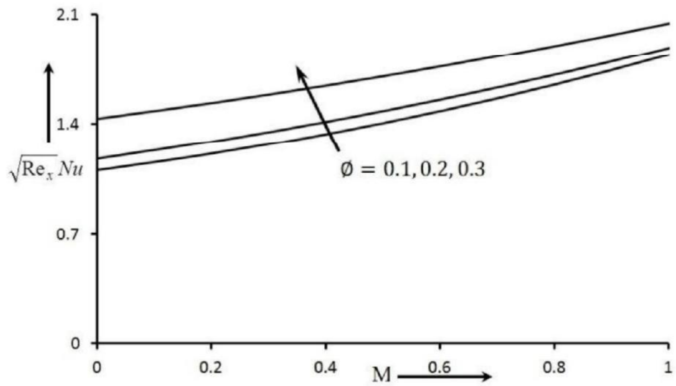

Fig. 9. Nusselt number of the nanofluid (Cu-Water) verses M for different values of ϕ when $\mathrm{Pr} = 1.0$, $Ec = 0.1$ **and** $\beta = 0.1$.

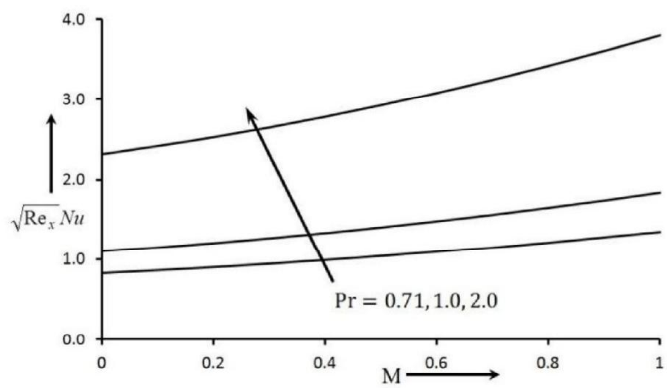

Fig. 10. Nusselt number of the nanofluid (Cu-Water) verses M for different values of Prwhen $\phi = 0.1$, $Ec = 0.1$ **and** $\beta = 0.1$.

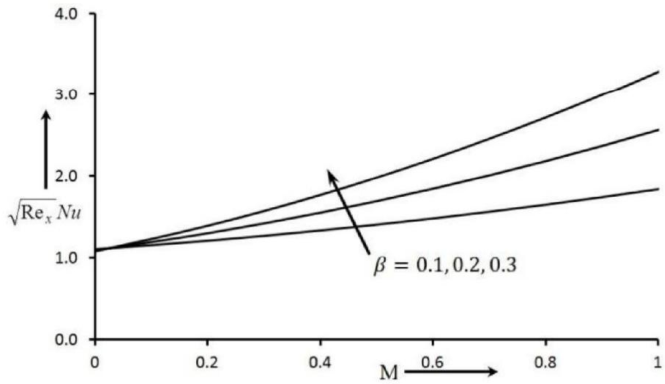

Fig. 11. Nusselt number of the nanofluid (Cu-Water) verses M for different values of β when $\phi = 0.1$, $Pr = 1.0$ **and** $Ec = 0.1$.

In fig. 2 and 3 the velocity against η (similarity variable) is drawn for different values of ϕ and M. It has been observed that when volume fraction of nanoparticle increases, the thickness of velocity boundary layer increases as higher values of volume fraction reduces the velocity profile width. An increment in magnetic parameter increases the velocity. This is because of the fact that magnetic parameter represents the ratio of magnetic induction to the viscous force. Hence, increase in the magnetic field parameter reduces the viscosity of the nanofluid, which results in thickening the velocity boundary layer.

The dimensionless temperature (θ) is drawn against η for different values of ϕ and β in figs. 4 and 5. It is noted that θ decreases as ϕ increases; whereas it increases with β. The presence of heat source in the boundary layer generates energy which causes the temperature of the fluid to increase.

The dimensionless temperature is plotted verses η for various values of M and Pr in figs. 6 and 7. It is observes here that θ increases with M or Pr. An increase in Pr increases the viscosity, which increases the temperature due to heat impinging on the surface, also it is interesting to note that at higher values of Prandtl number, the temperature attains a higher peak very sharply; and magnetic field increases the temperature of the fluid inside the boundary layer due to excess heating.

Fig. 8 shows the skin friction coefficient for different values of ϕ verses M. It is evident from the figure that skin friction increases with ϕ or M. Nusselt number (Nu) is drawn against M in figs. 9-11 for different values of ϕ, Pr and β respectively. Nusselt number increases with ϕ or Pr or β or M.

CONCLUSIONS

This problem deals with the flow of an incompressible viscous nanofluid over a stretching sheet. The governing equations are converted to similarity equations, and then the obtained coupled equations are solved by using perturbation analysis and MATLAB. It is observed that inclusion of volume fraction or magnetic field increases the velocity. Increment in Prandtl number or magnetic field extends the thermal boundary layer. Higher volume fraction results in thinner thermal boundary layer; whereas this phenomenon reverses for heat source.

NOMENCLATURE

x axial coordinate (m)	ρ_f density of fluid particles (kg/m^3)
y horizontal coordinate (m)	ρ_n density of nanoparticles (kg/m^3)
u axial velocity (m/s)	μ_f dynamic viscosity of fluid (Pa.s)
v transverse velocity (m/s)	k_n thermal conductivity of nanoparticles (W/m K)
p pressure (Pa)	k_f thermal conductivity of fluid particles (W/m K)
T temperature of the fluid (K)	$(\rho c_p)_f$ thermal capacitance of fluid particles (J/ m^{3}K)
T_∞ far field temperature (K)	$(\rho c_p)_n$ thermal capacitance of nano particles (J/ m^{3}K)
T_w wall field temperature (K)	ϕ volume fraction of nanoparticles
σ electrical conductivity (S/m)	f dimensionless stream function
Q heat generation coefficient (W m^{-3} K^{-1})	Pr Prandtl number
B_0 magnetic field coefficient (T)	Ec Eckert number
ϑ kinematic viscosity (m^2/s)	M Magnetic field parameter
c_p specific heat (J kg^{-1} K^{-1})	m injection parameter
ρ_{nf} density of nanofluid (kg/m^3)	θ dimensionless temperature
μ_{nf} dynamic viscosity of nanofluid (Pa.s)	β heat absorption parameter
α_{nf} thermal diffusivity of nanofluid (m^2/s)	f dimensionless stream function
$(\rho c_p)_{nf}$ thermal capacitance of nanofluid (J/ m^{3}K)	
k_{nf} thermal conductivity of nanofluid (W/m K)	

ACKNOWLEDGMENT

The author is very much thankful to Prof. (Ret.) Dr. S. S. Tak, Jai Narain Vyas University, Jodhpur (India) for offering his valuable suggestions and assistance to improve the paper.

REFERENCES

1. Muthtamilselvan, M., Kandaswamy, P. and Lee, J., *Communications in Nonlinear Science and Numerical Simulation,* **15**, 1501 (2010).
2. Tiwari, R. K. and Das, M. K., *International Journal of Heat and Mass Transfer*, **50**, 2002 (2007).
3. Ho, C. J., Chen, M. W. and Li, Z. W., *International Journal in Heat Mass Transfer*, **51**, 4506 (2008).
4. Abu-Nada, E., *International Journal of Heat Fluid Flow*, **29**, 242 (2008).
5. Oztop, H. F. and Abu-Nada, E., *International Journal of Heat and Fluid Flow*, **29**, 1326 (2008).

6. Abu-Nada, E. and Oztop, H. F., *International Journal of Heat Fluid Flow*, **30**, 669 (2009).
7. Congedo, P. M., Collura, S. and Congedo, P. M., *Proceedings of ASME Summer Heat Transfer Conference*, **3**, 569 (2009).
8. Aminossadati, S. M. and Ghasemi, B., *European Journal of Mechanics - B/Fluids*, **28**, 630 (2009).
9. Ghasemi, B. and Aminossadati, S. M., *Numerical Heat Transfer Part A: Applications*, **55**, 807 (2009).
10. Ghasemi, B. and Aminossadati, S. M., *International Journal of Thermal Sciences*, **49**, 1 (2010).
11. Ahmad, S. and Pop, I., *International Communications in Heat and Mass Transfer*, **37**, 987 (2010).
12. Hamad, M. A. A., *International Communication in Heat and Mass Transfer*, **38**, 487 (2011).
13. Wang, C. Y., *Journal of Applied Mathematics and Mechanics ZAMM*, **69**, 418 (1989).
14. Elbashbeshy, E. M. A. and Bazid, M. A. A., *Applied Mathematics and Computation*, **158**, 799 (2004).
15. Nazar, R., Amin, N., Filip, D. and Pop, I., *International Journal of Engineering Science,* **42**, 1241 (2004).
16. Choi, S. U. S., *The proceedings of 1995 ASME International Mechanical Engineering Congress and Exposition, San Francsco, USA, ASME,* **FED 231/MD**, 99 (1995).
17. Wong, K. F. V. and Leone, O. D., *Adv. Mech. Eng. Article*, **ID : 519659** (2010).
18. Khanafer, K., Vafai, K. and Lightstone, M., *J. Heat Mass Transfer*, **46**, 3639 (2003).
19. Maiga, S. E. B., Palm, S. J., Nguyen, C. T., Roy, G. and Galanis, N., *Int. J. Heat Fluid Flow*, **26**, 530 (2005).
20. Jou, R. Y. and Tzeng, S. C., *Int. Commun. Heat Transfer*, **33**, 727 (2006).
21. Hwang, K. S., Lee, Ji-H. and Jang, S. P., *Int. J. Heat Mass Transfer*, **50**, 4003 (2007).
22. Buongiorno, J.,, *ASME J. Heat Transfer*, **128**, 240 (2006).
23. Kuznetsov, A. V. and Nield, D. A., *Int. J. Therm. Sci.*, **49**, 243 (2010).
24. Rana, P. and Bhargava, R., *Communications in Nonlinear Science and Numerical Simulation*, **16**, 4318 (2011).
25. Chamkha, A. J. and Aly, A. M., *Chemical Engineering Communications*, **198**, 425 (2011).
26. Mukhopadhyay, S. and Layek, G. C., *Meccanica*, **47**, 863 (2011).

❑