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ON THE DIOPHANTINE EQUATION $4^X + 3^Y = Z^2$

DUNDI SARATH SEN REDDY

Asstt. Prof., Deptt. of Maths, Anurag College of Engineering, Aushapur, Ghatkesar, Medchal dist, Telangana

AND

DUNDI GOPALA NAIDU REDDY

Assistant Professor, Department of Mathematics, PVKR Junior College, Karlapalem, Guntur (dist), (A. P.), India

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In this paper we research for unique non negative integer solutions of the Diophantine equation $4^x + 3^y = z^2$ the solution (x, y, z) are (0,1,2) and (2, 2, 5).

KEYWORDS : Exponential Diophantine Equation, integer solution.

Mathematical Classification: AMS Mathematical subject classification (2010) : 11D61.

INTRODUCTION

A Linear Diophantine equation is an equation that sums two monomials of degree zero or one. An exponential Diophantine equation is one in which exponents on terms can be unknowns. A general theory of exponential Diophantine equations is not available. Majority of the equations are solved via adhoc methods such as Catalan's conjecture or even trail and error method.

For various problems and ideas one may refer [1] and [2], [3] to [10] has been studied for various methods of solving exponential Diophantine equations.

In this paper we research for unique non negative integer solutions of the Diophantine equation $4^x + 3^y = z^2$.

PRELIMINARIES

Catalan's conjecture: The Diophantine equation $a^x + b^y = 1$ has unique integer solution with min $\{a, b, x, y\} > 1$.The solution (a, b, x, y) is (3, 2, 2, 3).This was proved by Mihailescu in 2004.

METHOD OF ANALYSIS

In this section we prove that $4^x + 3^y = z^2$ has non negative integer solution. The solution $(x, y, z)$is (0, 1, 2) and (2, 2, 5). The second set of solution implies that (4, 2, 5) is the solution

(x, y, z) of the exponential Diophantine equation $2^x + 3^y = z^2$ where x, y and z are non-negative integers.

Theorem : (0, 1, 2) and (2, 2, 5) are solutions of (x, y, z) of the Diophantine equation $4^x + 3^y = z^2$ where x, y and z are non-negative integers.

Proof : We will divide the proof into three cases on y

Case (i) : Take $x \neq 0$

Suppose $y = 0$

Then $4^x + 1 = Z^2$

$4^x = Z^2 - 1$

$2^{2x} = (z - 1)(z + 1)$

Let $z - 1 = 2^u$ ... (1)

where u is a non negative integer

Implies $2^{2x} = (z + 1)\, 2^u$

$z + 1 = 2^u (2^{2x-u} - 1)$ ... (2)

(2) – (1) implies $2^u (2^{2x-u} - 1) = 2$

Implies $u = 0,\ 2^{2x} - 1 = 2$

This is impossible for positive values of x

Therefore $y \neq 0$

Case (ii) : $x = 0$

Write $4^x + 3^y = z^2$ as $2^{2x} + 3^y = z^2$

Implies $3^y = z^2 - 2^{2x}$

$3^y = (z - 2^x)(z + 2^x)$

Let $z - 2^x = 3^u$ ... (3)

where u is a non-negative integer

$Z + 2^x = 3^{y-u}$ ... (4)

(4) – (3) implies $2\,2^x = 3^u (3^{y-2u} - 1)$ ... (5)

If $x = 0$ then $u = 0$, we get

$2\,2^x = 3^y - 1,$

$y = 1$

Hence $(x, y, z) = (0, 1, 2)$

Case (iii) : $x \geq 2$

Proceeding as in case (ii) from (5) we have

$2\,2^x = 3^u (3^{y-2u} - 1)$

If $x = 2$ then $u = 0$ implies $y = 2$

In this case $z = 5$

Hence $(x, y, z) = (2, 2, 5)$

Corollary: (4, 2, 5) is a solution of (x, y, z) of the exponential Diophantine equation $2^x + 3^y = z^2$ where x, y, z are non-negative integers.

Proof: By theorem we have

$$4^2 + 3^2 = 5^2$$
$$2^4 + 3^2 = 5^2$$

Therefore (4, 2, 5) is a solution of the Diophantine equation $2^x + 3^y = z^2$ where x, y and z are non-negative integers.

CONCLUSION

In this paper we have found (0, 1, 2) and (2, 2, 5) are exact solutions of $4^x + 3^y = z^2$ in non-negative integers .One may also search for other set of solution for similar type of equations.

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