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NON-INVARIANT HYPERSURFACE OF HGF-STRUCTURE METRIC MANIFOLD

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Hypersurface of a manifold is a Submanifold of dimension one less then ambient manifold. The HGF Structure Metric manifold is defined and studied by Pandey[2] and others. In the present paper we define and study Non-Invariant Hypersurface of HGF structure metric manifold and give some fruitfull results.

INTRODUCTION

Let us consider M be a differentiable manifold of differentiability class C^{∞}. Let there exist a Vector valued linear function J of class C^{∞} satisfying the algebraic structure

$$J^2 = -a^2 I_n \qquad \dots (1.1)$$

where 'a' is a complex number. Then (M, J) is said to be a hyperbolic differentiable structure briefly known as HGF – structure defined by (1.1) and the manifold M is called HGF – manifold.

For different values of 'a' the equation (1.1) gives different algebraic structures such as:

(i) If $a \neq 0$ it is a hyperbolic π-Structure.

(ii) If $a = \pm 1$ it is an almost complex or an almost hyperbolic product structure.

(iii) If $a = \pm i$ it is an almost product or an almost hyperbolic complex structure.

(iv) If $a = o$ it is an almost tangent or a hyperbolic almost tangent structure.

Let the HGF structure be endowed with a hermite tensor g such that

$$g(\bar{X}, \bar{Y}) = a^2 g(X, Y)$$

or

$$g(JX, JY) = a^2 g(X, Y) \qquad \dots (1.2)$$

Then $\{J, g\}$ is said to give to M a hyperbolic differentiable metric structure and the manifold M is called a hyperbolic differentiable metric structure manifold.

In a hyperbolic metric structure if

$$(D_X J)Y = 0 \qquad \dots (1.3)$$

is satisfied, then M is said to be a hyperbolic kahler manifold.

and if

$$(D_X J)Y + (D_Y J)X = 0 \qquad \dots (1.4)$$

is satisfied then M is said to be hyperbolic nearly kahler manifold.

A Hypersurface of HGF metric manifold M is called a Non-Invariant hypersurface, if the transform of a tangent vector of the hypersurface under the action of (1, 1) tensor field defining the HGF structure is never tangent to the hypersurface .

Consider HGF manifold M and let $\tilde{M}$ be the hypersurface of M and let B be the differential of the immersion i of $\tilde{M}$ into M. Let X, Y, Z be tangent to $\tilde{M}$ and N a unit normal vector then we have,

$$JBX = BFX + u(X)N \qquad \dots (1.5)$$

where F is (1, 1) tensor field and u is a 1- form on $\tilde{M}$. If $u \neq 0$ then $\tilde{M}$ is called non-invariant hypersurface of M. If u is identically zero, then $\tilde{M}$ is said to be an invariant hypersurface that is, the tangent space of $\tilde{M}$ is invariant by J.

The metric g of an HGF metric manifold induces a Riemannian metric G on the hypersurface $\tilde{M}$ and is given by

$$G(X, Y) = g(BX, BY) \qquad \dots (1.6)$$

The Gauss and Weingarten formulae are given by

$$D_{BX}BY = B\ \bar{D}_X Y\ + h\ \ X, Y\ \ N \qquad \dots (1.7)$$

$$D_{BX}N = -BHX \qquad \dots (1.8)$$

For all $X, Y \in T\tilde{M}$, where D and $\bar{D}$ are the Riemannian and induced Riemannian connections on M and $\tilde{M}$ respectively. Here h is symmetric tensor of type (0, 2) called the second fundamental form of the hypersurface $\tilde{M}$ and H is a (1,1) tensor field on $\tilde{M}$ such that

$$h(X, Y) = g(HX, Y) \qquad \dots (1.9)$$

Let $$JN = -BU \qquad \dots (1.10)$$

where U is a vector field in $\tilde{M}$.

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Theorem 2.1. If $\tilde{M}$ is a non-invariant hypersurface of HGF Structure Metric Manifold then we get following relation *i.e.* we get (F, u, U, a^2) structure

$$F^2 = -a^2 I + (u \otimes U) \qquad \dots (2.1)$$

$$u \, o \, F = 0 \qquad \dots (2.2)$$

$$u(U) = -a^2 \qquad \dots (2.3)$$

$$F(U) = 0 \qquad \dots (2.4)$$

Proof : Operating equation (1.5) by J

$$J^2 BX = J(BFX) + u(X) JN$$

Now using equation (1.1) and (1.5) in above equation

$$-a^2 BX = BF^2 X + u(FX)N + u(X)\,JN$$

Using equation (1.10) in above equation and then equating tangential and normal parts, we get

$$-a^2 BX = BF^2 X - u(X) BU$$

and $$u\,(FX)N = 0$$

or $$F^2 = -a^2 I + (u \otimes U)$$

and $$u \, o \, F = 0$$

Operating equation (1.10) by J and using (1.1), we get

$$-a^2 N = -BFU + u\,(U)N$$

Equating tangential and normal part, we get

$$FU = 0 \text{ and } u(U) = -a^2$$

Theorem 2.2. The metric g of HGF metric manifold induces a Riemannian metric G on the hypersurface $\tilde{M}$ and is given by $G(X, Y) = g(BX, BY)$. Therefore in case of HGF metric manifold

$$G(FX, FY) = a^2 G(X, Y) - u(X)\,u(Y) \qquad \ldots (2.5)$$

Proof : From equation (1.2) we have ,

$$g(JBX, JBY) = a^2 g(BX, BY)$$

Using equation (1.5) in above equation

$$g(BFX + u\,(X)N, BFY + u(Y)N) = a^2 g(BX, BY)$$

$$g(BFX, BFY) + u(Y)g(BFX, N) + u(X)g(N, BFY) + u(X)\,u(Y)\,g(N, N) = a^2 g(BX, BY)$$

$$g(BFX, BFY) + u(X)\,u(Y) = a^2 g(BX, BY)$$

$$G(FX, FY) + u(X)\,u(Y) = a^2 G(X, Y)$$

$$G(FX, FY) = a^2 G(X, Y) - u(X)\,u(Y)$$

This completes the proof.

Theorem 2.3. In a Kahler HGF metric manifold, that is, HGF metric structure in which tensor J is parallel *i.e.* $(D_X J)\,Y = 0$, we have,

$$(\bar{D}_X F)Y = u(Y)HX - h\,(X, Y)\,U \qquad \ldots (2.6)$$

$$(\bar{D}_X u)Y = -h\,(X, FY) \qquad \ldots (2.7)$$

Proof : We have $(D_X J)\,Y = 0$

Therefore $(D_{BX} J)\,BY = 0$

$$D_{BX}J(BY) - J(D_{BX}BY) = 0$$

Using equation (1.5), we get

$$D_{BX}(BFY + u(Y)N) - J(D_{BX}BY) = 0$$

$$D_{BX}BFY + D_{BX}u(Y)N - J(D_{BX}BY) = 0$$

$$D_{BX}BFY + (D_{BX}u(Y))\,N + u(Y)D_{BX}N - J(D_{BX}BY) = 0$$

Using equation (1.7) and equation (1.8) in the above equation ,we get

$$B(\bar{D}_X FY) + h(X, FY)N + (D_{BX}u(Y))N - u(Y)BHX - J(B\bar{D}_X Y + h(X,Y)N) = 0$$

$$B(\bar{D}_X FY) + h(X, FY)N + (D_{BX}u(Y))N - u(Y)BHX - JB(\bar{D}_X Y) - h(X, Y)JN = 0$$

$$B(\bar{D}_X FY) + h(X, FY)N + (D_{BX}u(Y))N - u(Y)BHX - BF(\bar{D}_X Y)$$

$$-u(\bar{D}_X Y)N + h(X,Y)BU = 0 \qquad \ldots(2.7)$$

Tangential part from above equation (2.7) is given by

$$B(\bar{D}_X FY) - u(Y)\,BHX - BF(\bar{D}_X Y) + h(X, Y)BU = 0$$

or $$B(\bar{D}_X F)Y = u(Y)\,BHX - h(X, Y)\,BU$$

or $$(\bar{D}_X F)Y = u(Y)HX - h(X, Y)\,U$$

Normal part from equation (2.7) is given by

$$h(X, FY)N + (D_{BX}u\,(Y))\,N - u\,(\bar{D}_X Y)N = 0$$

or $$D_{BX}u(Y) - u(\bar{D}_X Y) = -h(X, FY)$$

or $$(\bar{D}_X u)Y = -h(X, FY)$$

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